A New Excitation Control for Multimachine Power Systems
I: Decentralized Nonlinear Adaptive Control Design and Stability Analysis

Haris E. Psillakis and Antonio T. Alexandridis

Abstract: In this paper a new excitation control scheme that improves the transient stability of multimachine power systems is proposed. To this end the backstepping technique is used to transform the system to a suitable partially linear form. On this system, a combination of both feedback linearization and adaptive control techniques are used to confront the nonlinearities. As shown in the paper, the resulting nonlinear control law ensures the uniform boundedness of all the state and estimated variables. Furthermore, it is proven that all the error variables are uniformly ultimately bounded (UUB) i.e. they converge to arbitrarily selected small regions around zero in finite-time. Simulation tests on a two generator infinite bus power system demonstrate the effectiveness of the proposed control.

Keywords: Multimachine power system control, adaptive control, backstepping design, decentralized control.

1. INTRODUCTION

Power systems are continuously growing in size and complexity with increasing interconnections. They consist of several generating units while the power demands vary incessantly. Additionally, small or large disturbances such as power changes or short-circuits (faults) may transpire. One of the most crucial operation demands is the maintenance of system stability. In particular, when a fault occurs, large currents and torques are produced and control action must be taken promptly if system stability is to be sustained. This is an imperative solution to the power system transient stability problem defined as that of assessing whether or not the system will reach an acceptable steady-state following the fault. However, power systems are large scale highly nonlinear systems that include a number of synchronous machines as producers. One of the main goals of the excitation control of each machine is the enhancement of power system stability.

Conventional excitation controllers are mainly designed by using linear control theory. Especially for the case of a single machine to infinite-bus power system, a method that has been extensively used is one based on the linearization around an operating point and the design of linear excitation controllers [1,2]. The main disadvantages of this design such as lack of reliability and robustness are well-known.

In the last decade, nonlinear control theory has also been widely used to account for the nonlinearities of the controlled power systems [3,4]. The majority of these controllers are based on the feedback linearization technique [5]. Feedback linearization is recently enhanced by using robust control designs such as $H_{\infty}$ control and $L_2$ disturbance attenuation [6-12]. In recent years, new approaches have been proposed for power stability designs according to other sophisticated schemes such as fuzzy logic control [13], adaptive control [14,15] and neuro-control [16-19]. Combinations of the above techniques are also proposed [20-22] in order to exploit the advantages of each method under the cost of the increase in complexity.

In this paper, we consider a multimachine power system wherein each machine is represented by its third order nonlinear dynamic model and the transmission net is described by the admittance matrix. On this model the well-known backstepping technique [23,24] is used in order to obtain the most possible partially linear form of the system. On this form we use the most simplified feedback linearization scheme in order to obtain a local feedback control law i.e., a control law that is dependent from the local measurable states of the system and a local measurable variable. Simultaneously, all the other nonlinearities that are dependent from locally immeasurable variables or variables that are not states are left on an unknown nonlinear term. An adaptive
control mechanism is then used to estimate suitable bounds of this unknown nonlinear term. Finally, we propose a nonlinear feedback controller (Theorem 1) based on both the effective adaptive operation and the suitable selection of some design constants.

By this control design we prove that the third error variable is driven in finite-time in a neighborhood of the origin of arbitrarily small dimensions. As soon as this happens, the other two error variables insert in finite-time in a sphere around the origin of arbitrarily small radius (Theorem 2). The boundedness of all signals is proved. The adaptation mechanism used belongs to the class of direct adaptive algorithms in the sense that it guarantees the uniform ultimate boundness (UUB) of the error variables while the estimated parameter errors remain bounded. Furthermore, as ascertained (Theorem 3), the power angle deviations converge to an even smaller region as time passes. This is crucial for the selection of the design constants since it leads to significantly lesser values for the control gains. Simulation results after a symmetrical three-phase short circuit fault on a two machine-infinite bus test system demonstrate the effectiveness of the proposed scheme.

2. DYNAMIC MODEL

After reducing the multimachine power system into a network with generator nodes only, the classical third-order single-axis dynamic generator model is used for the design of the excitation controller, whereas differential equations that represent dynamics with very short time constants have been neglected. In general, for a n-generator power system, the dynamic model of the i-th generator is

\[ \dot{\delta}_i(t) = \omega_i(t) - \omega_0, \]  
\[ \dot{\omega}_i(t) = -\frac{D_i}{M_i}(\omega_i(t) - \omega_0) + \frac{oh_0}{M_i}(P_{mi} - P_{ei}(t)), \]  
\[ \dot{E}_{qi}(t) = \frac{1}{T_d} (E_{fi}(t) - E_{qi}(t)), \]

where

\[ E_{qi}(t) = E_{qi}'(t) + (x_{di} - x_{di}') I_{di}(t), \]  
\[ E_{fi}(t) = k_i u_i(t), \]  
\[ I_{qi}(t) = \sum_{j=1}^{n} E_{qi}'(B_{ij} \sin \delta_j(t) + G_{ij} \cos \delta_j(t)), \]  
\[ I_{di}(t) = \sum_{j=1}^{n} E_{qi}'(G_{ij} \sin \delta_j(t) - B_{ij} \cos \delta_j(t)), \]  
\[ P_{ei}(t) = E_{qi}'(t) I_{qi}(t), \]  
\[ Q_{ei} = E_{qi}' I_{di}(t), \]  
\[ E_{qi}(t) = x_{adi}' I_{fi}(t), \]  
\[ V_{qj}(t) = E_{qi}'(t) - x_{di}' I_{di}(t), \]  
\[ V_{ad}(t) = x_{di}' I_{qi}(t), \]  
\[ V_i(t) = \sqrt{V_{ad}^2(t) + V_{qj}^2(t)}. \]

The symbols used in the above equations are explained in the Appendix.

3. BACKSTEPPING DESIGN

Introducing the first error variable for the i-th machine

\[ z_{i1} = \Delta \delta_i, \]

and viewing \( \Delta \omega_i \) as a virtual control, we define the second error variable

\[ z_{i2} = \Delta \omega_i - \alpha_{i1}(\Delta \delta_i), \]

where \( \alpha_{i1} \) is a function to be designed. Consider the first candidate Lyapunov function

\[ V_1 = \frac{1}{2} \sum_{i=1}^{n} z_{i1}^2. \]

Then, selecting

\[ \alpha_{i1}(\Delta \delta_i) = -c_{i1} z_{i1} = -c_{i1} \Delta \delta_i, \]

where \( c_{i1} > 0 \) is a constant that can be suitably selected and taking into account that

\[ \dot{z}_{i1} = \Delta \omega_i, \]

we have

\[ \dot{V}_1 = -\sum_{i=1}^{n} c_{i1} z_{i1}^2 + \sum_{i=1}^{n} z_{i1} z_{i2}. \]

For the second error variable the dynamics are

\[ \dot{z}_{i2} = -\frac{D_i}{M_i} \Delta \omega_i - \frac{oh_0}{M_i} \Delta P_{ei} - \frac{\partial \alpha_{i1}}{\partial \Delta \delta_i} \Delta \omega_i. \]

Viewing \( \Delta P_{ei} \) as a virtual control we introduce the third error variable

\[ z_{i3} = \Delta P_{ei} - \alpha_{i2}(\Delta \delta_i, \Delta \omega_i), \]

where \( \alpha_{i2} \) is a function to be designed. For the Lyapunov function

\[ V_2 = V_1 + \frac{1}{2} \sum_{i=1}^{n} z_{i2}^2, \]

we have

\[ \dot{V}_2 = -\sum_{i=1}^{n} c_{i1} z_{i1}^2 - \sum_{i=1}^{n} \frac{oh_0}{M_i} z_{i2} z_{i3} + \sum_{i=1}^{n} z_{i2} \left[ z_{i1} + \left( \frac{D_i}{M_i} + \frac{\partial \alpha_{i1}}{\partial \Delta \delta_i} \right) \Delta \omega_i - \frac{oh_0}{M_i} \alpha_{i2} \right]. \]

If one defines
\[
\alpha_{i2}(\Delta \delta_i, \Delta \omega_i) = \frac{M_i}{\omega_0} [z_{i1} + c_{i2} z_{i2}]
- \left( \frac{D_i}{M_i} + \frac{\partial \alpha_{i1}}{\partial \Delta \delta_i} \right) \Delta \omega_i.
\]

or equivalently in terms of \( \Delta \delta_i \) and \( \Delta \omega_i \) as

\[
\alpha_{i2}(\Delta \delta_i, \Delta \omega_i) = \frac{M_i}{\omega_0} (1 + c_{i1} c_{i2}) \Delta \delta_i
+ \frac{M_i}{\omega_0} (c_{i1} + c_{i2} - \frac{D_i}{M_i}) \Delta \omega_i,
\]

where \( c_{i2} > 0 \) is a constant that can be arbitrarily selected, then

\[
V'_2 = \sum_{i=1}^{n} c_{i1} z_{i1}^2 - \sum_{i=1}^{n} c_{i2} z_{i2}^2 - \sum_{i=1}^{n} \frac{\omega_0}{M_i} z_{i2} z_{i3}
\]

and therefore (24) leads to

\[
\frac{\partial \alpha_{i2}}{\partial \Delta \delta_i} = \frac{M_i}{\omega_0} (1 + c_{i1} c_{i2}),
\]

\[
\frac{\partial \alpha_{i2}}{\partial \Delta \omega_i} = \frac{M_i}{\omega_0} (c_{i1} + c_{i2} - \frac{D_i}{M_i}).
\]

For the third error variable it holds true that

\[
\dot{z}_{i3} = \ddot{f}_i(t) + \frac{1}{T_{d0i}} I_{qi} u_i(t) - \frac{\partial \alpha_{i2}}{\partial \Delta \delta_i} \Delta \omega_i
- \frac{\partial \alpha_{i2}}{\partial \Delta \omega_i} \left[ \frac{D_i}{M_i} \Delta \omega_i - \frac{\omega_0}{M_i} \Delta P_{ei} \right].
\]

where

\[
\ddot{f}_i(t) = E'_{qi}(t) I_{qi}(t)
- \frac{1}{T_{d0i}} \left[ E'_{qi}(t) + (x_{di} - x_{di}^0) I_{di}(t) \right] I_{qi}(t)
\]

is obviously a complex nonlinear function. However, for decentralized control purposes this is an unknown function since it cannot be reconstructed from the local \( i \)-th machine’s variables. Considering now, the Lyapunov function

\[
V_3 = V_2 + \frac{1}{2} \sum_{i=1}^{n} z_{i3}^2,
\]

its time derivative results in

\[
V'_3 = -\sum_{i=1}^{n} c_{i1} z_{i1}^2 - \sum_{i=1}^{n} c_{i2} z_{i2}^2 + \sum_{i=1}^{n} z_{i3} \left[ \ddot{f}_i(t) \right]
- \frac{\partial \alpha_{i2}}{\partial \Delta \delta_i} \Delta \omega_i
+ \frac{\partial \alpha_{i2}}{\partial \Delta \delta_i} \left[ \frac{D_i}{M_i} \Delta \omega_i + \frac{\omega_0}{M_i} \Delta P_{ei} \right].
\]

Using on (27), the feedback linearization technique, we select the input

\[
u_i(t) = T_{d0i} \left( -c_{i3} z_{i3} + \frac{\partial \alpha_{i2}}{\partial \Delta \delta_i} \Delta \omega_i + v_i \right)
- \frac{\partial \alpha_{i2}}{\partial \Delta \omega_i} \left( \frac{D_i}{M_i} \Delta \omega_i + \frac{\omega_0}{M_i} \Delta P_{ei} \right),
\]

where \( c_{i3} > 0 \) is a constant that can be suitably chosen.

4. CONTROLLER DESIGN AND STABILITY ANALYSIS

The proposed excitation control law of each machine, for \( k_{i1} = 1 \) as it is resulted from (30) is

\[
E_{fi}(t) = \frac{T_{d0i}}{I_{qi}} \left( k_{i1} \Delta \delta_i + k_{i2} \Delta \omega_i - k_{i3} \Delta P_{ei} + v_i \right),
\]

where \( v_i = v_j(t) \) is an arbitrary external input and the constant gains are given by

\[
k_{i1} = \frac{M_i}{\omega_0} (1 + c_{i1} c_{i2}),
\]

\[
k_{i2} = \frac{M_i}{\omega_0} \left[ c_{i3} - \frac{D_i}{M_i} \right] (c_{i1} + c_{i2} - \frac{D_i}{M_i}) + c_{i1} c_{i2} + 1.
\]

\[
k_{i3} = c_{i1} + c_{i2} + c_{i3} - \frac{D_i}{M_i}.
\]

For this control law, (29) becomes

\[
V'_3 = -\sum_{i=1}^{n} c_{i1} z_{i1}^2 - \sum_{i=1}^{n} c_{i2} z_{i2}^2 + \sum_{i=1}^{n} c_{i3} z_{i3}^2
+ \sum_{i=1}^{n} z_{i3} \left[ \ddot{f}_i(t) - \frac{\omega_0}{M_i} z_{i2} + v_i(t) \right].
\]

and the \( z_{i3} \)-dynamics are as follows

\[
\dot{z}_{i3} = -c_{i3} z_{i3} + \ddot{f}_i(t) + v_i(t).
\]

Let the function

\[
f_{i1}(t) = \ddot{f}_i(t) - \frac{\omega_0}{M_i} z_{i2} = E'_{qi}(t) I_{qi}(t)
- \frac{1}{T_{d0i}} P_{ei} - \frac{x_{di} - x_{di}^0}{T_{d0i}} I_{di}(t) I_{qi}(t) - \frac{\omega_0}{M_i} z_{i2}.
\]

Since the electrical power \( P_{ei} \) and the reactive power \( Q_{ei} \) of each generator as well as the electrical power flow through its transmission line are all bounded and given that the excitation voltage \( E_{fi} \) may raise up to 5 times of the \( E_{qi} \) when there is no load in the system,
we can easily establish that there are large enough unknown positive constants $\sigma_i, \xi_{i1}, \xi_{i2}, \xi_{i3}$ such that

$$
|f_i(t)| \leq \sigma_i + 3 \sum_{j=1}^{3} \xi_{ij} |z_{ij}|, \quad i = 1,2,\ldots,n.
$$

(38)

Now using adaptive control techniques we derive estimates for these constants that can be effectively used in the design of the control law $v_i(t)$ in such a way that the third error variable $z_{i3}$ converges in finite-time in an arbitrary small neighborhood of the origin while all signals remain bounded. Specifically we will prove the following theorem.

**Theorem 1:** For the $n$-machine system described by equations (1)-(13) under the assumption of (38), let the excitation input be chosen as in (31) with gains given by (32)-(34) and $v_i$ given by the nonlinear expression

$$
v_i = -\frac{\rho_i^2 z_{i3}}{\rho_i z_{i3} + l_i},
$$

(39)

where $l_i$ is a small positive scalar and

$$
\rho_i = |z_{i3} - \lambda_i||z_{i3}| + \hat{\sigma}_i(t) + 3 \hat{\xi}_{ij}(t)|z_{ij}|
$$

(40)

with $\lambda_i$ a constant related to the adaptive control design through the following estimates' update laws

$$
\dot{\hat{\sigma}}_i(t) = \begin{cases} \alpha_i |z_{i3}|, & \text{if } |z_{i3}| > \frac{2L_i}{\lambda_i}, \\ 0, & \text{otherwise} \end{cases}
$$

(41)

$$
\dot{\hat{\xi}}_{ij}(t) = \begin{cases} \gamma_{ii} |z_{i3}^2|, & \text{if } |z_{i3}| > \frac{2L_i}{\lambda_i}, \\ 0, & \text{otherwise} \end{cases}
$$

(42)

with $\gamma_{ii} > 0, j = 1,2,3, i = 1,2,\ldots,n$.

Then, there exists $T_i > t_0$ such that

$$
|z_{i3}(t)| \leq \frac{2L_i}{\lambda_i} \quad \forall t \geq T_i, \quad i = 1,2,\ldots,n
$$

(43)

and the other two error variables $z_{i1}(t), z_{i2}(t)$ are uniformly bounded for $t \in [t_0, T_i]$ and the signals $\dot{\hat{\sigma}}_i(t), \dot{\hat{\xi}}_{i1}(t), \dot{\hat{\xi}}_{i2}(t), \dot{\hat{\xi}}_{i3}(t)$ are all uniformly bounded for $t \geq t_0$.

**Proof:** For the $i$-th subsystem consider the nonnegative function

$$
V_i = \frac{z_{i1}^2}{2} + \frac{z_{i2}^2}{2} + \frac{z_{i3}^2}{2} + \frac{3}{2} \sum_{j=1}^{3} \xi_{ij}^2(t) + \frac{3}{2} \sum_{j=1}^{3} \xi_{ij}^2(t) + \frac{3}{2} \sum_{j=1}^{3} \xi_{ij}^2(t).
$$

(44)

For an excitation input given by eqs. (31)-(34) and update laws given by (41)-(42), the time derivative of $V_i$ for the case wherein

$$
|z_{i3}| > \frac{2L_i}{\lambda_i}
$$

is as follows

$$
\dot{V}_i = -c_{i1}^2 z_{i1}^2 - c_{i2}^2 z_{i2}^2 - \lambda_i z_{i3}^2 + z_{i3}^3 [f_i(t) + v_i(t)] + \frac{3}{2} \sum_{j=1}^{3} \xi_{ij}^2(t) + \frac{3}{2} \sum_{j=1}^{3} \xi_{ij}^2(t) + \frac{3}{2} \sum_{j=1}^{3} \xi_{ij}^2(t).
$$

(46)

Therefore, the following inequality results

$$
\dot{V}_i \leq -c_{i1} z_{i1}^2 - c_{i2} z_{i2}^2 - \lambda_i z_{i3}^2 + z_{i3}^3 [f_i(t) + v_i(t)] + \frac{3}{2} \sum_{j=1}^{3} \xi_{ij}^2(t) + \frac{3}{2} \sum_{j=1}^{3} \xi_{ij}^2(t) + \frac{3}{2} \sum_{j=1}^{3} \xi_{ij}^2(t).
$$

(47)

From (38) one can easily see that the fourth term on the right-hand side of (47) is always nonpositive and therefore (47) provides the following simplified inequality

$$
\dot{V}_i \leq -c_{i1} z_{i1}^2 - c_{i2} z_{i2}^2 - \lambda_i z_{i3}^2 + \frac{3}{2} \sum_{j=1}^{3} \xi_{ij}^2(t) + \frac{3}{2} \sum_{j=1}^{3} \xi_{ij}^2(t) + \frac{3}{2} \sum_{j=1}^{3} \xi_{ij}^2(t).
$$

(48)

Selecting $v_i(t)$ as in (39) we arrive at

$$
\dot{V}_i \leq -\lambda_i z_{i3}^2 - \frac{\rho_i^2 z_{i3}^2}{\rho_i z_{i3} + l_i} + \frac{3}{2} \sum_{j=1}^{3} \xi_{ij}^2(t) + \frac{3}{2} \sum_{j=1}^{3} \xi_{ij}^2(t) + \frac{3}{2} \sum_{j=1}^{3} \xi_{ij}^2(t).
$$

(49)

However, since from (45) we have $-\lambda_i z_{i3}^2 < -2l_i$ we result in

$$
\dot{V}_i \leq -2l_i + l_i \rho_i z_{i3} + l_i \rho_i z_{i3} + l_i = -2l_i + l_i = -l_i.
$$

(50)

From (50) it is clear that there exists $T_i \leq t_0 + V_i(t_0)/l_i$ such that for every $t \geq T_i$ it holds true that $|z_{i3}(t)| \leq \sqrt{2l_i/\lambda_i}$. The uniform boundedness of $z_{i1}(t), z_{i2}(t), z_{i3}(t), \dot{\sigma}_i(t), \dot{\xi}_{ij}(t), j = 1,2,3$ in the time interval $[t_0, T_i]$ is obvious from the definition of
\( V_i \) in (44). From (41)-(42), it can be seen that for \( t > T_i \)
we have that \( \dot{\sigma}_j(t) = \dot{\sigma}_j(T_i) (i=1,2,\ldots,n) \) and
\( \dot{\tilde{z}}_{ij}(t) = \dot{\tilde{z}}_{ij}(T_i) \) \( (j=1,2,3, \ i=1,2,\ldots,n) \). Hence, the
estimates \( \dot{\sigma}_j(t), \dot{\tilde{z}}_{ij}(t) \) are uniformly bounded for all
\( t \geq t_0 \).

Next, we demonstrate that after the third error variable enters into the chosen neighborhood, the
other two error variables will also enter in finite-time in a sphere of arbitrary small radius. Actually, the
radius of the sphere is directly related to the length of the interval, which ultimately bounds the third error
variable. Thus, we prove the following Theorem.

**Theorem 2:** For the \( n \)-machine system defined by
(1)-(13) and the excitation input given by (31)-(34)
and (39)-(42) there exists \( T_{i1} \geq T_i, (i=1,2,\ldots,n) \) such that
for every \( t \geq T_{i1} \) the \( z_{i1}, z_{i2} \) error variables lie inside the sphere
\[
S = \left\{ (z_{i1}, z_{i2}) \mid z_{i1}^2 + z_{i2}^2 \leq \left( \frac{\omega_0}{M_i} \sqrt{\frac{l_i}{m_i c_{i1}^2 \lambda_i}} \right)^2 \right\}
\]
with center the origin and radius
\[
R_{i1} = \frac{\omega_0}{M_i} \min \left\{ c_{i1}, (1-\epsilon_i) c_{i2} \right\} \sqrt{\frac{l_i}{m_i c_{i1}^2 \lambda_i}},
\]
where \( \epsilon_i \) is a design constant with \( 0 < \epsilon_i < 1 \).

**Proof:** Let the nonnegative function
\[
V_{i1} = \frac{z_{i1}^2 + z_{i2}^2}{2}.
\]
The dynamics for the \( z_{i1}, z_{i2} \) variables are given by
\[
\begin{align*}
\dot{z}_{i1} &= z_{i2} - c_i z_{i1} \\
\dot{z}_{i2} &= -z_{i1} - c_{i2} z_{i2} - \frac{\omega_0}{M_i} z_{i3}
\end{align*}
\]
and the time derivative of \( V_{i1} \) for \( t \geq T_i \) is
\[
\dot{V}_{i1} = -c_{i1} z_{i1}^2 - c_{i2} z_{i2}^2 - \frac{\omega_0}{M_i} z_{i3} z_{i1}
\leq -c_{i1} z_{i1}^2 - c_{i2} z_{i2}^2 - \frac{\omega_0}{M_i} \left[ \sqrt{\frac{2l_i}{\lambda_i}} \right] z_{i2}
\leq -c_{i1} z_{i1}^2 - c_{i2} (1-\epsilon_i) z_{i2}^2
- \epsilon_i c_{i2} \left[ z_{i2}^2 - \frac{\omega_0}{2 c_{i2} M_i} \left[ \sqrt{\frac{2l_i}{\lambda_i}} \right]^2 \right] + \frac{l_i \omega_0^2}{2 c_{i2} \lambda_i M_i^2}.
\]
Defining \( m_i = \min \{ c_{i1}, (1-\epsilon_i) c_{i2} \} \) we have that
\[
\dot{V}_{i1} \leq -m_i \left( z_{i1}^2 + z_{i2}^2 \right) + \frac{l_i \omega_0^2}{2 c_{i2} \lambda_i M_i^2}.
\]
Using the comparison principle [25] we have
\[
\dot{V}_{i1} \leq -2 m_i \left( V_i - \frac{l_i \omega_0^2}{4 m_i c_{i2} \lambda_i M_i^2} \right),
\]
which implies
\[
V_i(t) \leq V_i(T_i) e^{-2m_i (t-T_i)} + \frac{l_i \omega_0^2}{4 m_i c_{i2} \lambda_i M_i^2}.
\]
Therefore, there exists a \( T_{i1} \)
\[
T_{i1} = \max \left\{ T_i, T_i + \frac{1}{m_i} \ln \left( \frac{V_i(T_i)}{\frac{l_i \omega_0^2}{2 M_i} \sqrt{\frac{1}{m_i c_{i1}^2 \lambda_i}}} \right) \right\}
\]
so that for \( t \geq T_{i1} \) it holds true that
\[
V_i(t) \leq \frac{l_i \omega_0^2}{2 m_i c_{i2} \lambda_i M_i^2},
\]
i.e. the error variables \( z_{i1}, z_{i2} \) enter in finite-time inside the sphere
\[
S = \left\{ (z_{i1}, z_{i2}) \mid z_{i1}^2 + z_{i2}^2 \leq \left( \frac{\omega_0}{M_i} \sqrt{\frac{l_i}{m_i c_{i1}^2 \lambda_i}} \right)^2 \right\}
\]
with the center as the origin and radius
\[
R_{i1} = \frac{\omega_0}{M_i} \min \left\{ c_{i1}, (1-\epsilon_i) c_{i2} \right\} \sqrt{\frac{l_i}{m_i c_{i1}^2 \lambda_i}}.
\]

Apparantly, Theorem 2 directly gives a bound for \( \Delta \tilde{\sigma}_i = z_{i1} \)
\[
\left| \Delta \tilde{\sigma}_i(t) \right| \leq \frac{\omega_0}{M_i} \sqrt{\frac{l_i}{m_i c_{i1} c_{i2} \lambda_i} \lambda_i} \ \forall t \geq T_i.
\]
However, as \( t \to \infty \), Theorem 2 results in a tighter bound for \( \lim_{t \to \infty} \left| \Delta \tilde{\sigma}_i(t) \right| \) as shown by the following Corollary.

**Corollary 1:** For the \( n \)-machine system defined by
(1)-(13) and the excitation input given by (31)-(34)
and (39)-(42) the following bounds on the angle deviations hold true
\[
\lim_{t \to \infty} |\Delta \delta_i(t)| \leq R_{12} := \frac{\omega_0}{M_i c_{11}} \sqrt{\frac{l_i}{m_i e_i c_{12}^2 \lambda_i}}.
\]

**Proof:** Since \( z_{12} = \Delta \dot{\delta}_i + c_{11} \Delta \delta_i \) Theorem 2 gives

\[
|z_{12}(t)| \leq \frac{\omega_0}{M_i} \sqrt{\frac{l_i}{m_i e_i c_{12}^2 \lambda_i}} \quad \forall t \geq T_i.
\]

Therefore

\[
\Delta \dot{\delta}_i(t) + c_{11} \Delta \delta_i(t) \leq \frac{\omega_0}{M_i c_{11}} \sqrt{\frac{l_i}{m_i e_i c_{12}^2 \lambda_i}} \quad \forall t \geq T_i
\]

and

\[
\Delta \dot{\delta}_i(t) + c_{11} \Delta \delta_i(t) \geq -\frac{\omega_0}{M_i c_{11}} \sqrt{\frac{l_i}{m_i e_i c_{12}^2 \lambda_i}} \quad \forall t \geq T_i.
\]

Now using Lemma Growan-Bellman [26] we arrive at

\[
\Delta \dot{\delta}_i(T_i) e^{-c_{11}(t-T_i)} - \frac{\omega_0}{M_i c_{11}} \sqrt{\frac{l_i}{m_i e_i c_{12}^2 \lambda_i}} (1-e^{-c_{11}(t-T_i)}) \leq \Delta \delta_i(t)
\]

\[
\leq \Delta \dot{\delta}_i(T_i) e^{-c_{11}(t-T_i)} + \frac{\omega_0}{M_i c_{11}} \sqrt{\frac{l_i}{m_i e_i c_{12}^2 \lambda_i}} (1-e^{-c_{11}(t-T_i)}) \quad \forall t \geq T_i
\]

and taking the limit as \( t \to \infty \) the following bound occurs

\[
\lim_{t \to \infty} |\Delta \delta_i(t)| \leq \frac{\omega_0}{M_i c_{11}} \sqrt{\frac{l_i}{m_i e_i c_{12}^2 \lambda_i}}.
\]

**Remark 1:** Note that for \( \epsilon_i = 1/2 \) and \( c_{11} > c_{12}/2 \) we have the simple bound form

\[
\lim_{t \to \infty} |\Delta \delta_i(t)| \leq \frac{2 \omega_0}{M_i c_{11} c_{12}} \sqrt{\frac{l_i}{\lambda_i}}.
\]

(66)

However, an even superior bound for \( \lim_{t \to \infty} |\Delta \delta_i(t)| \) can be obtained. To this end, the following theorem is proved.

**Theorem 3:** For the \( n \)-machine system defined by (1)-(13) and the excitation input given by (31)-(34) and (39)-(42), the following bounds on the angle deviations hold true

\[
\lim_{t \to \infty} |\Delta \delta_i(t)| \leq R_i := \frac{\omega_0}{M_i (1 + c_{11} c_{12})} \sqrt{\frac{2 l_i}{\lambda_i}}.
\]

(67)

**Proof:** As \( t \to \infty \) the system tends to its steady state wherein

\[
\lim_{t \to \infty} \Delta \dot{\delta}_i = \lim_{t \to \infty} \Delta \dot{\omega}_i = 0.
\]

From (1) and (2) one can directly obtain

\[
\lim_{t \to \infty} \Delta \omega_i(t) = \lim_{t \to \infty} \Delta P_{ei}(t) = 0
\]

and therefore from (21) and (24) we have for \( z_{13} \)

\[
\lim_{t \to \infty} z_{13}(t) = -\frac{M_i}{\omega_0} (1 + c_{11} c_{12}) \lim_{t \to \infty} \Delta \delta_i(t).
\]

Finally, using (43) we obtain the bound for \( |\Delta \delta_i| \) as \( t \to \infty \)

\[
\lim_{t \to \infty} |\Delta \delta_i(t)| \leq R_i := \frac{\omega_0}{M_i (1 + c_{11} c_{12})} \sqrt{\frac{2 l_i}{\lambda_i}}.
\]

**Remark 2:** Using the inequality \( 1/(1+x) < 1/x < \sqrt{2}/x \ \forall x > 0 \), the bounds \( R_{11}, R_{12}, R_i \) given by (62), (66) and (67) respectively, are proved to be of decreasing order for the common case where \( c_{11} > 1 \), i.e. \( R_{11} > R_{12} > R_i \).

**Remark 3:** Comparing the bounds given by Theorems 2 and 3 it can be easily seen that if the bound given by Theorem 3 is used, the design results in smaller values for \( c_{11}, c_{12} \) and consequently in smaller values of the control gains \( k_{11}, k_{12}, k_{13} \).

**Remark 4:** The positive scalars \( c_{11}, c_{12} \) and \( c_{13} \) can be arbitrarily selected with reasonable limits coming from inequality (67) in accordance to desirable \( \lim_{t \to \infty} |\Delta \delta_i|, l_i \) and \( \lambda_i \).

### 5. CASE STUDY

The two generator infinite bus power system shown in Fig. 1, is used to demonstrate the efficiency of the proposed controller.

The system parameters are as follows:

- \( x_{11} = 0.129 \ \text{p.u.} \), \( x_{12} = 0.11 \ \text{p.u.} \), \( x_{13} = 0.55 \ \text{p.u.} \), \( x_{14} = 0.53 \ \text{p.u.} \), \( x_{15} = 0.6 \ \text{p.u.} \), \( T_{d101} = 6.9 \ \text{sec} \)
- \( x_{21} = 1.863 \ \text{p.u.} \), \( x_{22} = 0.257 \ \text{p.u.} \), \( D_1 = 5.0 \ \text{p.u.} \), \( M_1 = 8.0 \ \text{sec} \), \( M_2 = 10.2 \ \text{sec} \), \( D_2 = 3.0 \ \text{p.u.} \)
- \( x_{23} = 2.36 \ \text{p.u.} \), \( x_{24} = 0.319 \ \text{p.u.} \), \( T_{d201} = 7.96 \ \text{sec} \), \( k_{d1} = 1.0 \ \text{p.u.} \), \( k_{d2} = 1.0 \ \text{p.u.} \)

In the simulations, for a more accurate evaluation of the proposed controller, we take into account the physical limits of the excitation voltage, which are considered to be:

\[
|k_{ei} u_{f1}| \leq 5.0 \ \text{p.u.}, \quad |k_{ei} u_{f2}| \leq 5.0 \ \text{p.u.}
\]

The following case is simulated.

Fig. 1. Two machine infinite bus test system.
Fig. 2. Power angle deviations (in deg) and its calculated bounds of machine #1.

Fig. 3. Power angle deviations (in deg) and its calculated bounds of machine #2.

Fig. 4. Nominal speed deviation of machine #1.

Fig. 5. Nominal speed deviation of machine #2.

Fig. 6. Excitation voltage of machine #1.

Fig. 7. Excitation voltage of machine #2.

Fig. 8. Estimated parameters of machine #1.

Fig. 9. Estimated parameters of machine #2.
6. CONCLUSIONS

The proposed controller is completely decentralized with a rather simple structure

$$E_{f}(t) = \frac{T_{d0}}{T_{q1}} \left( K_{11} \Delta \delta_{i} + K_{12} \Delta \omega_{i} - K_{13} \Delta P_{ei} - v_{i} \right),$$

where the nonlinear input term $v_{i}$ is given by

$$v_{i} = -\frac{\rho_{i}^{2} z_{i3}}{\rho_{i} |z_{i3}| + I_{i}} \quad \text{(see Theorem 1)}$$

and constant gains given by (32)-(34).

The proposed control scheme involves only local measurements of $P_{ei}, \omega_{i}, \delta_{i}$ and the current $I_{qi}$ that can be calculated from the measurements.

As it is shown by an extensive analysis this control scheme ensures stability while it permits the selection of the control parameters in such a way that a trade-off between the gain values and the region width $R_{i1}$ is obtained.

APPENDIX

The nomenclature used is as follows.

- $\delta_{i1}$: power angle, in radian;
- $\omega_{i}$: rotor speed, in rad/sec;
- $\omega$: synchronous machine speed, in rad/sec;
- $P_{m}$: mechanical input power, in p.u;
- $P_{ei}$: active electrical power, in p.u.;
- $D_{i}$: damping constant, in p.u.;
- $M_{i}$: inertia coefficient, in seconds;
- $f_{i}$: transient EMF in the q-axis in p.u.;
- $E_{qf}$: equivalent EMF in excitation coil, in p.u.;
- $I_{di}$: d-axis transient short circuit time constant, in seconds;
- $I_{qi}$: excitation current, in p.u.;
- $I_{d1}$: d-axis current, in p.u.;
- $Q_{ei}$: reactive electrical power, in p.u.;
- $V_{di}$: generator terminal voltage, in p.u.;
- $k_{ei}$: gain of generator excitation amplifier, in p.u.;
- $u_{fi}$: input of the SCR amplifier, in p.u.;
- $x_{di}$: d-axis transient reactance, in p.u.;
- $x_{qi}$: q-axis reactance, in p.u.;
- $x_{ad}$: mutual reactance between the excitation coil and the stator coil, in p.u.;
- $Y_{ij} = G_{ij} + jB_{ij}$: the $i$th row and $j$th column element of nodal admittance matrix, in p.u.;
- $\Delta \delta_{i} = \delta_{i} - \delta_{i0}$;
- $\Delta \omega_{i} = \omega_{i} - \omega_{0}$;
- $\Delta P_{ei} = P_{ei} - P_{mi}$.

REFERENCES


Haris E. Psillakis received his Diploma in Electrical & Computer Engineering from the University of Patras, Greece, in 2000. Currently, he is working on his Ph.D. dissertation. His research interests include uncertain and stochastic systems, passive and nonlinear systems, and advanced control and stability applications on power and drive systems. Mr. Psillakis is a member of the National Technical Chamber of Greece.

Antonio T. Alexandridis received his Diploma in Electrical Engineering from the University of Patras, Greece, in 1981. In 1987, he received his Ph.D. degree from the Electrical & Computer Engineering Department of the W. Virginia University, USA. In 1988, he joined the Department of Electrical & Computer Engineering of the University of Patras where he is now an Associate Professor. During the last semester of 1998 he joined the Control Engineering Research Centre of the City University in London, UK, as a Visiting Researcher. His research interests include optimal control, eigenstructure assignment, passive and nonlinear systems, advanced control and stability applications on power and drive systems. Dr. Alexandridis is a member of IEEE and a member of the National Technical Chamber of Greece.