Efficient Computations for Evaluating Extended Stochastic Petri Nets using Algebraic Operations

Dong-Sung Kim, Hong-Ju Moon, Je-Hyeong Bahk, Wook Hyun Kwon, and Zygmunt J. Haas

Abstract: This paper presents an efficient method to evaluate the performance of an extended stochastic Petri net by simple algebraic operations. The reachability graph is derived from an extended stochastic Petri net, and then converted to a timed stochastic state machine, using a semi-Markov process. The n-th moments of the performance index are derived by algebraic manipulations with each of the n-th moments of transition time and transition probability. For the derivation, three reduction rules are introduced on the transition trajectories in a well-formed regular expression. Efficient computation algorithms are provided to automate the suggested method. The presented method provides a proficient means to derive both the numerical and the symbolic solutions for the performance of an extended stochastic Petri net by simple algebraic manipulations.

Keywords: Performance evaluation, computation, n-th moment, extended stochastic Petri net.

1. INTRODUCTION

For the proper performance evaluation of some systems, it is necessary to introduce time delays associated with transitions and/or places in Petri net models. This could be done, for example, through the use of stochastic Petri nets (SPN). A SPN is a Petri net in which each transition is associated with an exponentially distributed random variable that expresses the delay from the enabling condition to the firing of the transition [1]. SPNs are extended to a class of generalized stochastic Petri nets (GSPNs) by allowing immediate transitions [1]. An extended stochastic Petri net (ESPN) is a useful extension of a GSPN, which allows generally distributed transition delays for non-concurrent transitions [1, 2].

The Markovian analysis method based on the construction of the reachability graph is a standard analysis tool for the performance evaluation of Petri net models [3, 4]. Unfortunately, the application of this technique is severely limited by the complexity of the solution of the Markov model. For example, the method in [5] based on the Markov theory requires n^3 multiplications, n^2 differentiations, and many other computations to derive the mean first passage time of a semi-Markov process with n states.

The use of moment generating function (MGF) has been studied [6] as a performance evaluation method for the ESPN model. This method converts an ESPN model to a state machine Petri net model and derives the transfer functions for the passages that represent the desired performance measure. The MGF method provides a transient solution as well as a steady-state solution that is both systematic and useful for the derivations of numerous performance measures related to passages. However, the computations of the MGF are rather complex, requiring many Laplace transformations and differentiations of complex equations, as well as the application of Mason's rule to a complex graph to derive the transfer function [6]. Even though [7] has automated the Mason's rule for the SPN, the computation burden of Laplace transformations and differentiations still remains.

To generalize stochastic Deterministic Petri Nets (DSPNs) with the generally distributed times, Markov Regenerative Stochastic Petri Nets (MRSPNs) were introduced by H. Choi. However [8], H. Choi includes the restriction that, at most, only one generalized distributed time event can be enabled in each marking. Under this restricted assumption, the process subordinated in the two regeneration time points is a continuous-time Markov chain, and therefore the subordinated

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process is analyzed separately with the embedded process of the regeneration time events.

Recently, Telek et al. [9] provided the steady state analysis of MRSPNs with age memory policy applying Laplace Stieltjes transform, and Lindemann [10] suggested a Numerical Analysis Method for the steady state analysis of the concurrent deterministic cases. The transient analysis of the concurrent deterministic cases was solved after that by Lindemann in [11]. Puliafito et al. [12] provided a solution for the concurrent generally distributed cases, remedied with a restriction that the k transitions of general distributions are enabled simultaneously at an instant of time [12].

In most situations, a steady-state solution is enough for the performance evaluation of a Petri net model. Moreover, in many cases, the mean and the variance of a desired performance measure are sufficient. The mean and the variance of a passage of time can be obtained simply by the computations using the first and the second moments of time delays of the transitions that compose the passage. With the above motivation in mind, this paper, which is based on [13], proposes a simple and systematic method for the performance evaluation of an ESPN.

The proposed method applies the automata theory [14] and the probability theory to the Petri net theory. The method converts an ESPN model to a timed stochastic state machine with semi-Markovian properties (SMSMP) in a way similar to the MGF method in [6]. However, unlike [6] in which a transfer function is derived, in our scheme, a set of transition trajectories is derived to specify a performance measure, and the performance is then computed from the trajectory set represented by a well-formed regular expression. The n-th moment of the performance index is evaluated from the trajectory set by recursively applying three algebraic conversion rules to the regular expression. Automatic algorithms are also provided to evaluate the performance measures directly from a given SMPSM without deriving trajectories. The proposed method provides both a symbolic solution, as well as a numerical solution.

2. PERFORMANCE MODEL

An ESPN is a 7-tuple (P,T,I,O,H,m,F) [2, 6], where: $P=\{p_1,p_2,\cdots,p_n\}$ $(n>0)$ is a finite set of places; $T=\{t_1,t_2,\cdots,t_n\}$ $(s>0)$ is a finite set of transitions with $P\cup T\neq\varnothing$ and $P\cap T=\varnothing$; I: $P\times T\rightarrow N$ is an input function where $N=\{0,1,2,\cdots\}$; O: $P\times T\rightarrow N$ is an output function; H: $P\times T\rightarrow N$ is an inhibitor function; m: $P\rightarrow N$ is a marking whose ith component is the number of tokens in the P_i place and the initial marking is denoted by m_0; and $F: T\rightarrow R$, is a vector of firing time delays, specified as an extended distribution function.

The firing time delays with extended distribution functions are defined such that generalized distribution functions are allowed for non-concurrent transitions and only exponential distribution functions are permitted for concurrent transitions [6]. Two transitions are said to be concurrent at marking m if and only if firing one does not disable the other.

A transition $t\in T$ is enabled if and only if $m(p)\geq I(p,t)$ for all $p\in P$ and $m(p)<H(p,t)$ for every $p\in P$ s.t. $H(p,t)\neq 0$. An enabled transition t may fire at a marking m yielding the new marking m' according to the following equation:

$$m'(p)=m(p)+O(p,t)-I(p,t).$$

In a graphical representation, places are drawn as circles, untimed transitions are depicted using thin bars, and timed transitions are drawn as thick bars. There are $I(p,t)$ directed arcs from a place p to a transition t, $O(p,t)$ directed arcs from t to p, and $H(p,t)$ directed arcs from p to t with a small circle rather than an arrowhead. k parallel arcs can be drawn as an arc labeled with k.

Fig. 1 shows an example of an ESPN that corresponds to the machine-repairman model with a buffer. For a more detailed description of the problem, the reader is referred to [6]. In the ESPN model PN_R, the firing delays for the transition t_1 have a normal distribution $N(\alpha,\sigma^2)$. t_2 and t_5 have exponentially distributed firing delays with rates $n\lambda$ and μ, respectively, where n is the number of the tokens in the place p_2. Transitions t_3 and t_4 are immediate. Since transitions t_2 and t_5 are concurrent, their firing delays have exponential distributions, whereas the transition t_1 is not concurrent, and thus its firing delay may have any distribution, in this case normal distribution.

Now, we define the timed stochastic state machine model, which is our intermediate model for the performance evaluation. An ESPN model can be easily converted to a timed stochastic state machine model

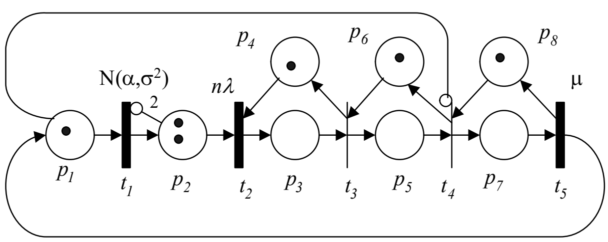

Fig. 1. ESPN PN_R for the three machine-repairman model with a buffer.

with help of the ESPN model the reachability graph.

A timed stochastic state machine is defined to be a 5-tuple $SM = (Q, E, T, P, S)$, where Q is the finite set of states, E is the finite set of events, $T \subset Q \times E \times Q$ is the transition relation, $P : T \rightarrow$ [0, 1] is the transition probability function, and $S : T \rightarrow$ *cumulative distribution function* is the stochastic transition time function. The timed stochastic state machine defined above is a semi-Markov process, and will be referred to as a SMPSM (state machine with semi-Markovian properties) throughout this paper. $P(q_1, \sigma, q_2)$ represents the transition probability with which the transition from the state q_1 to the state q_2 occurs by an event σ. $E(q)$ denotes the set of events that can occur in the state q; for each state $q \in Q$, $\sum_{\sigma \in E(q)} \sum_{q'} P(q, \sigma, q') = 1$. For consistency, when there is no event that can occur in a state q, we set $E(q) := \{\varepsilon\}$ and $P(q, \varepsilon, q) := 1$. For a transition t, $S(t)$ denotes the cumulative distribution function of the transition time of t when t is enabled.

The set of trajectories of transitions, T^* is defined as $T^* := \bigcup_{i=0}^{\infty} T^i$, where the element of T^0 can be thought of as a transition with the null event and is called the *null trajectory*. A null trajectory is denoted by η. $T_F(q_1, q_2)$ denotes the set of all the possible trajectories that begin from $q_1 (\in Q)$ and end at $q_2 (\in Q)$, visiting q_2 for the first time.

A SMPSM corresponds to a state machine PN [6], and an ESPN model can be converted to a SMPSM model, using the conversion procedure of an ESPN model to a state machine PN in [6]. More specifically, from an ESPN model, the reachability graph can be obtained. Then, by removing the vanishing markings, a SMPSM model for the ESPN model is achieved. The set of the tangible markings in the reachability graph is the set of the states in the SMPSM model, and the transition relation between the tangible markings in the reachability graph is the transition relation of the SMPSM model.

Consider a marking m of an ESPN, a set of enabled transitions $\{t_1, t_2, \cdots, t_d\}$ $(d \geq 1)$, and the random firing time delay X_j for a transition t_j $(1 \leq j \leq d)$. Let t_i' be the transition in the SMPSM model that corresponds to the transition t_i of the ESPN model. The transition probability $P(t_i')$ is as follows:

$$P(t_i') = P\{X_i \leq X_j, j \neq i\}, \\ = \int_0^{\infty} f_i(\tau) \prod_{1 \leq j \leq d, j \neq i} (1 - F_j(\tau)) d\tau, \quad (1)$$

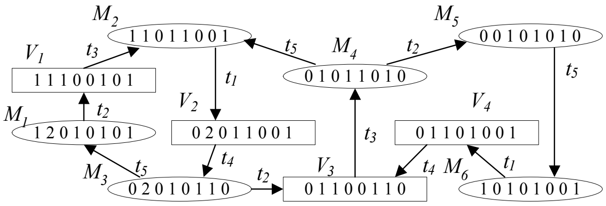

Fig. 2. Reachability graph for the ESPN PN_R.

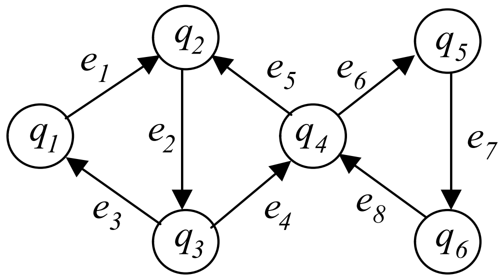

Fig. 3. SMPSM model M_R for the ESPN model PN_R.

where $f_j(\tau)$ is the probability density function and $F_j(\tau)$ is the cumulative distribution function of the firing time delay of the transition t_j. The cumu-lative distribution function $S(t_i')(\tau)$ for the transition t_i' is as follows:

$$S(t_i')(\tau) = P\{\min_{1 \leq j \leq d} (X_j) \leq \tau\} \\ = 1 - \prod_{1 \leq j \leq d} (1 - F_j(\tau)). \quad (2)$$

The reachability graph of the ESPN model in Fig. 1 is shown in Fig. 2, with the corresponding SMPSM model shown in Fig. 3. In the reachability graph, the tangible markings M_1, M_2, M_3, M_4, and M_5 are represented by ellipses and the vanishing markings V_1, V_2, V_3, and V_4 are represented by boxes. After removing the vanishing markings from the reachability graph for an ESPN model, the SMPSM model for the ESPN model is obtained, which for our example is shown in Fig. 3. The state q_i in the SMPSM M_R corresponds to the marking state M_i in the reachability graph.

In particular, the state q_1 in M_R corresponds to the marking M_1 in the reachability graph. Since the only transition t_2 is enabled solely at the marking M_1, $P(e_1) = 1$ and $S(e_1) = 1 - e^{-2\lambda t}$. The transition probabilities and the transition time distributions for e_2, e_7, and e_8 are obtained as follows: $P(e_2) = 1$, $S(e_2) = N(\alpha, \sigma^2)$, $P(e_7) = 1, S(e_7) = 1 - e^{-\mu t}$, and $P(e_8) = 1, S(e_8) = N(\alpha, \sigma^2)$.

The state q_3 corresponds to the marking M_3, and the transitions t_2 and t_5 are enabled simultaneously at the marking. Therefore, by applying (1) and (2), we obtain the transition probabilities and the transition time distributions for e_3 and e_4 as follows:

$P(e_3)=\frac{\mu}{2\lambda+\mu}$, $S(e_3)=1-e^{-(2\lambda+\mu)t}$, $P(e_4)=\frac{2\lambda}{2\lambda+\mu}$, and $S(e_4)=1-e^{-(2\lambda+\mu)t}$. Similarly, we can obtain $P(e_5)=\frac{\mu}{\lambda+\mu}$, $S(e_5)=1-e^{-(\lambda+\mu)t}$, $P(e_6)=\frac{\lambda}{\lambda+\mu}$, and $S(e_6)=1-e^{-(\lambda+\mu)t}$.

3. COMPUTATION OF THE PERFORMANCE MEASURE AND REDUCTION RULES

$p(t)$ denotes the probability of the transition $t\in T$ in a SMPSM and $|t|_n$ $(n\geq 1)$ denotes the n-th moment of the transition time of t. The definitions for $P(\cdot)$ and $|\cdot|_n$ are easily extended over to T^*.

For a transition trajectory $u=t_1t_2\cdots t_l$ $(t_l\in T$ for $1\leq i\leq l)$, $P(t_1t_2\cdots t_l)=P(t_1)P(t_2)\cdots P(t_l)$. Let x_i denote the transition time of a transition t_i. Then,

$$|t_1t_2\cdots t_l|_n=\int_{x_1}\cdots(x_1+x_2+\cdots+x_l)^n f(x_1,x_2,\cdots,x_l)dx_l\cdots dx_2 x_1,$$

where $f(x_1,x_2,\cdots,x_l)$ is the joint probability density function of x_i's $(1\leq i\leq l)$. From the definition, $|u|_0=1$ for $u\in T^*$. For consistency, $P(\eta):=1$, $|\eta|_n:=0$ $(n\geq 1)$, and $|\eta|_0:=1$. It is known that the following equation holds:

$$(x_1+x_2+\cdots+x_l)^n=\sum_{\substack{i_1+i_2+\cdots+i_k=n\\ i_1,i_2,\cdots,i_k\geq 0}}\frac{n!}{i_1!i_2!\cdots i_k!}x_1^{i_1}x_2^{i_2}\cdots x_k^{i_k}.$$

With the above equation and our definitions, we conclude that

$$P(u_1u_2\cdots u_k)=P(u_1)P(u_2)\cdots P(u_k) \tag{3}$$

and

$$|u_1u_2\cdots u_k|_n=\sum_{\substack{i_1+i_2+\cdots+i_k=n\\ i_1,i_2,\cdots,i_k\geq 0}}\frac{n!}{i_1!i_2!\cdots i_k!}|u_1|_{i_1}|u_2|_{i_2}\cdots|u_k|_{i_k} \tag{4}$$

$(n\geq 0)$ for $u_1,u_2,\cdots,u_k\in T^*$.

Now, consider a sample trajectory w in the SMPSM, which begins at q_1 and ends at q_2, visiting q_2 for the first time. $\tau(w)$ denotes the time for the first passage through q_2 for the sample trajectory w and $f_\tau(x)$ denotes its probability density function. The initial moment of the first passage time, T_1 is $\int_0^\infty xf_\tau(x)dx$. By Bayes's theorem of total probability, $f_\tau(x)=\sum_{u\in T_F(q_1,q_2)}f_\tau(x\mid w=u)P(u)$ and from the definition, $\int_0^\infty xf_\tau(x\mid w=u)dx=|u|$. Therefore,

$$T_1=\sum_{u\in T_F(q_1,q_2)}P(u)|u|.$$

In a similar way, the n-th moment of the first passage time, T_n is derived as follows:

$$\begin{aligned}T_n&=\int_0^\infty x^n f_\tau(x)dx\\ &=\sum_{u\in T_F(q_1,q_2)}\int_0^\infty x^n f_\tau(x\mid w=u)P(u)dx\\ &=\sum_{u\in T_F(q_1,q_2)}P(u)|u|_n.\end{aligned}$$

To derive T_n from a regular expression [14] which denotes $T_F(q_1,q_2)$, the n-th moment of a passage time, $\|s\|_n$ is defined for a regular expression *s,* which corresponds to a non-empty set of trajectories $D(s)$. The passage probability $\pi(s)$ is defined as well.

$$\pi(s):=\sum_{u\in D(s)}P(u)$$
$$\|s\|_n:=\sum_{u\in D(s)}P(u)|u|_n \quad \text{for } n\geq 0$$

From the above definition, it follows that $\|s\|_0=\pi(s)$ and $\|\eta\|_0=1$. For consistency, $\pi(\varnothing):=0$ and $\|\varnothing\|_0:=0$ for $n\geq 0$. Furthermore, it also follows from the above definition that $\|t\|_n=P(t)|t|_n$ for each $t\in T$. If s_f is a regular expression, which denotes $T_F(q_1,q_2)$, then $T_n=\|s_f\|_n$. And, $\pi(s_f)-1$ if there exists a passage from q_1 to q_2.

Next, we derive reduction rules for the computation of the passage probability and the passage time, for a passage represented by a well-formed regular expression. First, we introduce a special class of the regular expression, called the unambiguous regular expression (URE).

The URE is a regular expression that does not contain any duplicated expression as the unambiguous rational subsets [15]. Its formal definition is given as follows.

Definition 1 [Unambiguous regular expression] *Unambiguous regular expression over T and the sets that they denote are defined recursively as follows.*

1. *A regular expression* $\varnothing$ *is unambiguous.*
2. *A regular expression* η *is unambiguous.*
3. *A regular expression* $t \in T$ *is unambiguous.*
   Let r and s be unambiguous and denote the trajectory sets R and S, respectively.
4. *A regular expression (r+s) is unambiguous if* $R \cap S = \varnothing$.
5. *A regular expression (rs) is unambiguous if* $\forall u_1, u_2 \in R$, $v_1, v_2 \in S$, $u_1 v_1 = u_2 v_2$ *implies* $u_1 = u_2$ *and* $u_1 = u_2$.
6. *A regular expression* r^* *is unambiguous if* $r^{i-1}\, r$ *is unambiguous for* $i \ge 2$ *and* $r^i + r^j$ *is unambiguous for* $i \ne j$.

For convenience, we will use the same symbol to represent trajectory $u \in T^*$ and a regular expression which denotes $\{u\}$. From the definition of Kleene closure [14], $\varnothing^* = \eta$. For an index set *A*, the regular expression $r_{\alpha 1} + r_{\alpha 2} + r_{\alpha 3} + \cdots$ $(\alpha_i \in A)$ is denoted by $\sum_{\alpha \in A} r_\alpha$. For a regular expression *r*, the set of trajectories represented by *r* is denoted by *D(r)*.

The URE can be derived using general derivation methods for the regular expressions. Since all transitions are distinct, it is intuitively true that a regular expression is unambiguous if any trajectory is not considered more than once in the derivation of the regular expression. The regular expression derived by the following algorithm is unambiguous.

The following MR algorithm derives unambiguous regular expressions by an induction rule, where $\{q_i \mid 1 \le i \le N_s\}$ denotes the set of states.

MR Algorithm

- **Basis**: $r^0{}_{ij} = t_1 + \cdots + t_p$ *(or* $\varnothing$ *if there is no admissible transition), where* $\{t_1, \cdots, t_p\}$ *is the set of all admissible transitions from* q_i *to* q_j.

- **Induction Rule**: $r_{ij}^k = (r_{ik}^{k-1})(r_{kk}^{k-1})^*(r_{kj}^{k-1}) + r_{ij}^{k-1}$

i and *j* ($i \ne j$), r_{ij}^k ($r_{ij}^k + \eta$ for $i = j$) denote the set of all trajectories from q_i to q_j while avoiding any state numbered higher than *k* as the induction rule in [14]. Consequently, the regular expression $r_{ij}^{N_s}$ denotes the set of all the trajectories from q_i to q_j for $i \ne j$, and $r_{ij}^{N_s} + \eta$ denotes the set of all the trajectories from q_i to q_j. This induction rule is slightly different from the induction rule in [14] in the fact that η is added for *i=j* after the entire induction. In our version, $\eta \notin D(r_{ij}^k)$ for each r_{ij}^k. This property is necessary for derivation of a well-formed regular expression.

It can be proved that a regular expression derived by the above algorithm is unambiguous, since r_{ij}^0's are unambiguous and r_{ij}^k's obtained by the above induction rule are unambiguous when r_{ik}^{k-1}, r_{kk}^{k-1}, r_{kj}^{k-1}, and r_{ij}^{k-1} are unambiguous. In general, any regular expression derived in a natural way is unambiguous in general.

In what follows, we study the properties of $\pi(\cdot)$ and $\|\cdot\|_n$ over regular expressions. Since regular expressions are based on the three operations, concatenation, union, and Kleene closure, the conversions of $\pi(\cdot)$ and $\|\cdot\|_n$ are studied over these three operations.

Lemma 1: *For an URE* $s_1 s_2 \cdots s_k$,

$$\|s_1 s_2 \cdots s_k\|_n = \sum_{\substack{i_1+i_2+\cdots+i_k=n \\ i_1, i_2, \cdots, i_k \ge 0}} \frac{n!}{i_1!\, i_2! \cdots i_k!} \|s_1\|_{i_1} \|s_2\|_{i_2} \cdots \|s_k\|_{i_k} \qquad (n \ge 0)$$

Proof:
$D(s_1 s_2 \cdots s_k) = \{u_1 u_2 \cdots u_k \mid u_i \in D(s_i) \text{ for } 1 \le i \le k\}$. Since $s_1 s_2 \cdots s_k$ is unambiguous, all $u_1 u_2 \cdots u_k$'s are distinct. With this fact and the definitions, the remaining part of the proof is as follows.

When $D(s_i) \ne \varnothing$ for $1 \le i \le k$,

$$\|s_1 s_2 \cdots s_k\|_n = \sum_{u_1 \in D(s_1)} \sum_{u_2 \in D(s_2)} \cdots \sum_{u_k \in D(s_k)} P(u_1 u_2 \cdots u_k) |u_1 u_2 \cdots u_k|_n$$

Using the equations (3) and (4), the right-hand side of the above equation becomes:

$$\sum_{u_1 \in D(s_1)} \cdots \sum_{u_k \in D(s_k)} P(u_1) \cdots P(u_k) \sum_{\substack{i_1+i_2+\cdots+i_k=n \\ i_1, i_2, \cdots, i_k \ge 0}} \frac{n!}{i_1!\, i_2! \cdots i_k!} |u_1|_{i_1} |u_2|_{i_2} \cdots |u_k|_{i_k}$$

$$= \sum_{\substack{i_1+i_2+\cdots+i_k=n \\ i_1, i_2, \cdots, i_k \ge 0}} \frac{n!}{i_1!\, i_2! \cdots i_k!} \sum_{u_1 \in D(s_1)} P(u_1) |u_1|_{i_1} \cdots \sum_{u_k \in D(s_k)} P(u_k) |u_k|_{i_k}$$

$$= \sum_{\substack{i_1+i_2+\cdots+i_k=n \\ i_1, i_2, \cdots, i_k \ge 0}} \frac{n!}{i_1!\, i_2! \cdots i_k!} \|s_1\|_{i_1} \|s_2\|_{i_2} \cdots \|s_k\|_{i_k} \text{ for } n \ge 1.$$

Note that the above relation also holds for n=0.

Since $\|\varnothing\|_n$ for $n \ge 0$, the relation also holds for the cases that $D(s_i) = \varnothing$ for some $1 \le i \le k$. □

Corollary 1: *For an URE* $s_1 s_2$, $\pi(s_1 s_2) = \pi(s_1)\pi(s_2)$, $\|s_1 s_2\| = \pi(s_2)\|s_1\| + \pi(s_1)\|s_2\|$ *and*

$$\|s_1 s_2\|_2 = \pi(s_2)\|s_1\|_2 + \pi(s_1)\|s_2\|_2 + 2\|s_1\|\|s_2\|.$$

Lemma 2: *For an URE* $s_1 + s_2 + \cdots + s_k$,

$$\|s_1 + s_2 + \cdots + s_k\|_n = \|s_1\|_n + \|s_2\|_n + \cdots + \|s_k\|_n \quad (n \ge 0)$$

Proof: Since $s_1 + s_2 + \cdots + s_k$ is unambiguous, $D(s_i) \cap D(s_j) = \varnothing$ for $1 \le i, j \le k$ s.t. $i \ne j$. Therefore, if $D(s_i) \ne \varnothing$ for $1 \le i \le k$,

$$\begin{aligned}\|s_1 + s_2 + \cdots + s_k\|_n &= \sum_{\gamma \in \bigcup_{1 \le i \le k} D(s_i)} P(u_\gamma)|u_\gamma|_n \\ &= \sum_{\gamma \in D(s_1)} P(u_\gamma)|u_\gamma|_n + \cdots + \sum_{\gamma \in D(s_k)} P(u_\gamma)|u_\gamma|_n \\ &= \|s_1\|_n + \cdots + \|s_k\|_n.\end{aligned}$$

Since $\pi(\varnothing) = 0$ and $\|\varnothing\|_n = 0$, the relation also holds for the cases that $D(s_i) = \varnothing$ for some $1 \le i \le k$. □

Corollary 2: *For an URE* $s_1 + s_2$, $\pi(s_1 + s_2) = \pi(s_1) + \pi(s_2)$, $\pi(s_1 + s_2) = \pi(s_1) + \pi(s_2)$ $\|s_1 + s_2\| = \|s_1\| + \|s_2\|$ *and* $\|s_1 + s_2\|_2 = \|s_1\|_2 + \|s_2\|_2$.

Lemma 3: *For an URE* s^* *with* $\pi(s) < 1$, $\pi(s^*) = \dfrac{1}{1 - \pi(s)}$ *and for* $n \ge 1$,

$$\|s^*\|_n = \sum_{j=1}^{n} \frac{1}{(1 - \pi(s))^{j+1}} \Psi_j^n(s),$$

where

$$\Psi_j^n(s) = \sum_{\substack{i_1 + i_2 + \cdots + i_j = n \\ i_1, i_2, \cdots, i_j \ge 1}} \frac{n!}{i_1! i_2! \cdots i_j!} \|s_1\|_{i_1} \|s_2\|_{i_2} \cdots \|s_k\|_{i_j}$$

Proof: $D(s^*) = \bigcup_{k=0}^{\infty} D(s^k) = D(\sum_{k=0}^{\infty} s^k)$. From Lemma 2,

$$\|s^*\|_n = \left\| \sum_{k=0}^{\infty} s^k \right\|_n = \sum_{k=0}^{\infty} \|s^k\|_n \tag{5}$$

From Lemma 1,

$$\|s^k\|_n = \|ss \cdots s\|_n = \sum_{\substack{i_1 + i_2 + \cdots + i_k = n \\ i_1, i_2, \cdots, i_k \ge 0}} \frac{n!}{i_1! i_2! \cdots i_k!} \|s\|_{i_1} \|s\|_{i_2} \cdots \|s\|_{i_k} \tag{6}$$

For $n \ge 1$, (6) can be rewritten as follows.

$$(6) = \sum_{j=1}^{L} \|s\|_0^{k-j} \, {}_kC_j \sum_{\substack{i_1 + i_2 + \cdots + i_j = n \\ i_1, i_2, \cdots, i_j \ge 1}} \frac{n!}{i_1! i_2! \cdots i_j!} \|s\|_{i_1} \|s\|_{i_2} \cdots \|s\|_{i_j}, \tag{7}$$

where $L = \min(k,n)$. Since, by the definition of the binomial coefficient [16], ${}_kC_j = 0$ for $j > k$, the equation~(\ref{eqn:sj}) can be written as follows:

$$\|s^k\|_n = \sum_{j=1}^{L} \|s\|_0^{k-j} \, {}_kC_j \sum_{\substack{i_1 + i_2 + \cdots + i_j = n \\ i_1, i_2, \cdots, i_j \ge 1}} \frac{n!}{i_1! i_2! \cdots i_j!} \|s\|_{i_1} \|s\|_{i_2} \cdots \|s\|_{i_j}. \tag{8}$$

Substituting equation (8) into equation (5), we obtain that

$$\sum_{k=0}^{\infty} \|s^k\|_n = \sum_{j=1}^{n} \sum_{k=0}^{\infty} \frac{(k+j)!}{k!\, j!} \|s\|_0^k \, \Psi_j^n(s). \tag{9}$$

Now substituting

$$\sum_{k=0}^{\infty} \frac{(k+j)!}{k!} x^k = \frac{j!}{(1-x)^{j+1}}, \tag{10}$$

which holds when $0 \le x < 1$, into the equation (9), we obtain the Lemma for $n \ge 1$.

For n=0, the equation (6) becomes $\pi(s)^k$. Therefore,

$$\pi(s^*) = \sum_{k=0}^{\infty} \pi(s)^k = \frac{1}{1 - \pi(s)}.$$

The above proof also holds for $s = \varnothing$. □

Corollary 3: *For an URE* s^* *with* $\pi(s) < 1$,

$$\|s^*\| = \frac{1}{(1 - \pi(s))^2} \|s\| \quad \text{and}$$

$$\|s^*\|_2 = \frac{1}{(1 - \pi(s))^2} \|s\|_2 + \frac{2}{(1 - \pi(s))^3} \|s\|^2.$$

It would be impossible for a Kleene closure operation s^* with $\pi(s)=1$ to be contained in a regular expression that denotes a set of first passage trajectories, since every intermediate loop in a trajectory has another branch with a positive transition probability.

On the basis of Lemma 1, Lemma 2, and Lemma 3, the following theorem is obtained immediately as the main result for the proposed performance evaluation method.

Theorem 1: *Let s_f be an URE which denotes $T_F(q_1,q_2)$. The n-th moment of the first passage time from q_1 to q_2 in an SMPSM, $\|s_f\|_n$, can be represented by P(t) and $|t|_n$'s ($n \geq 1$) for each $t \in T$ through the conversions of the regular expression with the three operations.*

Theorem 1 together with Lemma 1, Lemma 2, and Lemma 3 provides the basis of the proposed performance evaluation method.
It should be noted that the proposed algorithm cannot be applied to MRSPNs and DSPNs. Since MRSPNs and DSPNs allow for concurrent transitions with generally distributed firing time, the transition time x_i of a transition t_i is not dependent on the adjacent x_i, so that $f(x_1 x_2 \cdots\cdots x_l) = f(x_1) f(x_2) \cdots\cdots f(x_k)$. Therefore, the equation (3) is not satisfied in the case of MRSPNs and DSPNs.

4. PERFORMANCE EVALUATION PROCEDURE

The performance of various systems such as manufacturing systems and communication systems can be evaluated according to the procedure shown in Fig. 4.

For a given system and a performance measure to be used for evaluation, the system is first modeled as an ESPN. The ESPN model is then converted to an SMPSM model as presented in Section 2. From the SMPSM model, the set of transition trajectories is obtained, according to the required performance measure, and the trajectories are represented by UREs. By applying the three reduction rules to the UREs, the performance measure is then evaluated. In this section, we will provide an algorithm, which will facilitate the derivation of the set of transition trajectories and the application of the conversion rules in an integrated manner, so that the performance measure can be evaluated directly from the SMPSM model.

The steps in the performance evaluation process will depend on the particular performance measure. Throughout the balance of this section, we will assume that the performance evaluation procedures are:

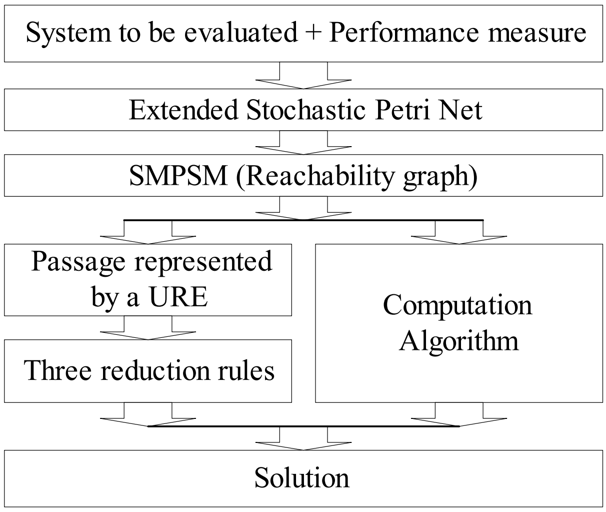

Fig. 4. Flow chart of the performance evaluation procedure.

the first passage of time, the recurrent time, and the steady-state probability.

4.1. The first passage of time

The first passage of time is the time duration from leaving one state, q_1, until the first time the system visits another state, q_2. The n-th moment of the first passage of time can be obtained by applying Theorem 1 to the SMPSM model together with Lemma 1, Lemma 2, and Lemma 3. The procedures to derive the URE, which denotes $T_F(q_1, q_2)$ and to apply the three conversion rules to the URE are combined and automated by the following algorithm.

The algorithm for the first passage of time, which is referred to as the FPT algorithm, is obtained by applying Theorem 1 to the MR algorithm together with Lemma 1, Lemma 2, and Lemma 3. By letting $p_{ij}^k := \pi(r_{ij}^k)$ and $a_{ij}^k(n) := \|r_{ij}^k\|_n$ and applying the conversion rules, the FPT algorithm is obtained. Let $Q = \{q_i \,|\, 1 \leq i \leq N_s\}$ be the set of the states in the SMPSM model under consideration. The FPT algorithm gives the n-th moment of the first passage of time from a state $q_I \in Q$ to a state $q_F \in Q$ when $q_I \neq q_F$.

FPT Algorithm

(1) Basis: Let A_{ij} denote the set of all admissible transitions from q_i to q_j.

$$a_{ij}^0(n) = \begin{cases} 0, & \text{if} \quad q_i = q_F. \\ \sum_{t \in A_{ij}} P(t)_i \, |t|_n & \text{Otherwise.} \end{cases}$$

(When $A_{ij} = \varnothing, a_{ij}^0(n) = 0.$)

(2) Induction Rule:

$$a_{ij}^{k}(n)=\sum_{i_1=0}^{n}\sum_{i_2=0}^{n-i_1}\frac{n!}{i_1!i_2!(n-i_1-i_2)}a_{ik}^{k-1}(i_1)\Phi_{k-1}^{k}(i_2)a_{kj}^{k-1}$$
$$\times(n-i_1-i_2)+a_{ij}^{k-1}(n),$$

where

$$\Phi_{k-1}^{k}(i_2)=\begin{cases}\dfrac{1}{1-p_{kk}^{k-1}} & i_2=0.\\ \displaystyle\sum_{j=1}^{i_2}\frac{1}{(1-p_{kk}^{k-1})^{j+1}}\sum_{\substack{l_1+\cdots+l_j=i_2\\ l_1,\cdots,l_j\geq 1}}\frac{i_2!}{l_1!\cdots l_j!}a_{kk}^{k-1}(l_1)\cdots a_{kk}^{k-1}(l_j), & Otherwise.\end{cases}$$

(3) For i, j s.t. $q_i=q_I$ and $q_j=q_F$, $a_{ij}^{N_S}(n)$ is the n-th moment of the first passage of time from q_I to q_F.

The recursive equation for $a_{ij}^{k}(n)$ becomes somewhat complicated as n increases. The FPT algorithm is most efficient for calculation of low order moments. For instance, in the evaluation of the mean and the variance of the first passage of time, only $a_{ij}^{k}(n)$ −s for $n\leq 2$ are of interest, and the recursive equations are simplified as follows:

$$p_{ij}^{k}=\frac{p_{ik}^{k-1}p_{kj}^{k-1}}{1-p_{kk}^{k-1}}+p_{ij}^{k-1},$$

$$b_{ij}^{k}=\frac{p_{kj}^{k-1}}{1-p_{kk}^{k-1}}b_{ik}^{k-1}+\frac{p_{ik}^{k-1}p_{kj}^{k-1}}{(1-p_{kk}^{k-1})^2}b_{kk}^{k-1}+\frac{p_{ik}^{k-1}}{1-p_{kk}^{k-1}}b_{kj}^{k-1}$$
$$+b_{ij}^{k-1},$$

and

$$c_{ij}^{k}=\frac{p_{kj}^{k-1}}{1-p_{kk}^{k-1}}c_{ik}^{k-1}+\frac{p_{ik}^{k-1}p_{kj}^{k-1}}{(1-p_{kk}^{k-1})^2}c_{kk}^{k-1}+\frac{p_{ik}^{k-1}}{1-p_{kk}^{k-1}}c_{kj}^{k-1}$$
$$+c_{ij}^{k-1}+2\frac{p_{kj}^{k-1}}{(1-p_{kk}^{k-1})^2}b_{ik}^{k-1}b_{kk}^{k-1}+2\frac{p_{ik}^{k-1}}{(1-p_{kk}^{k-1})^2}b_{kk}^{k-1}b_{kj}^{k-1}$$
$$+2\frac{1}{1-p_{kk}^{k-1}}b_{ik}^{k-1}b_{kj}^{k-1}+2\frac{p_{ik}^{k-1}p_{kj}^{k-1}}{(1-p_{kk}^{k-1})^3}(b_{kk}^{k-1})^2,$$

where $p_{ij}^{k}:=a_{ij}^{k}(0)$, $b_{ij}^{k}:=a_{ij}^{k}(1)$, and $c_{ij}^{k}:=a_{ij}^{k}(2)$.

Since the computational complexity of the MR algorithm is $O(N_S^3)$, the above algorithm is also $O(N_S^3)$ for the computation of the low order moments. Moreover, by using the FPT algorithm, both symbolic solutions as well as numerical solutions can be obtained.

4.2 Recurrence time

The recurrence time is the time for a system to regenerate a state; i.e., it is the first passage of time from a state q_F to itself. Therefore, the recurrence time can be evaluated in a similar way to the evaluation of the first passage of time.

One possibility for evaluating the recurrence time of a state q_F in an SMPSM is to use the FPT algorithm. To do so, an extra state q'_F is introduced and all the transitions from the state q_F are altered to occur from the state q'_F. Then, the first passage of time from q'_F to q_F is the recurrence time for q_F, which can be obtained by the FPT algorithm as previously indicated.

Let us derive a formula for the n-th moment of the recurrence time. Let r_{iF}^{Ns} denote the set of trajectories from a state q_i to the state q_F as the first visit and r_{Fi}^{0} denote the set of admissible transitions from the state q_F to a state q_i. r_{FF}^{0} denotes the set of admissible transitions from the state q_F to the state itself. Consider the SMPSM with the additional state q'_F. Then, $T_F(q'_F, q_F)$ can be denoted by s_r as follows:

$$s_r=\sum_{\substack{1\leq i\leq N_S,\\ i\neq F}}r_{Fi}^{0}r_{iF}^{N_S}+r_{FF}^{0}.$$

By applying Lemma 1 and Lemma 2,

$$\begin{aligned}\|s_r\|_n&=\sum_{\substack{1\leq i\leq N_S,\\ i\neq F}}\sum_{\substack{i_1+i_2=n,\\ i_1,i_2\geq 0}}\frac{n!}{i_1!i_2!}\sum_{t\in A_{Fi}}P(t)|t|_{i_1}\left\|r_{iF}^{N_S}\right\|_{i_2}+\sum_{t\in A_{FF}}P(t)|t|_n\\ &=\sum_{\substack{1\leq i\leq N_S,\\ i\neq F}}\sum_{l=0}^{n}\frac{n!}{l!(n-l)!}\left\|r_{iF}^{N_S}\right\|_l\sum_{t\in A_{Fi}}P(t)|t|_{n-l}+\sum_{t\in A_{FF}}P(t)|t|_n.\end{aligned}\tag{11}$$

Therefore, we obtain the n-th moment of the recurrence time from the FPT algorithm as follows.

$$\|s_r\|=\sum_{\substack{1\leq i\leq N_S,\\ i\neq F}}\sum_{l=0}^{n}\frac{n!}{l!(n-l)!}a_{iF}^{N_S}(l)\sum_{t\in A_{Fi}}P(t)|t|_{n-l}+\sum_{t\in A_{FF}}P(t)|t|_n\tag{12}$$

4.3 Steady-state probability

The steady-state probability of a state q_F in an SMPSM can be derived from its mean recurrence time and its special mean recurrence time, which is defined as a mean recurrence time on a graph with all the transition delays set to zero, except that the transition delays from the state q_F are timed transitions. The special mean recurrence time for q_F is calculated as follows:

$$T_R^{'} = \sum_{1 \le i \le N_S} \sum_{t \in A_{Fi}} P(t)|t| \quad (13)$$

From (12), the mean recurrence time for q_F is:

$$\begin{aligned} T_R &= \sum_{\substack{1 \le i \le N_S, \\ i \ne F}} \{ \sum_{t \in A_{Fi}} P(t)|t| + a_{iF}^{N_S}(1) \sum_{t \in A_{Fi}} P(t) \} \\ &+ \sum_{t \in A_{FF}} P(t)|t| \quad (14) \\ &= \sum_{\substack{1 \le i \le N_S, \\ i \ne F}} \{ \sum_{t \in A_{Fi}} a_{iF}^{N_S}(1) \sum_{t \in A_{Fi}} P(t) \} + T_R^{'}. \end{aligned}$$

The steady-state probability for q_F can now be obtained by calculating $T_R^{'} / T_R$ with the help of (13) and (14).

5. EXAMPLE

In this section, we detail how the methods proposed in this paper can be efficiently applied to evaluate the performance of a system. Two examples are presented. The first example shows the effectiveness of the FPT algorithm.

Example 1 (The recurrence time and the steady-state probability of a machine-repairman model)

The recurrence time and the steady-state probability are evaluated for the machine-repairman model using a buffer, which has been considered in Section 2. Table 1 summarizes the transition probabilities and the transition times in the SMPSM M_R model, where F_i is the cumulative distribution of the transition delay for a transition e_i, and p_i is the transition probability for a transition e_i. In this example, it was assumed that λ = 0.5/hour, μ = 1/hour, α = 0.5 hour, and σ =0.1 hour, as in [6]. The numerical values of the transition probability, and the first and the second moments of the transition delay are calculated and shown in Table 1.

We consider here the recurrence time of q_1. By applying the RT algorithm, the first moment of the recurrence time, $T_R(1)$, and the second moment of the recurrence time, $T_R(2)$, are derived. The basis for the RT algorithm is given as follows:

$a_{12}^0(0) = p_1$, $a_{12}^0(0) = p_1$, $a_{23}^0(0) = p_2$, $a_{31}^0(0) = p_3$, $a_{34}^0(0) = p_4$, $a_{42}^0(0) = p_5$, $a_{45}^0(0) = p_6$, $a_{56}^0(0) = p_7$, $a_{64}^0(0) = p_8$ and $a_{ij}^0(0) = 0$ otherwise; $a_{12}^0(1) = p_1|e_1|$, $a_{23}^0(1) = p_2|e_2|$, $a_{31}^0(1) = p_3|e_3|$, $a_{34}^0(1) = p_4|e_4|$, $a_{42}^0(1) = p_5|e_5|$, $a_{45}^0(1) = p_6|e_6|$, $a_{56}^0(1) = p_7|e_7|$, $a_{64}^0(1) = p_8|e_8|$, and $a_{ij}^0(1) = 0$ otherwise; $a_{23}^0(2) = p_2|e_2|_2$, $a_{31}^0(2) = p_3|e_3|_2$, $a_{34}^0(2) = p_4|e_4|_2$, $a_{42}^0(2) = p_5|e_5|_2$, $a_{45}^0(2) = p_6|e_6|_2$, $a_{56}^0(2) = p_7|e_7|_2$, $a_{64}^0(2) = p_8|e_8|_2$, and $a_{ij}^0(2) = 0$ otherwise.

We then apply the FPT algorithm to obtain the $a_{i1}^6(n)$ -s for $0 \le n \le 2$. Since $A_{1j} = \varnothing$ except for j=2 . Equation (12) provides that

$$\|s_r\| = \sum_{l=0}^{n} \frac{n!}{l!(n-l)!} a_{21}^{N_S}(l) p_1 |e_1|_{n-l}.$$

Therefore, we have that $T_R(1)$ = 4.75 hours and $T_R(2) = 43.9 hour^2$. The above mean recurrence time of q_1, agrees with the result of [6]. Finally, we can

Table 1. Transitions of the M_R model.

Transition	F_i	p_i	$\lvert e_i \rvert$	$\lvert e_i \rvert_2$	Numerical Values p_i	Numerical Values $\lvert e_i \rvert$	Numerical Values $\lvert e_i \rvert_2$
e_1	$1-e^{-2\lambda t}$	1	$\frac{1}{2\lambda}$	$\frac{1}{2\lambda^2}$	1	1	2
e_2	$N(\alpha,\sigma^2)$	1	α	$\alpha^2+\sigma^2$	1	0.5	0.26
e_3	$1-e^{-(2\lambda+\mu)t}$	$\frac{\mu}{2\lambda+\mu}$	$\frac{1}{2\lambda+\mu}$	$\frac{2}{(2\lambda+\mu)^2}$	0.5	0.5	0.5
e_4	$1-e^{-(2\lambda+\mu)t}$	$\frac{2\lambda}{2\lambda+\mu}$	$\frac{1}{2\lambda+\mu}$	$\frac{2}{(2\lambda+\mu)^2}$	0.5	0.5	0.5
e_5	$1-e^{-(\lambda+\mu)t}$	$\frac{\mu}{\lambda+\mu}$	$\frac{1}{\lambda+\mu}$	$\frac{2}{(\lambda+\mu)^2}$	2/3	2/3	8/9
e_6	$1-e^{-(\lambda+\mu)t}$	$\frac{\lambda}{\lambda+\mu}$	$\frac{1}{\lambda+\mu}$	$\frac{2}{(\lambda+\mu)^2}$	1/3	2/3	8/9
e_7	$1-e^{-\mu t}$	1	$\frac{1}{\mu}$	$\frac{2}{\mu^2}$	1	1	2
e_8	$N(\alpha,\sigma^2)$	1	α	$\alpha^2+\sigma^2$	1	0.5	0.26

compute the variance of the recurrence time of q_1 is $T_R(2) - T_R(1)^2 = 21.3\ hour^2$.

Through symbolic manipulation of the FPT algorithm and (12), the mean recurrence time and the variance of the recurrence time of q_1 can be derived in terms of λ, μ, α, and σ.

The symbolic values in Table 1 are assigned as the basis for the FPT algorithm and after applying the induction rule of the algorithm and the Equation (12), the obtained mean recurrence time of q_1, $T_R(1)$, is as follows:

$$T_R(1) = (\frac{2\lambda^2}{\mu^2} + \frac{2\lambda}{\mu} + 1)\alpha + \frac{2\lambda^2}{\mu^3} + \frac{2\lambda^2}{\mu^2} + \frac{2\lambda}{\mu} + 1.$$

In the MGF-based approach introduced by [6], a transfer function is used. The transfer function W for the transition delay performance measure is given by:

$$W_i = p_i M_i \quad W = \frac{W_1 W_2 W_3 (1 - W_6 W_7 W_8)}{1 - W_4 W_5 W_2 - W_6 W_7 W_8},$$

where $W_i = p_i M_i$ and M_i is the moment generating function for the transition delay of the transition e_i. From W, one can compute all the moments in particular,

$$T_R(1) = \left. \frac{\delta}{\delta s} W(s) \right|_{s=0}$$

and

$$T_R(2) = \left. \frac{\delta^2}{\delta s^2} W(s) \right|_{s=0}.$$

The MGF-based approach is useful when the complete distribution for a performance measure is required. However, the derivations of $\frac{\delta}{\delta s} W(s)$ and $\frac{\delta^2}{\delta_S} W(s)$ are rather complex.

The steady-state probability of the state q_1 is obtained as the ratio of T'_R to $T_R(1)$.

From (13), $T'_R = p_1 |e_1| = 1$. Therefore, the steady-state probability of the state q_1 is $T'_R / T_R = 1/4.75 = 0.21$.

The following example derives the mean service time of a station in a timer-controlled token bus network. It shows that the proposed method is helpful for deriving the analytic solution even when the model has a variable.

Example 2 (Service time of a timer-controlled token bus network)

The performance model is based on [17], and the highest access class of the IEEE 802.4 token bus network is considered.

The operation of a station $i (1 \le i \le N)$ in the timer-controlled token bus network is modeled by an ESPN as in [17]. Fig. 5 shows the ESPN model PN_{tb}. The detailed description of the model is referred to in [17].

When a station receives the right for a medium (p_3 is marked), the internal timer is set to its initial value H_i (p_5 is marked). Before the timer expires, a station may transmit messages. Place p_2 represents the transmission queue of the station with a buffer capacity of K_i. The number of tokens in p_2 is the number of frames waiting to be transmitted. The frames are arrived as a Poisson process with parameter λ_i (firing of t_1) and transmitted with service times of exponential distribution with parameter μ_i (firing of t_4). If there are no more frames to be transmitted or the timer expires (t_5), the right for the medium is passed to the next station (p_8). If the timer expires during transmission of the current data frame (when p_4 is not marked), the right for the medium is passed to the next station following completion of that transmission.

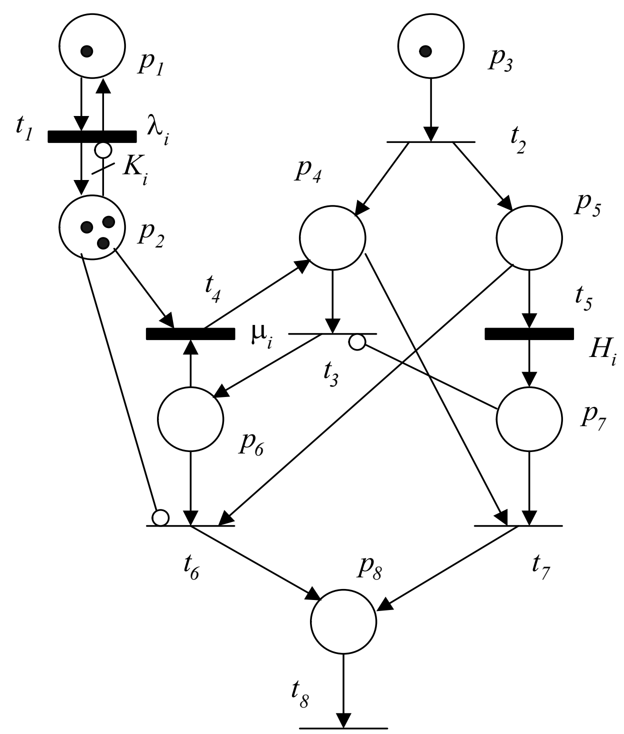

Fig. 5. An ESPN model of the timer-controlled token bus network: PN_{tb}.

The mean service time at a station i, T_s^i can be represented as follows:

$$T_s^i = \sum_{j=0}^{K_i} p_{a,j}^i T_{s,j}^i, \tag{15}$$

where $p_{a,j}^i$ denotes the probability that there are j frames in the transmission queue of the station i when the token is arrived at the station i and $T_{s,j}^i$ denotes the mean service time of the station i in the case where there are j frames in the transmission queue of the station i when the token is arrived at the station i.

The mean service time can be obtained only if $T_{s,j}^i$ is computed [17]. For simplicity of the analysis, this paper assumes the exhaustive service. This assumption excludes the firing of the transition t_5. Under this assumption, the reachability becomes as is shown in Fig. 6. The states (x_1, x_2, x_3) of the reachability graph represent x_1, x_2, and x_3 tokens in p_2, p_4, and p_6, respectively. In the reachability graph, $T_{s,j}^i$ is the mean first passage time from the marking $M_j^{'}(M_j)$ to the marking M_0.

The SMPSM model for PN_{tb} can be obtained from the reachability graph as shown in Fig. 7. The state $q(1 \le q \le K_i)$ in SM_{tb} corresponds to the

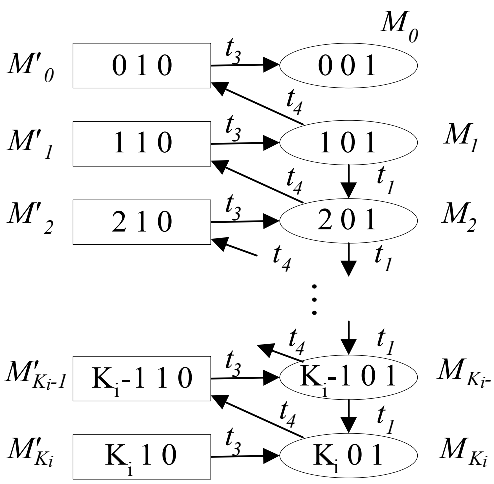

Fig. 6. The reachability graph for PN_{tb}.

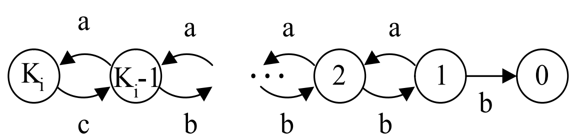

Fig. 7. The SMPSM model SM_{tb} for PN_{tb}.

marking M_q in the reachability graph. Since the transitions $t_{q,q-1}$ from the state q to the state $q-1(1 \le q \le K_i - 1)$ have the same $P(t_{q,q-1})$ and $S(t_{q,q-1})$, they are denoted by b. The transitions $P(t_{q,q+1})$ from the state q to the state $q+1(1 \le q \le K_i - 1)$ have the same $P(t_{q,q+1})$ and $S(t_{q,q+1})$, and they are denoted by a.

From (1) and (2), $P(a) = \frac{\lambda_i}{\lambda_i + \mu_i}$, $P(b) = \frac{\mu_i}{\lambda_i + \mu_i}$, $|a| = |b| = \frac{1}{\lambda_i + \mu_i}$, $|a|_2 = |b|_2 = \frac{2}{(\lambda_i + \mu_i)^2}$, $P(c) = 1$, $|c| = \frac{1}{\mu_i}$, and $|c|_2 = \frac{1}{\mu_i^2}$.

Let $S_f(K_i, j)$ denote the URE of the first passage from the state j to the state 0 in SM_{tb}. Then,

$$T_{s,j}^i = \left\| S_f(K_i, j) \right\| \tag{16}$$

$S_f(K_i, 1)$ can be derived by induction. $S_f(1, 1)$, $S_f(2, 1)$, and the induction rule is obtained as follows:

$$S_f(1, 1) = c. \tag{17}$$

$$S_f(2, 1) = (ac)*b. \tag{18}$$

$$S_f(K_i + 1, 1) = (aS_f(K_i, 1)*b. \tag{19}$$

$$S_f(K_i, J + 1) = S_f(K_i - 1, j)S_f(K_i, 1). \tag{20}$$

By applying Lemma 1, Lemma 2, and Lemma 3 to (17), (18), and (19),

$$\left\| S_f(K_{i,} 1) \right\| = \frac{1}{\mu_i^{K_i}} \sum_{i=1}^{K_i} \lambda_i^{K_i - 1} \mu_i^{l-1}. \tag{21}$$

Applying Lemma 1 to (20),

$$\left\| S_f(K_{i,} j + 1) \right\| = \sum_{p=0}^{j} \left\| S_f(K_i - p, 1) \right\|. \tag{22}$$

From (21), we obtain

$$\left\| S_f(K_{i,} j) \right\| = \sum_{p=0}^{j-1} \frac{1}{\mu_i^{K_i}} \sum_{l=1}^{K_i - p} \lambda_i^{K_i - p - l} \mu_i^{p+l-1}. \tag{23}$$

The variance of the service time can also be obtained from (17), (18), (19), and (20) by applying Lemma 1, Lemma 2, and Lemma 3.

The result derived in this section well corresponds to that in [17]. In addition, the performance of the timer-controlled token bus network was derived analytically using only simple algebraic manipulations without applying Mason's rule to complex graphs, and Laplace transformations and differentiations on complex transfer functions.

6. CONCLUSIONS

In this paper, we have introduced an efficient method to evaluate the performance of an ESPN through simple algebraic manipulations. The proposed method applies the automata theory and the probability theory to the Petri net theory. A performance measure is represented by a well-formed regular expression for the transition trajectories concerned, and the computation of the n-th moments of the regular expression is performed by applying three simple reduction rules. In addition to a numerical solution, a symbolic solution can be obtained as well.

The evaluations of the first passage of time, the recurrence time, and the steady-state probability were presented, and an algorithm was provided to automate the entire computation. When the reachability graph of an ESPN model has N_S tangible marking states, the complexities of the algorithms are $O(N_S^3)$ and the algorithm requires $O(N_S^2)$ memory spaces for calculation of the moments of low orders. Each step of the algorithm consists of simple algebraic operations, and does not need matrix inversion, integration, or differentiation.

Two additions, one subtraction, five multiplications, and one division are required at each step to derive the mean value for a performance measure. The method is efficient for computation of low order moments, however, it is inadequate for high orders.

As a future direction of this work, it may be pertinent to consider developing a direct application of the proposed method to an ESPN model without the intermediate conversion step to an SMPSM model.

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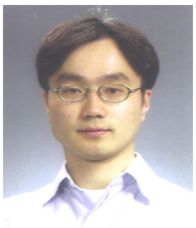

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