

Global Finite-Time Convergence of TCP Vegas without Feedback Information Delay

Joon-Young Choi, Kyungmo Koo, Jin S. Lee, and Steven H. Low

Abstract: We prove that TCP Vegas globally converges to its equilibrium point in finite time assuming no feedback information delay. We analyze a continuous-time TCP Vegas model with discontinuity and high nonlinearity. Using the upper right-hand derivative and applying the comparison lemma, we cope with the discontinuous signum function in the TCP Vegas model; using a change of state variables, we deal with the high nonlinearity. Although we ignore feedback information delay in analyzing the model of TCP Vegas, the simulation results illustrate that TCP Vegas in the presence of feedback information delay shows very similar dynamic trends to TCP Vegas without feedback information delay. Consequently, dynamic properties of TCP Vegas without feedback information delay can be used to estimate those of TCP Vegas in the presence of feedback information delay.

Keywords: Comparison lemma, congestion control, finite-time convergence, TCP Vegas, nonlinear systems.

1. INTRODUCTION

TCP Vegas was introduced as a TCP implementation modified from TCP Reno in [1], and extensive experiments have been conducted to compare the performance of Vegas with that of Reno [1-4]. TCP Vegas was systematically analyzed and presented as a model of discrete-time dynamic system without feedback information delay in [5]. The TCP Vegas model is discontinuous and highly nonlinear, which has caused a theoretical difficulty in analyzing the dynamic properties of TCP Vegas. A sufficient condition for local asymptotic stability of TCP Vegas was presented in the presence of feedback information delay in [6], and a modified version of TCP Vegas was proposed to improve the local stability condition by

adding a derivative term of queuing delay to the original TCP Vegas. The work in [6], however, was based on an approximated model assuming the time varying RTT(round trip time) as a constant value and ignoring the nonlinear and discontinuous properties of TCP Vegas dynamics, and showed only local stability results.

Global stability issues have been studied in other congestion control schemes and some initial results have been reported recently in [7-12] under various assumptions and limitations. The global asymptotic stability of the multi-link multi-source network with TCP Vegas sources was proved without feedback information delay in [13], but they used approximated models to avoid the theoretical difficulties associated with the original discontinuous TCP Vegas.

In this paper, we consider the multi-link multi-source network with TCP Vegas sources without feedback information delay. We rigorously analyze a continuous-time model of TCP Vegas with dynamics both in sources and in links without any approximation of the original TCP Vegas model. The continuous-time model has high nonlinearity and discontinuity due to a signum function in the right-hand side of the model equation. To deal with the discontinuous signum function in TCP Vegas model equations, we adopt the upper-right hand derivative and apply the comparison lemma [14]. In addition, to deal with the high nonlinearity in the TCP Vegas model, we make a change of state variables. We prove that assuming no feedback information delay, TCP Vegas globally converges to its equilibrium point in finite time without any other conditions.

Manuscript received August 24, 2005; revised August 16, 2006; accepted September 15, 2006. Recommended by Editorial Board member Dong-Ho Cho under the direction of past Editor Keum-Shik Hong. This work was supported by IT Scholarship Program supervised by IITA (Institute for Information Technology Advancement) & MIC (Ministry of Information and Communication), Korea.

Joon-Young Choi is with the Department of Electronic Engineering, Pusan National University, 30 Jangjeon-dong, Geumjeong-gu, Busan 609-735, Korea (e-mail: jyc@pusan.ac.kr).

Kyungmo Koo and Jin S. Lee are with the Department of Electrical Engineering, Pohang University of Science and Technology, San 31, Hyoja-Dong, Pohang 790-784, Korea (e-mails: {pumpkins, jsoo}@postech.ac.kr).

Steven H. Low is with the Departments of Computer Science and Electrical Engineering, California Institute of Technology, Pasadena CA 91125 USA (e-mail: slow@caltech.edu).

Although we ignore feedback information delay in analyzing the model of TCP Vegas, the simulation results illustrate that TCP Vegas in the presence of feedback information delay shows very similar dynamic trends to TCP Vegas without feedback information delay. Consequently, the analysis results on TCP Vegas without considering feedback information delay are of significance in practice, and can be used to analyze and estimate the stability and performance of TCP Vegas in the presence of feedback information delay.

This paper is organized as follows. Section 2 describes the multi-link multi-source network model of TCP Vegas. Section 3 analyzes the boundedness property of TCP Vegas. Section 4 proves the global finite-time convergence of TCP Vegas. Section 5 provides simulation results and discussions. Section 6 makes conclusions.

2. NETWORK MODEL

We consider a network of L communication links shared by N sources, where $L \leq N$. Each link indexed by l has a finite transmission capacity c_l ($1 \leq l \leq L$) and is assumed to have infinite buffering storage. Each source is indexed by i ($1 \leq i \leq N$). The $L \times N$ routing matrix R is defined by its (l, i) elements:

$$R_{li} = \begin{cases} 1 & \text{if source } i \text{ uses link } l \\ 0 & \text{otherwise.} \end{cases}$$

The matrix R is assumed to be fixed and of full row rank to prevent the network from degenerating. Associated with the link l is the queuing delay $p_l(t)$ and with the source i is the source rate $x_i(t)$. Ignoring the feedback information delays in the interconnection, we assume at time t that the source i observes as a feedback signal the aggregated queuing delay in its path

$$q_i(t) := \sum_l R_{li} p_l(t) = R_i^T p(t) \quad \forall 1 \leq i \leq N, \quad (1)$$

where R_i is the i -th column of R and $p^T = [p_1, p_2, \dots, p_L]$, and link l observes the aggregated source rate

$$y_l(t) := \sum_i R_{li} x_i(t) = R^l x(t) \quad \forall 1 \leq l \leq L, \quad (2)$$

where R^l is the l -th row of R and $x^T = [x_1, x_2, \dots, x_N]$. The RTT $T_i(t)$ is defined for each source i as $T_i(t) = d_i + q_i(t)$, where d_i is the constant round trip propagation time.

A model of TCP Vegas with its associated queue management was presented as a discrete-time dynamic system in [5]:

$$p_l(t+1) = \left[p_l(t) + \frac{1}{c_l} \left(\sum_i R_{li} x_i(t) - c_l \right) \right]^+, \quad (3)$$

$$w_i(t+1) = \left[w_i(t) + \frac{1}{T_i(t)} \text{sgn}(\alpha_i d_i - x_i(t) q_i(t)) \right]^+, \quad (4)$$

where $x_i(t) := \frac{w_i(t)}{T_i(t)}$, $w_i(t)$ is the window size of source i , α_i is the congestion control parameter of Vegas algorithm, $[z]^+ = \max\{0, z\}$, and $\text{sgn}(z) = -1$ if $z < 0$, 0 if $z = 0$, 1 if $z > 0$. As in [5], the congestion control parameters α_i and β_i of Vegas algorithm are assumed to be $\alpha_i = \beta_i$ for simplicity. Using the Euler's method, the discrete-time model of TCP Vegas (3) and (4) is transformed to a continuous-time dynamic system:

$$\dot{p}_l = \frac{1}{c_l} \left(\sum_i R_{li} x_i(t) - c_l \right)_{p_l}^+, \quad (5)$$

$$\dot{w}_i = \frac{1}{T_i(t)} \text{sgn}(\alpha_i d_i - x_i(t) q_i(t)), \quad (6)$$

where

$$(h)_z^+ := \begin{cases} h & \text{if } z > 0 \\ \max(0, h) & \text{if } z = 0. \end{cases}$$

The corresponding equilibrium point (x_i^*, p_l^*) of (5) and (6) satisfies

$$\begin{cases} x_i^* q_i^* = \alpha_i d_i & \forall 1 \leq i \leq N \\ \left(\sum_i R_{li} x_i^* - c_l \right)_{p_l}^+ = 0 & \forall 1 \leq l \leq L \end{cases} \quad (7)$$

and the equilibrium point turns out to be unique as shown in the following lemma.

Lemma 1: The equilibrium point of TCP Vegas described by (5) and (6) is unique.

Proof: See [13]. \square

To facilitate the analysis, making the change of variables in (5) and (6) as

$$\begin{aligned} \tilde{x}_i &= x_i - x_i^*, \\ s_i &= \frac{1}{w_i} - \frac{q_i}{\alpha_i d_i T_i}, \end{aligned} \quad (8)$$

and using the fact that $\text{sgn}(\alpha_i d_i - x_i q_i) = \text{sgn}(s_i)$, we

obtain the resulting system with the new variables \tilde{x}_i and s_i :

$$\dot{\tilde{x}}_i = -\frac{\dot{q}_i(t)}{T_i(t)}x_i(t) + \frac{1}{T_i^2(t)}\text{sgn}(s_i(t)), \quad (9)$$

$$\dot{s}_i = -\frac{1}{w_i^2(t)T_i(t)}\text{sgn}(s_i(t)) - \frac{\dot{q}_i(t)}{\alpha_i T_i^2(t)}. \quad (10)$$

Before analyzing the stability of (9) and (10), it is necessary to consider the existence of solution of the initial-value problem (9) and (10). Since the right-hand sides of (9) and (10) don't satisfy the Lipschitz condition with respect to state variables because of the discontinuous signum function, the existence of solution is not guaranteed in the standard sense [14]. Fortunately, however, the discontinuity comes with only finite jump at $s_i=0$, and it holds that the solution of (9) and (10) exists for any initial condition in the sense of Filippov [15]. As another property of the solution of TCP Vegas model, the following lemma shows the absolute continuity of the solution and this property will be used in the subsequent analysis.

Lemma 2: $p_l(t)$, $w_i(t)$, $\tilde{x}_i(t)$, and $s_i(t)$ developed by (5), (6), (9), and (10) respectively are absolutely continuous with respect to time t .

Proof: This lemma holds from Theorem 1 of Chapter 0 in [16]. \square

3. BOUNDEDNESS

In this section, we investigate the boundedness properties of $x_i(t)$, $w_i(t)$, $q_i(t)$, $s_i(t)$, and $p_l(t)$ with respect to time t . First, we assume that the initial window size $w_i(0) > 0$ for all $1 \leq i \leq N$, which is justified by the slow-start phase [1], during which w_i keeps increasing until either the packet is lost somewhere in the network or when some congestion feedback signal arrives from the network. When that happens, the congestion control phase takes over, at which instant the current $w_i > 0$ is set to $w_i(0)$ for the TCP Vegas. We now show in the following lemma that $w_i(t)$ for all $1 \leq i \leq N$ and $p_l(t)$ for all $1 \leq l \leq L$ are bounded below.

Lemma 3:

(i) $w_i(t)$ is bounded below for all $t \geq 0$ and for all $1 \leq i \leq N$ as

$$w_i(t) \geq w_i^{LB} := \min(w_i(0), \alpha_i d_i) > 0. \quad (11)$$

(ii) $p_l(t) \geq 0$ for all $t \geq 0$ and for all $1 \leq l \leq L$.

Proof:

(i) Whenever $0 < w_i < \alpha_i d_i$, it holds that $s_i = \frac{1}{w_i}$

$$\left(1 - \frac{q_i w_i}{\alpha_i d_i T_i}\right) > 0 \quad \text{and} \quad \dot{w}_i = \frac{1}{T_i} \text{sgn}(s_i) > 0, \quad \text{from}$$

which we have $w_i \geq \min(w_i(0), \alpha_i d_i) > 0$ for all $t \geq 0$ and for all $1 \leq i \leq N$.

(ii) Since $\dot{p}_l = \left[\frac{1}{c_l}(y_l - c_l)\right]_{-p_l}^+$ and $p_l(0) \geq 0$ for

all $1 \leq l \leq L$, it is obvious that $p_l(t) \geq 0$ for all $t \geq 0$ and for all $1 \leq l \leq L$. \square

We now show in the following that $x_i(t)$, $w_i(t)$, $q_i(t)$, and $s_i(t)$ are bounded above for all $1 \leq i \leq N$, and $p_l(t)$ is bounded above for all $1 \leq l \leq L$.

Lemma 4:

(i) $x_i(t)$ is bounded above for all $t \geq 0$ and for all $1 \leq i \leq N$ as

$$x_i(t) \leq w_i^{UB} := \max\left(x_i(0), 2 \max_{l \in R_i^T} c_l, \frac{1}{d_i}\right).$$

(ii) $w_i(t)$ is bounded above for all $t \geq 0$ and for all $1 \leq i \leq N$ as

$$w_i(t) \leq w_i^{UB} := \max\left(w_i(0), x_i^{UB}(d_i + 1), \alpha_i d_i(d_i + 1)\right).$$

(iii) $q_i(t)$ is bounded above for all $t \geq 0$ and for all $1 \leq i \leq N$ as

$$q_i(t) \leq q_i^{UB} := \max\left(q_i(0), \frac{\max_{l \in R_i^T} |R^l|}{\min_{l \in R_i^T} c_l} \max_{i \in R^l} w_i^{UB}\right).$$

(iv) $s_i(t)$ is bounded above for all $t \geq 0$ and for all $1 \leq i \leq N$ as

$$s_i(t) \leq s_i^{UB} := \frac{1}{w_i^{LB}} + \frac{1}{\alpha_i d_i}.$$

(v) $p_l(t)$ is bounded above for all $t \geq 0$ and for all $1 \leq l \leq L$ as

$$p_l(t) \leq p_l^{UB} := \max\left(p_l(0), \frac{|R^l|}{c_l} \max_{i \in R^l} w_i^{UB}\right),$$

where $l \in R_i^T$ means that link l belongs to the links that the source i passes through, and $i \in R^l$ means that the source i belongs to the sources that passes through the link l .

Proof: See Appendix A. \square

4. CONVERGENCE

In this section, we investigate the global finite-time convergence of TCP Vegas described by (9) and (10). To deal with the discontinuity in (9) and (10), we introduce the upper right-hand derivative $D^+V(t)$ defined by

$$D^+V(t) := \limsup_{h \rightarrow 0^+} \frac{V(t+h) - V(t)}{h},$$

and the comparison lemma [14] stated as follows.

Lemma 5: Consider the scalar differential equation

$$\dot{u} = f(t, u), \quad u(t_0) = u_0,$$

where $f(t, u)$ is continuous in t and locally Lipschitz in u , for all $t \geq 0$ and all $u \in J \subset \mathfrak{R}$. Let $[t_0, T)$ (T could be infinity) be the maximal interval of existence of the solution $u(t)$, and suppose $u(t) \in J$ for all $t \in [t_0, T)$. Let $v(t)$ be a continuous function whose upper right-hand derivative $D^+v(t)$ satisfies the differential inequality

$$D^+v(t) \leq f(t, v(t)), \quad v(t_0) \leq u_0$$

with $v(t) \in J$ for all $t \in [t_0, T)$. Then, $v(t) \leq u(t)$ for all $t \in [t_0, T)$.

The following theorem shows that TCP Vegas described by (9) and (10) is globally finite-time convergent, and $x_i(t)$ and $p_l(t)$ converge to their equilibrium points in finite time with any initial condition.

Theorem 1: TCP Vegas (9) and (10) is globally finite-time convergent, that is, $x_i(t)$ for all $1 \leq i \leq N$ and $p_l(t)$ for all $1 \leq l \leq L$ converge to their equilibrium points in finite time.

Proof: We consider a Lyapunov function candidate

$$V(s, \tilde{x}) = \sum_{i=1}^N \alpha_i |s_i| + \frac{1}{2} \sum_{l=1}^L \frac{1}{c_l} (R^l \tilde{x})^2, \quad (12)$$

which is absolutely continuous from Lemma 2 and bounded by Lemma 4 for all $t \geq 0$. Note that the continuity of (12) with respect to time t is a necessary condition to apply the comparison lemma and we can choose the set $J \subset \mathfrak{R}$ in the comparison lemma such that $V(t) \in J$ for all $t \geq 0$ because of the boundedness of $V(t)$. Equation (12) is not differentiable with the standard derivative when $s_i = 0$, but even when $s_i = 0$, we can calculate the upper right-hand derivative of (12) along the trajectories of (9) and (10):

$$\begin{aligned} D^+V(s, \tilde{x}) &= \sum_{i=1}^N \left(\alpha_i \frac{s_i \dot{s}_i}{|s_i|} \right) + \sum_{l=1}^L \frac{1}{c_l} (R^l \tilde{x}) (R^l \dot{\tilde{x}}) \\ &= - \sum_{i=1}^N \left(\frac{\alpha_i}{w_i^2 T_i} + \frac{\dot{q}_i}{T_i^2} \operatorname{sgn}(s_i) \right) + \sum_{l=1}^L \dot{p}_l (R^l \tilde{x}) \\ &= - \sum_{i=1}^N \left(\frac{\alpha_i}{w_i^2 T_i} + \frac{\dot{q}_i}{T_i^2} \operatorname{sgn}(s_i) \right) + \sum_{l=1}^L \left(\sum_{i=1}^N R_{li} \dot{p}_l \right) \tilde{x}_i \\ &= - \sum_{i=1}^N \left(\frac{\alpha_i}{w_i^2 T_i} + \frac{\dot{q}_i}{T_i^2} \operatorname{sgn}(s_i) \right) + \sum_{i=1}^N \dot{q}_i \tilde{x}_i \\ &= - \sum_{i=1}^N \left(\frac{\alpha_i}{w_i^2 T_i} + \frac{\dot{q}_i}{T_i^2} \operatorname{sgn}(s_i) \right) \\ &\quad + \sum_{i=1}^N \dot{q}_i \left(-\frac{\dot{q}_i}{T_i} x_i + \frac{1}{T_i^2} \operatorname{sgn}(s_i) \right) \\ &= - \sum_{i=1}^N \left(\frac{\alpha_i}{w_i^2 T_i} \right) - \sum_{i=1}^N \left(-\frac{x_i}{T_i} \dot{q}_i^2 \right) \\ &= - \sum_{i=1}^N \left(\frac{\alpha_i}{w_i^2 T_i} \right) - \sum_{i=1}^N \frac{x_i}{T_i} (R_i^T \dot{p})^2 \\ &= - \sum_{i=1}^N \left(\frac{\alpha_i}{w_i^2 T_i} \right) - \dot{p}^T \left(\sum_{i=1}^N \frac{x_i}{T_i} R_i R_i^T \right) \dot{p} \\ &\leq - \sum_{i=1}^N \left(\frac{\alpha_i}{w_i^2 T_i} \right), \end{aligned} \quad (13)$$

where the last inequality is derived by using $\dot{p}^T \left(\sum_{i=1}^N \frac{x_i}{T_i} R_i R_i^T \right) \dot{p} \geq 0$. On the other hand, since $w_i(t) \leq w_i^{UB}$ and $q_i(t) \leq q_i^{UB}$ by Lemma 4, we have the inequality

$$\sum_{i=1}^N \left(\frac{\alpha_i}{w_i^2 T_i} \right) \geq \sum_{i=1}^N \left(\frac{\alpha_i}{w_i^{UB^2} (q_i^{UB} + d_i)} \right) := \varepsilon > 0 \quad (14)$$

from which we obtain the differential inequality

$$D^+V(s, \tilde{x}) \leq -\varepsilon.$$

Applying the comparison lemma, we achieve that

$$V(s(t), \tilde{x}(t)) \leq V(s(0), \tilde{x}(0)) - \varepsilon t, \quad (15)$$

which implies that $V(s, \tilde{x})$ reaches zero before or at time $t = \frac{V(s(0), \tilde{x}(0))}{\varepsilon}$ [14,p.553]. $V(s, \tilde{x}) = 0$ implies a set of equations described with the variables x_i

and $q_i : s_i = 0$, that is, $x_i q_i = \alpha_i d_i$ for $1 \leq i \leq N$ and $R^l \tilde{x} = \sum_i R_{li} x_i - c_l = 0$ for $1 \leq l \leq L$, which are identical to (7). Consequently, it is proved by Lemma 1 that $x_i(t)$ for $1 \leq i \leq N$ and $p_l(t)$ for $1 \leq l \leq L$ converge to their equilibrium points in finite time. \square

It is worth while to take a close look at how the congestion control parameter a_i affects the performance of TCP Vegas. It is shown from (7) that the equilibrium points of w_i and $T_i (= d_i + q_i)$ in (13) increase as a_i increases, which implies that the

terms $\frac{\alpha_i}{w_i^2 T_i}$ and $\frac{x_i}{T_i} \left(= \frac{w_i}{T_i^2} \right)$ in (13) decrease in the

steady state as a_i increases. Therefore, the sizes of negative terms in (13) decrease as a_i increases, and it is concluded that the convergent time of TCP Vegas lengthens as a_i increases, which will be illustrated in the simulation section. Note, however, that an extremely small a_i may cause sensitivity to the measurement noise because the queuing delay, the feedback information to be measured at each source, has the equilibrium point proportional to a_i .

5. SIMULATION

In this section, we present the simulation results to illustrate the global finite-time convergence of TCP Vegas by using both MATLAB and *ns-2* network simulator. We simulate TCP Vegas described by (9) and (10) without feedback information delay using MATLAB, and simulate TCP Vegas in the presence of feedback information delay with *ns-2* simulator.

We conduct the simulation with two sets of scenarios, denoted by Scenario 1 and 2. Scenario 1 and 2 simulate a multi-link multi-source network that consists of two links and four heterogeneous sources as shown in Fig. 1. Each source transfers FTP packets to its counterpart with a packet size of 1KB. Link 1 is shared by source 1, 2, and 3 and has a capacity of 50pkts/ms; link 2 is shared by source 1 and 4 and has a capacity of 30pkts/ms. Source 1 sends packets through both link 1 and 2, and has a round trip latency of 50ms. The round trip latencies of source 2, 3, and 4 are 10ms, 20ms, and 20ms, respectively. The routers maintain a FIFO queue with a capacity of 20000pkts. We set the congestion control parameter as follows: $\alpha=3$ pkts/RTT in Scenario 1, $\alpha=6$ pkts/RTT in Scenario 2, and $\alpha=\beta$ in *ns-2*.

Table 1 and 2 summarize the calculated equilibrium point of each source for each scenario. Figs. 2 and 3 show the simulation results of Scenario 1 and 2, respectively, where the solid lines indicate the *ns-2* results and the broken lines indicate the MATLAB results. The results from MATLAB (broken lines) in Figs. 2 and 3 show that TCP Vegas modeled by (9)

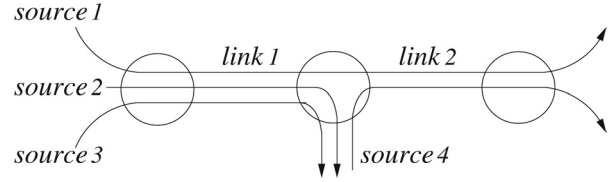


Fig. 1. Network topology.

Table 1. Scenario 1: Equilibrium points.

$c_1=50$ $c_2=30$ $\alpha=3$	d (ms)	Window Size (pkts)	Sending rate (pkts/ms)	Queuing delay (ms)
Source 1	50	1076	18.53	8.09
Source 2	10	135	10.49	2.86
Source 3	20	480	20.98	2.86
Source 4	20	289	11.47	5.23

Table 2. Scenario 2: Equilibrium points.

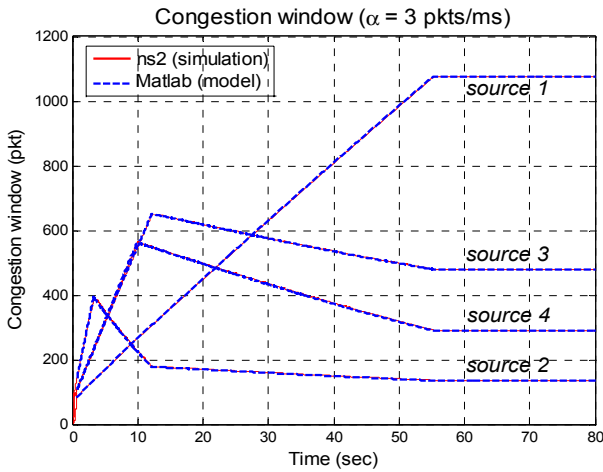
$c_1=50$ $c_2=30$ $\alpha=6$	d (ms)	Window Size (pkts)	Sending rate (pkts/ms)	Queuing delay (ms)
Source 1	50	1226	18.53	16.18
Source 2	10	165	10.49	5.72
Source 3	20	540	20.98	5.72
Source 4	20	349	11.47	10.46

and (10) converges to the calculated equilibrium point in finite time irrespective of the homogeneity of network sources. Moreover, *ns-2* simulation results in Figs. 2 and 3 verify that TCP Vegas, even in the presence of feedback information delay, follows the similar trend to the model without feedback information delay. Consequently, the simulation results illustrate that, even in the presence of feedback information delay, TCP Vegas globally converges in finite time to its equilibrium point.

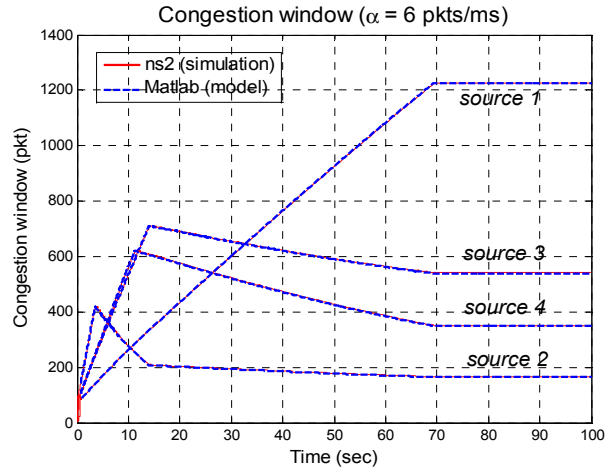
Regarding how the congestion control parameter α affects the performance of TCP Vegas, it is shown from Figs. 2 and 3 that the convergent time increases as the parameter α increases.

6. CONCLUSIONS

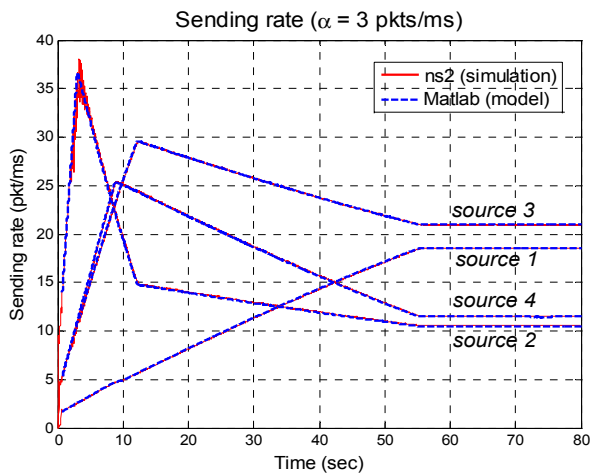
Considering a multi-link multi-source network with TCP Vegas sources without feedback information delay, we analyze the dynamic properties of TCP Vegas based on a continuous-time model without any approximation of the model, namely, keeping the high nonlinearity and discontinuous signum function of TCP Vegas model. We prove that assuming no feedback information delay, TCP Vegas is globally finite-time convergent to its equilibrium point. As illustrated in the simulation results, the dynamic behavior of TCP Vegas in the presence of feedback information delay shows similar phase to that of TCP



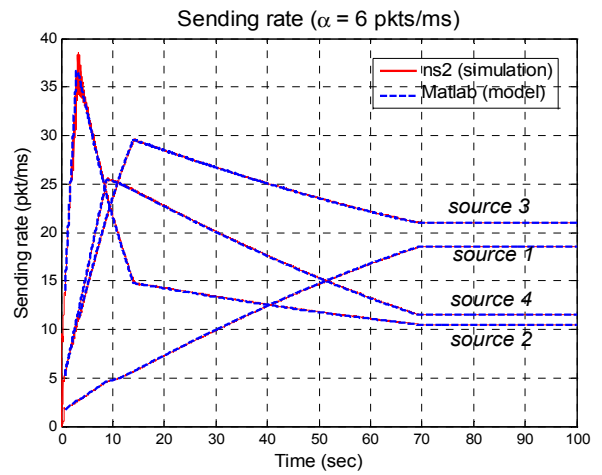
(a) Window size.



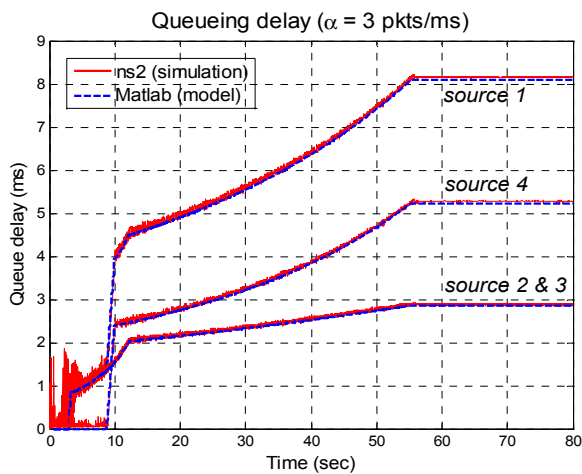
(a) Window size.



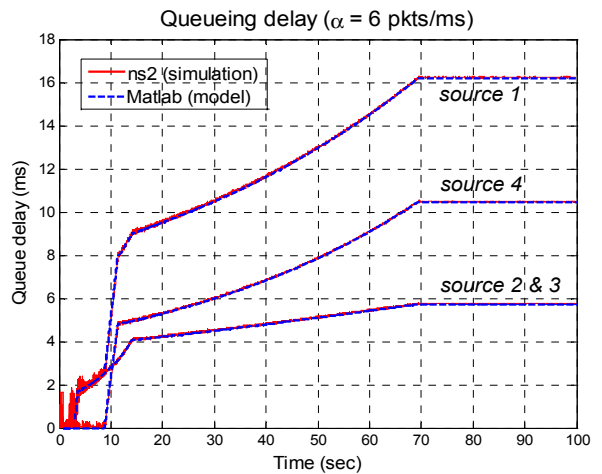
(b) Sending rate.



(b) Sending rate.



(c) Queueing delay.



(c) Queueing delay.

Fig. 2. Window size, sending rate, and queueing delay when $\alpha = 3\text{pkts}/\text{RTT}$, $d_1 = 50\text{ms}$, $d_2 = 10\text{ms}$, $d_3 = 20\text{ms}$, $d_4 = 20\text{ms}$, $c_1 = 50\text{pkts}/\text{ms}$, and $c_2 = 30\text{pkts}/\text{ms}$.

Fig. 3. Window size, sending rate, and queueing delay when $\alpha = 6\text{pkts}/\text{RTT}$, $d_1 = 50\text{ms}$, $d_2 = 10\text{ms}$, $d_3 = 20\text{ms}$, $d_4 = 20\text{ms}$, $c_1 = 50\text{pkts}/\text{ms}$, and $c_2 = 30\text{pkts}/\text{ms}$.

Vegas without feedback information delay. This observation presents a significance that dynamic properties of TCP Vegas without feedback

information delay can be used to estimate those of TCP Vegas in the presence of feedback information delay.

As a future work, we will consider establishing a method to obtain an accurate and useful estimate of the finite convergence time of TCP Vegas by using only a priori available system information.

APPENDIX A

Proof of Lemma 4

Proof: (i) Assume that there exists an unbounded $x_i(t)$ among $1 \leq i \leq N$ and derive a contradiction. Since $x_i(t)$ is unbounded, there exists a $t_k \geq 0$ for every $K > 0$ such that $x_i(t_k) > K$. If we choose a K such that $K > x_i(0)$, then, $x_i(t)$ being continuous from Lemma 2, there exists a $t_K^* \geq 0$ such that $x_i(t_K^*) = K$ and $x_i(t) \geq K$ for all $t \in [t_K^*, t_K]$. Now choose a K such that

$$K > \max \left(x_i(0), 2 \max_{l \in R_i^T} c_l, \frac{1}{d_i} \right). \quad (16)$$

Since $x_i(t) \geq K > 2 \max_{l \in R_i^T} c_l$ for all $t \in [t_K^*, t_K]$,

$$\dot{p}_l(t) = \frac{1}{c_l} (y_l(t) - c_l) \geq \frac{1}{c_l} (x_i(t) - c_l) > 1$$

for all $t \in [t_K^*, t_K]$ and $l \in R_i^T$, and since $\dot{q}_i = R_i^T \dot{p}$,

$$\dot{q}_i(t) > 1 \quad \forall t \in [t_K^*, t_K]. \quad (17)$$

On the other hand, note from (9) that

$$\begin{aligned} \dot{x}_i(t) &= -\frac{\dot{q}_i(t)}{T_i(t)} x_i(t) + \frac{1}{T_i^2(t)} \text{sgn}(s_i(t)) \\ &\leq -\frac{1}{T_i(t)} \left(\dot{q}_i(t) x_i(t) - \frac{1}{d_i} \right) \quad \forall t \geq 0. \end{aligned} \quad (18)$$

Since $\dot{q}_i(t) > 1$ for all $t \in [t_K^*, t_K]$ from (17) and $x_i(t) \geq K > \frac{1}{d_i}$ for all $t \in [t_K^*, t_K]$ from (16), then (18) yields

$$\dot{x}_i(t) \leq -\frac{1}{T_i(t)} \left(1 \cdot \frac{1}{d_i} - \frac{1}{d_i} \right) = 0 \quad (19)$$

for all $t \in [t_K^*, t_K]$. But this contradicts the fact that $x_i(t) \geq K$ and $x_i(t)$ is bounded above by K for all $t \geq 0$ and for all $1 \leq i \leq N$.

(ii) Assume that there exists an unbounded $w_i(t)$ among $1 \leq i \leq N$ and derive a contradiction. Since $w_i(t)$ is unbounded, there exists a $t_k \geq 0$ for every $K > 0$ such that $w_i(t_k) > K$. If we choose a K such

that $K > w_i(0)$, then, $w_i(t)$ being continuous from Lemma 2, there exists a $t_K^* \geq 0$ such that $w_i(t_K^*) = K$ and $w_i(t) \geq K$ for all $t \in [t_K^*, t_K]$. Now choose a K such that

$$K > \max \left(w_i(0), x_i^{UB}(d_i + 1), \alpha_i d_i (d_i + 1) \right), \quad (20)$$

where x_i^{UB} is an upper bound of $x_i(t)$, which was shown in (i). From the definition of $x_i(t)$, we have

$$q_i(t) = \frac{w_i(t)}{x_i(t)} - d_i \quad \forall t \in [t_K^*, t_K], \quad (21)$$

which implies that

$$q_i(t) \geq \frac{x_i^{UB}}{x_i(t)} (d_i + 1) - d_i > (d_i + 1) - d_i = 1 \quad (22)$$

for all $t \in [t_K^*, t_K]$ because $w_i(t) \geq K > x_i^{UB}(d_i + 1)$ for all $t \in [t_K^*, t_K]$ from (20). On the other hand, noting that $w_i(t) > \alpha_i d_i (d_i + 1)$ for all $t \in [t_K^*, t_K]$ from (20) and $\frac{q_i(t)}{T_i(t)} = \frac{q_i(t)}{d_i + q_i(t)} > \frac{1}{(d_i + 1)}$ for all $t \in [t_K^*, t_K]$ from (22), we have

$$\begin{aligned} s_i(t) &= \frac{1}{w_i(t)} - \frac{q_i(t)}{\alpha_i d_i T_i(t)} \\ &< \frac{1}{\alpha_i d_i (d_i + 1)} - \frac{1}{\alpha_i d_i (d_i + 1)} = 0 \end{aligned}$$

for all $t \in [t_K^*, t_K]$, and,

$$\dot{w}_i(t) = \frac{1}{T_i(t)} \text{sgn}(s_i(t)) < 0$$

for all $t \in [t_K^*, t_K]$. But this contradicts the fact that $w_i(t_k) > K$ for all $t \in [t_K^*, t_K]$ and $w_i(t)$ is bounded above by K for all $t \geq 0$ and for all $1 \leq i \leq N$.

(iii) Assume that there exists an unbounded $q_i(t)$ among $1 \leq i \leq N$ and derive a contradiction. Since $q_i(t)$ is unbounded, there exists a $t_k \geq 0$ for every $K > 0$ such that $q_i(t_k) > K$. If we choose an K such that $K > q_i(0)$, then, $q_i(t)$ being continuous from Lemma 2, there exists a $t_K^* \geq 0$ such that $q_i(t_K^*) = K$ and $q_i(t) \geq K$ for all $t \in [t_K^*, t_K]$. Now

choose an K such that

$$K > \max \left(q_i(0), \frac{\max_{l \in R_i^T} |R^l|}{\max_{l \in R_i^T} c_l} \max_{i \in R^l} w_i^{UB} \right),$$

where w_i^{UB} is the upper bound of $w_i(t)$ as shown in (ii) and $|R^l|$ is the number of users passing through link l . Then we obtain

$$\begin{aligned} \dot{q}_i(t_K^*) &= \sum_{l \in R_i^T} \frac{1}{c_l} \left(\sum_{i \in R^l} \frac{w_i}{T_i} - c_l \right) \Big|_{t=t_K^*} \\ &\leq \sum_{l \in R_i^T} \frac{1}{c_l} \left(\sum_{i \in R^l} \frac{w_i^{UB}}{K} - c_l \right) < 0, \end{aligned}$$

which contradicts the fact that $q_i(t) \geq K$ for all $t \in [t_K^*, t_K]$ and $q_i(t)$ is bounded above by K for all $t \geq 0$ and for all $1 \leq i \leq N$.

(iv) From the definition of $s_i(t)$, we obtain

$$|s_i| = \left| \frac{1}{w_i} - \frac{q_i}{\alpha_i d_i T_i} \right| \leq \left| \frac{1}{w_i} \right| + \left| \frac{q_i}{\alpha_i d_i T_i} \right| \leq \frac{1}{w_i^{LB}} + \frac{1}{\alpha_i d_i}$$

where w_i^{LB} is the lower bound of $w_i(t)$ as shown in (i) of Lemma 3.

(v) Assume that there exists an unbounded $p_l(t)$ among $1 \leq l \leq L$ and derive a contradiction. Since $p_l(t)$ is unbounded, there exists a $t_M \geq 0$ for every $M > 0$ such that $p_l(t_M) > M$. If we choose an M such that $M > p_l(0)$, then $p_l(t)$ being continuous from Lemma 2, there exists a $t_M^* \geq 0$ such that $p_l(t_M^*) = M$ and $p_l(t) \geq M$ for all $t \in [t_M^*, t_M]$. Now choose an M such that

$$M > \max \left(p_l(0), \frac{|R^l|}{c_l} \max_{i \in R^l} w_i^{UB} \right),$$

where w_i^{UB} (for all $1 \leq i \leq N$) is the upper bound of $w_i(t)$ as shown in (ii) and $|R^l|$ is the number of users passing through link l . Then we obtain

$$\begin{aligned} \dot{p}_l(t_M^*) &= \frac{1}{c_l} \left(\sum_{i \in R^l} \frac{w_i}{T_i} - c_l \right) \Big|_{t=t_M^*} \\ &\leq \frac{1}{c_l} \left(\sum_{i \in R^l} \frac{w_i^{UB}}{M} - c_l \right) < 0, \end{aligned}$$

which contradicts the fact that $p_l(t) \geq M$ for all $t \in [t_M^*, t_M]$ and $p_l(t)$ is bounded above by M for all $t \geq 0$ and for all $1 \leq l \leq L$. \square

REFERENCES

- [1] L. S. Brakmo and L. L. Peterson, "TCP Vegas: End to end congestion avoidance on a global internet," *IEEE Journal on Selected Areas in Communications*, vol. 13, no. 8, pp. 1465-1480, 1995.
- [2] J. S. Ahn, P. B. Danzig, Z. Liu, and L. Yan, "Evaluation of TCP Vegas: Emulation and experiment," *Proc. of SIGCOMM'95*, 1995.
- [3] J. Mo, R. La, V. Anantharam, and J. Walrand, "Analysis and comparison of TCP Reno and Vegas," *Proc. of IEEE Infocom*, March 1999.
- [4] U. Hengartner, J. Bolliger, and Th. Gross, "TCP Vegas revisited," *Proc. of IEEE Infocom*, March 2000.
- [5] S. H. Low, L. L. Peterson, and L. Wang, "Understanding TCP Vegas: A duality model," *Journal of the ACM*, vol. 49, no. 2, pp. 207-235, 2002.
- [6] H. Choe and S. H. Low, "Stabilized Vegas," *Proc. of IEEE Infocom*, April 2003.
- [7] F. Paganini, "A global stability result in network flow control," *System & Control Letters*, vol. 46, no. 3, pp. 153-163, 2002.
- [8] S. Deb and R. Srikant, "Global stability of congestion controllers for the internet," *IEEE Trans. on Automatic Control*, vol. 48, no. 6, pp. 1055-1060, June 2003.
- [9] T. Alpcan and T. Basar, "Global stability of an end-to-end congestion control scheme for general topology networks with delay," *Proc. of the 42nd IEEE Conference on Decision and Control*, Maui, Hawaii, Dec. 2003.
- [10] S. Liu, T. Basar, and R. Srikant, "Controlling the internet: A survey and some new results," *Proc. of the 42nd IEEE Conference on Decision and Control*, Maui, Hawaii, Dec. 2003.
- [11] Z. Wang and F. Paganini, "Global stability with time-delay of a primal-dual congestion control," *Proc. of the 42nd IEEE Conference on Decision and Control*, Maui, Hawaii, Dec. 2003.
- [12] L. Ying, G. E. Dullerud, and R. Srikant, "Global stability of internet congestion controllers with heterogeneous delays," *IEEE/ACM Trans. on Networking*, vol. 14, no. 3, pp. 579-590, June 2006.
- [13] J. S. Lee and K. Koo, "Global stability of TCP Vegas without network delay," *Proc. of the 43rd IEEE Conf. Decision and Control*, Atlantis, Bahamas, Dec. 2004.
- [14] H. K. Khalil, *Nonlinear Systems*, 3rd edition, Prentice Hall, 2002.

- [15] A. F. Filippov, "Differential equations with discontinuous right hand side," *Amer. Math. Soc. Translations*, vol. 42, no. 2, pp. 199-231, 1964.
- [16] J. P. Aubin and A. Cellina, *Differential Inclusions*, Springer-Verlag, Berlin Heidelberg, 1984.



Joon-Young Choi received the B.S., M.S. and Ph.D. degrees in Electronic and Electrical Engineering from Pohang University of Science and Technology (POSTECH), Pohang, Korea in 1994, 1996, and 2002, respectively. From 2002 to 2004, he worked as a Senior Engineer at

Electronics and Telecommunication Research Institute (ETRI), Daejeon, Korea. From 2004 to 2005, he worked as a Visiting Associate in the departments of Computer Science and Electrical Engineering at California Institute of Technology (CALTECH), Pasadena, CA. Since 2005, he has been an Assistant Professor in the School of Electrical Engineering at Pusan National University, Busan, Korea. His research interests include nonlinear control, internet congestion control, embedded systems and automation.



Kyungmo Koo received the B.S. degree in Electrical Engineering from Pohang University of Science and Technology (POSTECH), Korea in 2001. He is now a Ph.D. candidate in Electrical Engineering and Computer Science at POSTECH, Korea since 2001. During his studies, he spent one

year at the California Institute of Technology as a Visiting Researcher in 2004. His research interests are in the control and optimization of nonlinear systems.



Jin S. Lee received the B.S. degree in Electronics Engineering from Seoul National University, Seoul, Korea, in 1975, the M.S. degree in Electrical Engineering and Computer Science from the University of California, Berkeley, in 1980, and the Ph.D. degree in System Science from the University of California, Los Angeles,

in 1984. From 1984 to 1985, he worked as a Member of Technical Staff at AT&T Bell laboratories, Holmdel, NJ, and from 1985 to 1989, as a Senior Member of Engineering Staff at GE Advanced Technology Laboratories, MT. Laurel, NJ. Since 1989, he has been a Professor at Pohang University of Science and Technology (POSTECH), Pohang, Korea. From 1998 to 2000, he served as the Head of the Electrical Engineering Department, POSTECH and from 2000 to 2003, as the Dean of Research Affairs at POSTECH. From 2003 to 2004, he was with the Computer Science Department, the California Institute of Technology, Pasadena, for his sabbatical leave. His research interests include nonlinear systems and control, robotics, and automation.



Steven H. Low received the B.S. from Cornell University, and Ph.D. from Berkeley, both in Electrical Engineering. He is a Professor of the Computer Science and Electrical Engineering Departments at Caltech. He was with AT&T Bell Laboratories, Murray Hill, from 1992 to 1996, the University of Melbourne, Australia from 1996 to

2000, and was a Senior Fellow of the University of Melbourne from 2000 to 2004. He was a co-recipient of the IEEE William R. Bennett Prize Paper Award in 1997 and the 1996 R&D 100 Award. He was on the Editorial Board of IEEE/ACM Transactions on Networking from 1997 to 2006 and on that of Computer Networks Journal from 2003 to 2005. He is on the Editorial Boards ACM Computing Surveys, NOW Foundations and Trends in Networking, and is a Senior Editor of the IEEE Journal on Selected Areas in Communications. He is a Member of the Networking and Information Technology Technical Advisory Group for the US President's Council of Advisors on Science and Technology (PCAST). His research interests are in the control and optimization of networks and protocols.