Application of CDM to MIMO Systems: Control of Hot Rolling Mill

Young-Chol Kim and Myung-Joon Hur

Abstract: This paper deals with a design problem of a decentralized controller with a strongly connected two-input two-output multivariable system. To this end, we present a classical design approach which consists of two main steps: one is to decompose the multivariable plant into two single-input single-output systems by means of the Individual Channel Design (ICD) concept, the other is to design controller of each channel by the Coefficient Diagram Method (CDM) so that it satisfies, especially, time domain specifications such as settling time, overshoot etc.. A design procedure was proposed and then was applied to a 2×2 hot rolling mill plant. Simulation results showed that the proposed method has excellent control performances.

Keywords: individual channel design, coefficient diagram method, and multivariable system

I. Introduction

Most large-scale systems such as power plants, steel making processes or aircraft manufacturing factories are multivariable systems. Although many modern control approaches have given us an elegant theory of both synthesis and analysis, it is known that these approaches are not satisfactory and practical for industrial applications.

The analysis and design of multivariable control systems are based mainly on state-space methods such as \( H_\infty \) theory and LQG/LTR, which can be described, mathematically. However, there were not enough examples to convince us of their practicality for industries. So, for practical purposes, the classical frequency domain design techniques, for example, \( \mu \)-verse Nyquist Array technique \([6]\) has been adopted. However, it is sometimes not convenient to satisfy time domain specifications like settling time, overshoot, and rise time etc..

In this paper, we propose a new practical design method, which is a combination of the CDM (Coefficient Diagram Method)\([5]\) and the ICD (Individual Channel Design) concept \([1]-[4]\). A most popular case in the MIMO systems, a two-input two-output \((2\times2)\) plant is considered. The objective is to develop a method for designing a decentralized controller for a \((2\times2)\) plant under the time domain specification given above.

The ICD is a classical method which decomposes a MIMO (Multi-input Multi-output) system into a set of SISO systems. We will show that the ICD allows a good decoupling compensator design, which is quite different from the conventional input decoupler.

The CDM is a very effective design method for a SISO problem when the specification to be satisfied is imposed on the time domain performance. The design procedure consists of three steps. First, decomposing the given \((2\times2)\) plant into two SISO plants by means of the ICD, the CDM is applied to each channel to obtain the feedback controllers, in which they shall result in both the stability of the overall system and the desired transient behavior. In the final step, feed-forward compensators, whose input is the reference input of the opposite channel, are designed. The improvement of the tracking performance via the this compensator is remarkable if the exact model is known and does not make the closed-loop stability worse, even though there may exist a mismatching problem between model and plant.

As an example for application to show the usefulness of the proposed method, we chose a simplified model of 2-input 2-output Hot Rolling Mill plant, which is currently operating at a steel processing company in Korea. The actual performance of the existing controllers is very poor, and thus the company wants to improve it without big changes in control configuration. We will show how the newly designed controller works through simulations.

II. Problem statements

We consider the following multivariable control system which consists of decentralized feedback controllers, newly proposed decoupling compensators, and internally coupled 2-input 2-output plant as shown in Fig. 1.

![Fig. 1. Multivariable control system with new decoupling Compensators.](image_url)

The plant transfer function \( G(s) \) is

\[
G(s) = \begin{bmatrix}
    r_{11}(s) & r_{12}(s) \\
    r_{21}(s) & r_{22}(s)
\end{bmatrix}
\]

(1)

In Fig. 1, \( Ac_1(s), \ Bc_1(s), \ BA_1(s), \ ACA_2(s), \ BCA_2(s), \ BA_2(s), \) are all polynomials of decentralized controllers to be designed. Both \( P_1(s) \) and \( P_2(s) \) are new decoupling compensators in the form of rational function. Suppose that the design specifications are transient responses such as overshoot and settling time.

The CDM is basically the same as the model matching method in the sense that for given a plant \( G(s) \) and chosen a...
target model $T(s)$, the problem is to find a controller so that the overall transfer function matches $T(s)$. But, the CDM can deal with lower order design problems, which do not satisfy the solvability condition of Diophantine equation. In these model matching approaches, two-parameter configuration is mainly used because it has several advantages compared with the other two degree of freedom configurations [8]. The objective in this paper is to extend the CDM for SISO to the MIMO cases, specifically 2×2 systems. We are going to propose a CDM design procedure of a 2×2 decentralized controller for the MIMO system shown in Fig. 1. We will apply the decomposition using the ICD. Through the procedure, the 2×2 overall system can be expressed by exact two single loops. Then we will address how to design each feedback controller although the given plant is strongly connected. It will be shown that the proposed decoupling compensator $P_i$ does not affect the closed loop stability at all. Finally, the proposed method will be examined in a looper control system.

III. Preliminary results and design procedures

1. Channel decomposition

In this section, we will decompose the control system without the decoupling compensators into 2 Channels based on the ICD scheme. It results in the block diagrams shown in Fig. 2. Let $C_i(s)$ be the transfer function of the Channel 1 relating the output $y_i(s)$ to the reference input $r_i(s)$, and let $C_j(s)$ of Channel 2 be that of the output $y_j(s)$ to the reference input $r_j(s)$.

The following equations can be derived. For $(i, j) \in \{1,2\}$,

\[ C_i(s) = k_i(s) g_i(s)(1 - \gamma(s)) h_i(s) \]

\[ \gamma(s) = \frac{g_i(s)g_j(s)}{g_i(s)g_j(s)} \]

\[ k_j(s) = \frac{Bc_j(s)}{Ac_j(s)} \]

\[ h_j(s) = \frac{k_j(s) g_j(s)}{1 + k_j(s)g_j(s)} \]

\[ d_i(s) = \frac{1}{Bc_j(s)} g_i(s) h_i(s) \]

\[ D_j(s) = Ba_j(s)d_j(s)r_j(s) \]

It is noted that the magnitude of $\gamma(s)$ is a measure of the loop signal interaction and $h_j(s)$ denotes the subsystem of Channel 1. For example, the large value of $\gamma(s)$ implies that $G(s)$ is a strongly interconnected system. The signal $D_j(s)$ from $r_j(s)$ through some transfer functions is regarded as the disturbance in Channel 1. It means that the interaction from the opposite input can be modeled as an output disturbance by means of the ICD decomposition.

From Fig. 2, the closed-loop relations between the inputs and outputs can be described as follows:

\[ y_1(s) = T_{11}(s)r_1(s) + T_{12}(s)r_2(s) \]

\[ y_2(s) = T_{21}(s)r_1(s) + T_{22}(s)r_2(s) \]

Fig. 2. Decomposed channels of the control system without decoupler.

where, for $(i, j) \in \{1,2\}$,

\[ T_{i1}(s) = B_{c_i} \frac{g_i(1 - \gamma h_i)}{Ac_i (1 + k_i g_j(1 - \gamma h_j))} \]

\[ T_{i2}(s) = B_{a_i} \frac{g_j h_i}{Ac_i (1 + k_i g_j(1 - \gamma h_j))} \]

2. Decentralized feedback controller design by the CDM

We now design decentralized feedback controllers using the CDM. As shown in Fig. 2, we see that subsystem $h_2$ of Channel 1 is dependent on controller $k_2$ of Channel 2, and vice versa for $h_1$ of Channel 2. Hence, each channel cannot be designed independently in principle, since the magnitude of $\gamma(s)$ is large.

If the magnitude of $\gamma(s)$ is very small, it is feasible to design those, independently, in a practical sense. For the purpose of solving the case where the plant is strongly coupled internally, we present some design guides as follows:

**Assumption 1:** The given plant is of minimum-phase and the polar plots of $[\gamma(j\omega)h_j(j\omega)]$ are not near the point $(1,0)$ in a complex plane.

**Assumption 2** Let the closed-loop bandwidth of each channel be $\omega_{k_1}$, $\omega_{k_2}$, respectively. Assume that $\omega_{k_1} < \omega_{k_2}$.

**Remark 1:** Assumption 1 implies that the bandwidth of subsystem $h(s)$, $\omega_k$, is approximately equal to $\omega_{k_1}$.

**Remark 2:** Assumption 2 implies that the gain crossover frequency of an open loop transfer function of Channel 1, $\omega_{k_1}$, is significantly less than that of Channel 2, $\omega_{k_2}$. That is, $\omega_{k_1} < \omega_{k_2}$ [1].

Under the above conditions, we give brief guidelines for CDM design as follows.

1) It is noted from eq. (5) that if the gain of the controller $k_j(s)$ is large, then the subsystem $h_i(s)$ approaches unity.

2) If the design specifications satisfy Assumption 2 (or Remark 2), then the above 1 means that the controller $k_j(s)$ of Channel 1 can be designed without regard to $k_i(s)$ of Channel 2 because $h_i(s)$ approaches unity.

These properties allow us to adopt the CDM as a tool for designing the multivariable control systems since some prop-
erties in a given frequency domain can be interpreted as some other properties in a time domain heuristically, even though it may not be true mathematically.

(3) The bandwidth of $T(s)$ is generally related to the speed of response. The larger the bandwidth, the faster the response is. Thus, Assumption 2 and Remark 2 imply that the time domain response of Channel 1 is slower than Channel 2, and vice versa.

(4) Based on the concepts above, if the response of Channel 2 requires to be significantly faster than that of Channel 1, the controller $k_2(s)$ can be designed almost independently of $k_2(s)$ of Channel 2 because $h_2(s)$ approaches unity. Once the controller $k_1(s)$ of Channel 1 is designed, $h_1(s)$ in Channel 2 must be known accordingly. Therefore, $k_2(s)$ of Channel 2 can be obtained by CDM.

3. Design of new decoupling compensators

Since conventional input decouplers compensate the loop signal interactions inside the feedback loops, the stability and the performance of the overall control system may be highly sensitive to model uncertainty. Thus, we propose new decoupling compensators, $P_{12}(s)$ and $P_{21}(s)$, by exchanging signal transmission paths like Fig. 1. Since these $P_{12}(s)$ and $P_{21}(s)$ are excited by signals from outside of the feedback loops, they will not affect the overall stability. On the other hand, the overall performance will be affected. This will be proven in the following theorem.

Theorem 1: Suppose that $P_{12}(s)$ and $P_{21}(s)$ are proper stable rational functions. Then the feedback system in Fig. 1 is totally stable if the following characteristic polynomial $\Delta(s)$ is Hurwitz.

$$\Delta(s) = \Delta_c(s)\Delta_r(s)\det[I + G(s)C(s)]$$

where, $\Delta_c(s)$ and $\Delta_r(s)$ are characteristic polynomials of $G(s)$ and $C(s)$, respectively and

$$C(s) = \begin{bmatrix} \frac{B_1(s)}{A_1(s)} & 0 \\ 0 & \frac{B_2(s)}{A_2(s)} \end{bmatrix}$$

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix}$$

Proof: It is easy to see that the overall system in Fig. 1 can be written as follows.

$$y(s) = [I + G(s)C(s)]^{-1} G(s)P(s)r(s)$$

where, $y(s) = [y_1(s) \ y_2(s)]'$, $r(s) = [r_1(s) \ r_2(s)]'$.

$$P(s) = \begin{bmatrix} 0 & P_{12}(s) \\ P_{21}(s) & 0 \end{bmatrix}.$$ 

Since $P_i(s)$ are assumed to be stable, letting $r_i(s) = P_i(s)r(s)$.

(14) can be rewritten by

$$y(s) = [I + G(s)C(s)]^{-1} G(s) r_i(s)$$

The eq. (15) is equivalent to the standard multivariable feedback system without feedforward term. Therefore, remainder part of proof is completed by Chen [9].

![Fig. 3. Decomposed channels with decoupling compensators.](image)

Now, we are going to propose a design method for the decoupling compensators $P_{12}(s)$ and $P_{21}(s)$. The overall system including decoupling compensators in Fig. 1 can be also decomposed into 2 Channels as shown in Fig. 3, in which the loop interactions have been transformed by equivalent disturbances of each channel. The relations of inputs $r_i(s)$ of Channel $i$ to outputs $y_j(s)$ of Channel $j$, for $i \neq j$ are as follows:

$$y_j(s) = (Ba_i(s)P_{ij}(s)C_j(s) + Ba_i(s)d_i(s))r_i(s)$$

(16)

$$y_j(s) = (Ba_i(s)P_{ij}(s)C_j(s) + Ba_i(s)d_i(s))r_i(s)$$

(17)

If $P_{ij}(s)$ and $P_{ji}(s)$ are selected such that (16) and (17) become zero, then two channels will be disconnected completely. Therefore, we have

$$P_{12}(s) = \frac{d_1(s)}{C_1(s)}$$

(18)

$$P_{21}(s) = \frac{d_2(s)}{C_2(s)}$$

(19)

According to Theorem 1, these decouplers should be stable. If the right hand sides of (18) and (19) do not satisfy the stability, then they may be designed, as like filter so as to have the similar frequency magnitudes as those of $P_{12}(s)$ and $P_{21}(s)$.

IV. Control of hot rolling mill

In this section, the proposed decentralized control scheme will be examined. As an illustrative example, we consider a Hot Rolling Mill control system. This system is sometimes called a Looper System since the loopers in the systems play important roles to sustain and/or regulate the tension of the strip, which passes through each roll stands. Fig. 4 shows a block diagram of a simplified model of an actual looper system, which consists of two PI controllers and conventional input decouplers. Wherein $k_{p1}$, $k_{p2}$, $k_{i1}$, $k_{i2}$, $k_{f1}$, $k_{f2}$ are the gain parameters of PI controllers, and $c_{11}$, $c_{12}$, $c_{21}$, $c_{22}$ are parameters of input decouplers. $s_i$, $e_i$, $g_i$, $f_i$, $k_i$, $k_n$ are the parameters of Mill Motor and Looper Motor, and $t_n$, $t_k$ are parameters of
The settling times of the Mill Motor speed is 0.3 ~ 0.5 sec, and that of the Looper Motor angle is 45 ~ 60 sec as shown in Fig. 5. These facts suggest that the Mill Motor response is significantly faster than the Looper Motor response. So, letting the Mill Motor be Channel 1, and the Looper Motor be Channel 2, we can design each channel almost independently, which is already stated in Remark 5.

In this paper, we assume that the desired design specifications are as follows:

Subject to plant parameter perturbations of ±20%,
1) settling time of Channel 1 is less than 0.3 sec,
2) settling time of Channel 2 is less than 50 sec,
3) overshoots should be less than 20%.
In addition, it is required that the controller be easily reduced the steady state errors due to plant uncertainty.

2. Swapping and channel decomposition

The plant in Fig. 4 is given by the following 2×2 \( G(s) \):

\[
G = \begin{bmatrix}
g_{11} & g_{12} \\
g_{21} & g_{22}
\end{bmatrix} = \begin{bmatrix}
\frac{-4.31(s + 0.7541)}{(s + 24.9694)(s + 9.5917)(s + 5.6639)} & 17.2 \\
\frac{11.525}{(s + 50.0212)(s + 9.5489)(s + 5.6799)} & \frac{0.8492}{(s + 50.0212)(s + 9.5489)(s + 5.6799)}
\end{bmatrix}
\]

From (3), calculating the magnitude of \( \gamma(s) \), which is a measure of the loop signal interaction, we have

\[
\gamma(s)\big|_{s=0} = -4
\]

The negative sign of \( \gamma \) means that the interaction effect is minus, and the value (4 >> 1) means that the interaction effect is considerably large. Thus, this system needs swapping, which means to exchange the control variables to other controlled variables in the same system as shown in Fig. 6. That is, the swapped plant is written by

\[
g_{11} = \frac{-4.31(s + 0.7541)}{(s + 24.9694)(s + 9.5917)(s + 5.6639)}
\]

Equations relating to the swapped system are

\[
\gamma_s(s) = \frac{g_{11}(s)g_{22}(s)}{g_{12}(s)g_{21}(s)}
\]
3. The design of feedback controllers for each channel
Since the response of Channel 1 is faster than that of Channel 2, we let the subsystem \( h_1(s) \) of Channel 1 be approximately equal to 1. Then Channel 2 will be

\[
C_{2u}(s) = g_{21}(s)(1 - \gamma_u(s)h_2(s)) \equiv g_{21}(s)(1 - \gamma_u(s))
\]

\[
\frac{0.39682}{s^2 + 25s}
\]

Using CDM, a feedback controller for Channel 2 was designed as in (25) and (26). Fig. 7 shows the coefficient diagram of Channel 2, on which the real line curve indicates coefficients of the resulting characteristic polynomial of channel 2, that is, \( \delta_2 = 0.333s^3 + 8.499s^2 + 4.175s + 0.3968 \). The detail design procedure refers to [5,10].

\[
Ac_2(s) = 0.333s + 0.1667 \quad (25)
\]

\[
Bc_2(s) = 1, \quad Ba_2(s) = 1 \quad (26)
\]

Next, we design a feedback controller for Channel 1. Since \( k_2(s) \) has been determined by (25) and (26), \( C_{1u}(s) \) in (23b) can be obtained by

\[
C_{1u}(s) = g_{12}(s)(1 - \gamma_u(s)h_2(s)) \equiv g_{12}(s)(1 - \gamma_u(s))
\]

\[
\frac{10.94s + 1.407}{s^2 + 65.35s^2 + 822.5s^2 + 2793s + 264.6}
\]

Also using CDM, a feedback controller for Channel 1 is designed as in (28) and (29).

\[
Ac_1(s) = 0.001086s + 0.0001401 \quad (28)
\]

\[
Bc_1(s) = 0.1799s + 0.9737 \quad (29)
\]

4. The design of new decoupling compensators
From eq. (18) and (19), the new decoupling compensators are determined by

\[
P_{12}(s) = \frac{56.82s^3 + 2551s^2 + 32710s + 111300}{s^3 + 99.6s^2 + 754.7s + 1326} \quad (31)
\]

\[
P_{21}(s) = \frac{1.186s^3 + 60.18s^2 + 44.7s}{s^3 + 25.49s^2 + 12.2s + 0.9481} \quad (32)
\]

Substituting all these compensators above into Fig. 1, we
see that the resulting step responses satisfy the specification elegantly as shown in Fig. 10.

Fig. 10. Step responses of multivariable control system with decoupling compensators.

5. Examinations of robustness and ease of tuning

To examine the robustness of our decentralized multivariable control system we assume that there are perturbations of ±20% in all of the plant parameters ($t_v, t_p, s_p, e, g, f, t_q, k_v, k_n$). In such cases, Fig. 11 represents the worst case step responses, where we can find that Channel 1 maintains the robustness but in Channel 2, the steady state error of about ±25% occurs.

Since the plant perturbation is reflected by the constant steady-state error of Channel 2 with no overshoot, from (11) we have

$$y_2(\infty) = T_{22}(0) r_2(\infty) + T_{12}(0) r_1(\infty).$$

(33)

Although the individual controller in CDM is designed so that $T_{ii}(0) = 1, \ i = 1,2$ at a nominal model, this will not hold due to parameter perturbations. Then the steady state error of channel 2 is expressed by

$$e_{2ss} = y_2(\infty) - r_2(\infty) = \left(1 - T_{22}(0)\right) r_2(\infty) + T_{12}(0) r_1(\infty).$$

(34)

We see that the zero steady state error cannot be obtained in case where even $T_{ii}(0) = 1, \ i = 1,2$ hold. It is one of feedback effect that $T_{ii}(0)$ is insensitive to the model changes as long as feedback gain is sufficient large. This means that $T_{ii}(0) = 1, \ i = 1,2$. Therefore, the steady state error can be reduced remarkably if we make $T_{22}(0) = 0$. It will be good choice that if feedback controllers are fixed because they relate to the closed loop stability. On the other hand, the purpose can be achieved by only slightly modifying $P_{12}(s)$ in (31). Replacing the lowest parameter of the numerator of (31) by $111300 + \beta$, we have (35).

$$P_{12}(s) = \frac{8.39s + (83.97 + \beta)}{0.384s + 1}. $$

(36)

The new decoupling compensator in (36) gives rise to almost the same responses as that in (31) with $\beta = 16.03$ in +20% perturbation case and $\beta = -16.47$ in -20% case.

Fig. 11. The worst case response scenario with perturbations of ±20% (Solid:+20%, Dotted:-20%).

Fig. 12. Recovered responses by tuning $\beta$ of $P_{12}(s)$ (Solid:+20%, Dotted:-20%).

V. Concluding remarks

A new design approach of a decentralized controller for a MIMO system has been presented. As in a most popular case, a two-input two-output plant was considered. The design objective that has been emphasized is to find a satisfactory controller to time repose specifications, such as settling time,
overshoot, rise time, steady state errors etc. To this end, the CDM and the ICD concepts have been extended. The proposed method was examined by applying to a simplified model of an actual 2x2 Hot Rolling Mill plant, which is in use at a steel process company in Korea. Through some simulations, we have demonstrated that this approach may be very useful in designing controllers that should meet the time domain specifications.

References

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