

# Robust and Non-fragile $H_\infty$ Control for Descriptor Systems with Parameter Uncertainties and Time Delay

Jong-Hae Kim and Do-Chang Oh

**Abstract:** This paper describes a robust and non-fragile  $H_\infty$  controller design method for descriptor systems with parameter uncertainties and time delay, as well as a static state feedback controller with multiplicative uncertainty. The controller existence condition, as well as its design method, and the measure of non-fragility in the controller are proposed using linear matrix inequality (LMI) technique, which can be solved efficiently by convex optimization. Therefore, the presented robust and non-fragile  $H_\infty$  controller guarantees the asymptotic stability and disturbance attenuation of the closed loop systems within a prescribed degree in spite of parameter uncertainties, time delay, disturbance input and controller fragility.

**Keywords:** LMI, non-fragile control, parameter uncertainty, singular systems, time delay.

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## 1. INTRODUCTION

The problem of  $H_\infty$  control for standard state-space has received considerable interest over the last decade. Although  $H_\infty$  control theory has been perfectly developed, most works have been developed based on state space equations. Recently, much attention has been given to the extensions of the results of  $H_\infty$  control theory for state-space systems to descriptor systems. State space models are very useful, but the state variables thus introduced do not provide a physical meaning. Hence, the descriptor form is a natural representation of linear dynamical systems, and makes it possible to analyze a larger class of systems than state space equations do [1,2], because state space equations cannot represent algebraic restrictions between state variables and some physical phenomena, like impulse and hysteresis, which are important in circuit theory. Although  $H_\infty$  control theory in descriptor systems has been developed over the last decade, there are no papers considering non-fragile  $H_\infty$  controller design methods for descriptor systems.

It is generally known that feedback systems designed for robustness with respect to plant

parameters, or designed for the optimization of a single performance measure, may require very accurate controllers. An implicit assumption that is inherent to those control methodologies is that the controller is designed to be implemented precisely. However, the controller implementation is subject to A/D conversion, D/A conversion, finite word length and round-off errors in numerical computations, in addition to the requirement of providing the practicing engineer with safe-tuning margins. Therefore, it is necessary that any controller should be able to tolerate some uncertainty in parameters. Since controller fragility is basically the performance deterioration of a feedback control system due to inaccuracies in controller implementation, the non-fragile control problem has been an important issue [3-8].

In a recent paper, Keel *et al.* [3] have shown that the resulting controllers exhibit a poor stability margin if not implemented exactly. So, some researchers have developed non-fragile controller design algorithms [3-8]. Haddad *et al.* [4] proposed a non-fragile controller design method via quadratic Lyapunov bounds. And, Famularo *et al.* [5] considered the LQ robust non-fragile static state feedback controller design method in the presence of uncertainties in both plant and controller. However, Famularo *et al.* [5] did not obtain the value of the non-fragility directly, but predetermined the value of non-fragility before finding a controller. Also Dorato *et al.* [6] dealt with the problem on the design of non-fragile compensators via symbolic quantifier elimination. However, the existing works have been focused on the non-descriptor systems.

Since the stability analysis and control of dynamic systems with time delay are problems of recurring interest as time delay often is the cause for instability

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Jong-Hae Kim is with the Division of Electronic Engineering, Sun Moon University, 100, Kalsan-ri, Tangjeong-myeon, Asan-si, Chungnam 336-708, Korea (e-mail: kjhae@sunmoon.ac.kr).

Do-Chang Oh is with the Department Electronics & Information Engineering, Konyang University, 26 Naedong, Nonsan, 320-711, Korea (e-mail: docoh@konyang.ac.kr).

and poor performance of control systems, the study of time delay systems has received considerable attention over the past years [9,10, and references therein]. This fact motivates the author to develop robust and non-fragile  $H_\infty$  control of singular systems with parameter uncertainties and time delay.

In this paper, we propose a robust and non-fragile  $H_\infty$  state feedback controller design method for descriptor systems and static state feedback controller with multiplicative uncertainty. The condition of controller existence, the design method of robust and non-fragile  $H_\infty$  controller, and the measure of non-fragility in controller are presented via linear matrix inequality (LMI) technique. The measure of non-fragility is related to the performance of controller gain variations. Also, the sufficient condition can be rewritten as an LMI form in terms of transformed variables through singular value decomposition, some changes of variables, and Schur complements. Since the proposed controller design algorithm is an LMI form in terms of all variables, the solutions can be obtained simultaneously.

The following notations will be used in this paper.  $(\cdot)^T$ ,  $(\cdot)^{-1}$ ,  $\deg(\cdot)$ ,  $\det(\cdot)$  and  $\text{rank}(\cdot)$  denote the transpose, inverse, degree, determinant, and rank of a matrix. And  $I$ ,  $I_r$ ,  $x_r(t)$ , and  $\mathbf{R}^T$  denote an identity matrix with proper dimensions, an identity matrix with  $r \times r$  dimension,  $r \times 1$  dimensional vector, and  $r \times 1$  dimensional real vector, respectively. \* represents the transposed elements in the symmetric positions.

## 2. PROBLEM FORMULATION

Consider an uncertain singular system with time delay

$$\begin{aligned} E\dot{x}(t) &= [A + \Delta A(t)]x(t) + [A_d + \Delta A_d(t)]x(t-d) \\ &\quad + B_1u(t) + B_2w(t), \\ z(t) &= Cx(t), \\ x(t) &= 0(t < 0), \end{aligned} \quad (1)$$

where  $x(t) \in \mathbf{R}^n$  is the state variable,  $z(t) \in \mathbf{R}^l$  is the controlled output variable,  $u(t) \in \mathbf{R}^m$  is the control input variable,  $w(t) \in \mathbf{R}^p$  is the disturbance input variable,  $E$  is a singular matrix with  $\text{rank}(E) = r \leq n$ , time delay  $d$  is the positive real number, and all matrices have proper dimensions. The parameter uncertainties are defined as follows:

$$\Delta A(t) = DF_1(t)H, \quad \Delta A_d(t) = D_dF_2(t)H_d, \quad (2)$$

where  $D$ ,  $D_d$ ,  $H$ ,  $H_d$  are known matrices and unknown matrices,  $F_i(t)$ ,  $i = 1, 2$ , are satisfied with

$$F_i(t)^T F_i(t) \leq I. \quad (3)$$

Although one finds the controller

$$u(t) = Kx(t), \quad (4)$$

the actual controller implemented is assumed as

$$u(t) = [I + \alpha\Phi(t)]Kx(t), \quad (5)$$

where  $K$  is the nominal controller gain,  $\alpha$  is the positive constant, the term  $\alpha\Phi(t)K$  represents controller gain variations and  $\Phi(t)$  is defined as

$$\Phi(t)^T \Phi(t) \leq I. \quad (6)$$

Here, the value of  $\alpha$  indicates the measure of non-fragility against controller gain variations. Now, the closed loop system from the descriptor system (1) and the controller (5) is given by

$$\begin{aligned} E\dot{x}(t) &= [A + \Delta A(t) + B_1(I + \alpha\Phi(t))K]x(t) \\ &\quad + [A_d + \Delta A_d(t)]x(t-d) + B_2w(t), \\ z(t) &= Cx(t). \end{aligned} \quad (7)$$

Also, we introduce  $H_\infty$  performance measure

$$\int_0^\infty [z(t)^T z(t) - \gamma^2 w(t)^T w(t)] dt. \quad (8)$$

The objective of the controller is described as follows:

- (i) The closed loop system (7) is asymptotically stable.
- (ii) The closed loop system guarantees, under zero initial condition,  $H_\infty$  norm bound of (7) for all non-zero  $w(t) \in L_2[0, \infty)$ .

Here, we introduce some properties of singular systems with time delay.

**Definition 1**[11]: For any given two matrices  $E \in \mathbf{R}^{n \times n}$ ,  $A^* \in \mathbf{C}^{n \times n}$ , the pencil  $(E, A^*)$  is called regular if there exists a constant scalar  $\alpha \in \mathbf{C}$  such that  $|\alpha E + A^*| \neq 0$  or the polynomial  $|sE - A^*| \neq 0$ . Here,  $A^* = A + A_d e^{-j\omega d}$ .

**Lemma 1**[11]:  $(E, A + A_d e^{-j\omega d})$  is regular if and only if two nonsingular matrices  $Q^*$ ,  $P^*$  may be chosen such that

$$\tilde{E} \equiv Q^* E P^* = \text{diag}(I_{n_1}, N) \quad (9)$$

and

$$\tilde{A} \equiv Q^* A P^* = \text{diag}(A_1^*, I_{n_2}), \quad (10)$$

where  $A^* = A + A_d e^{-j\omega d}$ ,  $n_1 + n_2 = n$ ,  $A_1^* \in \mathbf{C}^{n_1 \times n_1}$  and  $N \in \mathbf{R}^{n_2 \times n_2}$  is nilpotent.

**Remark 1**[11]: The following statements are

equivalent:

- (a)  $E\dot{x}(t) = Ax(t) + A_d x(t-d)$  is impulse free,
- (b)  $\text{rank}(E) = \text{deg}(\det(sE - A^*))$ ,
- (c)  $N$  in Lemma 1 is a null matrix,
- (d)  $(sE - A^*)^{-1}$  is proper.

### 3. MAIN RESULTS

The sufficient condition in terms of LMI form and robust and non-fragile  $H_\infty$  controller design method are considered in this section. In the following, we present a sufficient condition for the existence of non-fragile  $H_\infty$  controller.

**Theorem 1:** Consider a closed loop uncertain descriptor system with time delay (7). If there exists invertible symmetric matrix  $P$ , feedback gain  $K$ , and positive scalars  $\alpha$ ,  $\varepsilon_1$ ,  $\varepsilon_2$ , and  $\varepsilon_3$  satisfying

$$E^T P = P^T E \geq 0, \quad (11)$$

$$\begin{bmatrix} \Pi & P^T A_d & P^T B_2 \\ * & -I + \frac{1}{\varepsilon_2} H_d^T H_d & 0 \\ * & * & -\gamma^2 I \end{bmatrix} < 0 \quad (12)$$

then the controller (4) is a robust and non-fragile  $H_\infty$  controller guaranteeing not only asymptotic stability in the presence of time delay, parameter uncertainties, disturbance input and controller gain variations but also  $H_\infty$  norm bound of the closed loop descriptor systems. Here,  $\Pi$  is defined as follows:

$$\begin{aligned} \Pi = & A^T P + P^T A + C^T C + I + P^T B_1 K + K^T B_1^T P \\ & + \varepsilon_1 P^T D D^T P + \frac{1}{\varepsilon_1} H^T H + \varepsilon_2 P^T D_d D_d^T P \\ & + \alpha \varepsilon_3 P^T B_1 B_1^T P + \frac{\alpha}{\varepsilon_3} K^T K. \end{aligned}$$

**Proof:** For asymptotic stability of the closed loop uncertain descriptor system with time delay (7), if we take a Lyapunov functional

$$V(x(t)) = x(t)^T E^T P x(t) + \int_{t-d}^t x(t)^T x(t) dt \quad (13)$$

with  $E^T P = P E \geq 0$ , then the time derivative of (13) is given by

$$\begin{aligned} \dot{V}(x(t)) = & \dot{x}(t)^T E^T P x(t) + x(t)^T P^T E \dot{x}(t) \\ & + x(t)^T x(t) - x(t-d)^T x(t-d). \end{aligned} \quad (14)$$

The matrix inequality (12) from the Lyapunov functional (13) and  $H_\infty$  performance measure (8)

implies

$$z(t)^T z(t) - \gamma^2 w(t)^T w(t) + \dot{V}(x(t)) < 0. \quad (15)$$

Therefore, using the following lemma

$$\begin{aligned} & 2x(t)^T P^T D F_1(t) H x(t) \\ & \leq \varepsilon_1 x(t)^T P^T D D^T P x(t) + \frac{1}{\varepsilon_1} x(t)^T H^T H x(t), \\ & 2x(t)^T P^T D_d F_2(t) H_d x(t-d) \\ & \leq \varepsilon_2 x(t)^T P^T D_d D_d^T P x(t) + \frac{1}{\varepsilon_2} x(t-d)^T H_d^T H_d x(t-d), \\ & 2\alpha x(t)^T P B_1 \Phi(t) K x(t) \\ & \leq \alpha \varepsilon x(t)^T P B_1 B_1^T P x(t) + \frac{\alpha}{\varepsilon} x(t)^T K^T K x(t), \end{aligned}$$

the inequality (15) is equal to

$$\begin{bmatrix} x(t) \\ x(t-d) \\ w(t) \end{bmatrix}^T \begin{bmatrix} \Pi & P^T A_d & P^T B_2 \\ * & -I + \frac{1}{\varepsilon} H_d^T H_d & 0 \\ * & * & -\gamma^2 I \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-d) \\ w(t) \end{bmatrix} < 0. \quad (16)$$

Hence, the inequality (16) implies the sufficient condition (12).  $\square$

However, it is not easy to solve Theorem 1, because the sufficient condition of (12) is not an LMI form and the equality condition is included in (11). In order to make a perfect LMI condition in terms of finding all variables and eliminate equality condition, the obtained sufficient conditions are changed in the following Theorem 2 by proper manipulations. Moreover, the robust and non-fragile  $H_\infty$  controller design method for uncertain descriptor systems with time delay is presented.

**Theorem 2:** If there exists positive definite matrix  $Q_1$ , an invertible symmetric matrix  $Q_4$ , matrices  $Q_3$ ,  $Y_1$ ,  $Y_2$ , and positive scalar  $\beta_1$ ,  $\beta_2$ ,  $\rho$  satisfying

$$\begin{bmatrix} \Gamma_1 & \Gamma_2 & A_{d1} & A_{d2} & B_{21} & Q_1 C_1^T + Q_3 C_2^T \\ * & \Gamma_3 & A_{d3} & A_{d4} & B_{22} & Q_4 C_2^T \\ * & * & -I & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 \\ * & * & * & * & -\rho I & 0 \\ * & * & * & * & * & -I \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix}$$

$$\begin{bmatrix} Q_1 & Q_3^T & Q_1 H_1^T + Q_3^T H_2^T & Y_1^T & 0 \\ 0 & Q_4 & Q_4 H_2^T & Y_2^T & 0 \\ 0 & 0 & 0 & 0 & H_{d_1}^T \\ 0 & 0 & 0 & 0 & H_{d_2}^T \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -I & 0 & 0 & 0 & 0 \\ * & -I & 0 & 0 & 0 \\ * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & -\beta_2 I & 0 \\ * & * & * & * & -\varepsilon_2 I \end{bmatrix} < 0 \quad (17)$$

then the matrix expressed by

$$K = [Y_1 P_1 + Y_2 P_3 \quad Y_2 P_4] \quad (18)$$

is a controller gain in robust and non-fragile  $H_\infty$  controller satisfying asymptotic stability, regular, impulse-free, and  $H_\infty$  norm bound in the presence of controller gain variations, time delay and disturbance input. Here,  $A_1$ ,  $A_4$ ,  $A_{d_1}$ ,  $A_{d_2}$ ,  $A_{d_3}$ ,  $A_{d_4}$ ,  $B_{11}$ ,  $B_{12}$ ,  $B_{21}$ ,  $B_{22}$ ,  $C_1$ ,  $C_2$ ,  $D_1$ ,  $D_2$ ,  $D_{d_1}$ ,  $D_{d_2}$ ,  $H_1$ ,  $H_2$ ,  $H_{d_1}$ , and  $H_{d_2}$  are decomposed matrices with proper dimensions from the uncertain descriptor system matrices. And some notations are defined as follows:

$$\begin{aligned} \Gamma_1 &= A_1 Q_1 + Q_1 A_1^T + B_{11} Y_1 + Y_1^T B_{11}^T + \varepsilon_1 D_1 D_1^T \\ &\quad + \varepsilon_2 D_{d_1} D_{d_1}^T + \beta_1 B_{11} B_{11}^T, \\ \Gamma_2 &= Q_3^T A_4^T + B_{11} Y_2 + Y_2^T B_{12}^T + \varepsilon_1 D_1 D_2^T \\ &\quad + \varepsilon_2 D_{d_1} D_{d_2}^T + \beta_1 B_{11} B_{12}^T, \\ \Gamma_3 &= A_4 Q_4 + Q_4 A_4^T + B_{12} Y_2 + Y_2^T B_{12}^T + \varepsilon_1 D_2 D_2^T \\ &\quad + \varepsilon_2 D_{d_2} D_{d_2}^T + \beta_1 B_{12} B_{12}^T, \\ P_3 &= -P_4 Q_3 P_1, \\ \beta_1 &= \alpha \varepsilon_3, \\ \beta_2 &= \frac{\varepsilon_3}{\alpha}, \\ \rho &= \gamma^2. \end{aligned} \quad (19)$$

**Proof:** Using Schur complements [12] and some changes of variables, the matrix inequality (12) is transformed into

$$\begin{bmatrix} \Psi & A_d & B_2 & Q^T C^T & Q^T & Q^T H^T & Y^T & 0 \\ * & -I & 0 & 0 & 0 & 0 & 0 & H_d^T \\ * & * & -\rho I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & * & -\beta_2 I & 0 \\ * & * & * & * & * & * & * & -\varepsilon_2 I \end{bmatrix} < 0, \quad (20)$$

where  $\Psi$  is defined

$$\begin{aligned} \Psi &= Q^T A^T + A Q + B_1 Y + Y^T B_1^T + \varepsilon_1 D D^T \\ &\quad + \varepsilon_2 D_d D_d^T + \beta_1 B_1 B_1^T. \end{aligned}$$

To obtain an LMI sufficient condition in terms of finding all variables and eliminate the equality in (11), we make use of singular value decomposition and changes of variables. Without loss of generality, we assume that the uncertain descriptor system matrices of (1) have the following singular decomposition form [1].

$$\begin{aligned} E &= \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} A_1 & 0 \\ 0 & A_4 \end{bmatrix}, \quad A_d = \begin{bmatrix} A_{d_1} & A_{d_2} \\ A_{d_3} & A_{d_4} \end{bmatrix}, \\ B_1 &= \begin{bmatrix} B_{11} \\ B_{12} \end{bmatrix}, \quad B_2 = \begin{bmatrix} B_{21} \\ B_{22} \end{bmatrix}, \quad C = \begin{bmatrix} C_1 & C_2 \end{bmatrix}, \\ D &= \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}, \quad D_d = \begin{bmatrix} D_{d_1} \\ D_{d_2} \end{bmatrix}, \\ H &= \begin{bmatrix} H_1 & H_2 \end{bmatrix}, \quad H_d = \begin{bmatrix} H_{d_1} & H_{d_2} \end{bmatrix}, \end{aligned} \quad (21)$$

where all matrices have appropriate dimensions. Also, if we set

$$P = \begin{bmatrix} P_1 & 0 \\ P_3 & P_4 \end{bmatrix} \quad (22)$$

in order to satisfy (11) and other solutions have the following structure

$$\begin{aligned} Y &= \begin{bmatrix} Y_1 & Y_2 \end{bmatrix}, \\ Q &= P^{-1} = \begin{bmatrix} Q_1 & 0 \\ Q_3 & Q_4 \end{bmatrix}, \end{aligned} \quad (23)$$

then (20) is changed to (17) by applying (21)-(23) to (20). Therefore, (17) is an LMI form in terms of all variables,  $Q_1$ ,  $Q_3$ ,  $Q_4$ ,  $Y_1$ ,  $Y_2$ ,  $\varepsilon_1$ ,  $\varepsilon_2$ ,  $\beta_1$ ,  $\beta_2$ , and  $\rho$ .

**Remark 2:** In the case of  $E = I$ , the problem can

be solved directly from an LMI condition in (20). Therefore, the proposed design algorithm can be solved in non-descriptor systems with an LMI condition. Thus, the result is the general design method.

**Remark 3:** (17) is an LMI form in terms of all variables,  $Q_1$ ,  $Q_3$ ,  $Q_4$ ,  $Y_1$ ,  $Y_2$ ,  $\varepsilon_1$ ,  $\varepsilon_2$ ,  $\beta_1$ ,  $\beta_2$ , and  $\rho$ . Therefore, robust and non-fragile  $H_\infty$  state feedback controller can be calculated directly using the LMI Toolbox [12]. Also, the measure of non-fragility in controller can be calculated by  $\alpha = \sqrt{\beta_1/\beta_2}$  and the value of disturbance attenuation can be obtained by  $\gamma = \sqrt{\rho}$ .

**Remark 4:** In order to get a minimum value of  $\gamma$  or maximum value of  $\alpha$ , the LMI feasibility problem in Theorem 2 can be reformulated. To obtain the minimum value of  $\gamma$ , the optimization problem is rewritten as

*Maximize  $\rho$  subject to LMI (19).*

In the case of maximum value of  $\alpha$ , the optimization problem is modified by

*Maximize  $\beta_1$  subject to LMI (19).*

*Minimize  $\beta_2$  subject to LMI (19).*

However, it is difficult to obtain the value of minimum value of  $\gamma$  and maximum value of  $\alpha$  at the same time. Therefore, one of the future researches is to develop synthesis algorithms that take into account certain structured uncertainties in the controllers and search for the best solution that guarantees a compromise between optimality  $\gamma$  and fragility  $\alpha$ .

**Example:** To demonstrate the proposed controller design algorithm, a descriptor system with parameter uncertainties and time delay is considered as follows:

$$\begin{aligned} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{x}(t) &= \left\{ \begin{bmatrix} -2 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} F_1(t) \begin{bmatrix} 0.1 & 0.1 & 0.1 \end{bmatrix} \right\} x(t) \\ &+ \left\{ \begin{bmatrix} 0.5 & 0.4 & 0.5 \\ 0.3 & 0.6 & 0.4 \\ 0.5 & 0.3 & 0.4 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix} F_2(t) \begin{bmatrix} 0.1 & 0.1 & 0.1 \end{bmatrix} \right\} x(t-d) \\ &+ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix} w(t), \\ z(t) &= \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} x(t). \end{aligned} \quad (24)$$

All solutions can be calculated at the same time from the LMI Toolbox [12] because the proposed optimization problem of Theorem 2 is an LMI form in terms of finding all variables. The solutions satisfying Theorem 2 are as follows:

$$\begin{aligned} Q &= \begin{bmatrix} 1.7611 & -0.6518 & 0 \\ -0.6518 & 0.3344 & 0 \\ 0.3367 & 1.4549 & 0.6805 \end{bmatrix}, \\ Y &= \begin{bmatrix} -9.7125 & -13.7311 & -12.2399 \end{bmatrix}, \\ \beta_1 &= 5.2725, \\ \beta_2 &= 28.6563, \\ \varepsilon_1 &= 8.0468, \\ \varepsilon_2 &= 16.5810, \\ \rho &= 16.5237. \end{aligned} \quad (25)$$

Therefore,  $H_\infty$  norm bound and the measure of non-fragility are obtained from the changes of variables (19) as follows:

$$\begin{aligned} \alpha &= \sqrt{\beta_1/\beta_2} = 0.5584, \\ \gamma &= \sqrt{\rho} = 4.0649. \end{aligned} \quad (26)$$

Here, the meaning of  $\alpha$  implies that the obtained robust and non-fragile  $H_\infty$  controller guarantees asymptotic stability and  $H_\infty$  norm bound of the closed loop systems in spite of controller gain variations within 55.84%. Hence, the robust and  $H_\infty$  control law by (18) is

$$u(t) = \begin{bmatrix} 41.9440 & 118.9359 & -17.9855 \end{bmatrix} x(t). \quad (27)$$

For computer simulations, if we take  $F_1(t) = F_2(t) = \sin t$ ,  $\Phi(t) = \cos t$ ,  $d = 5$  and the value of  $w(t)$  as defined in Fig. 1, then the trajectories of

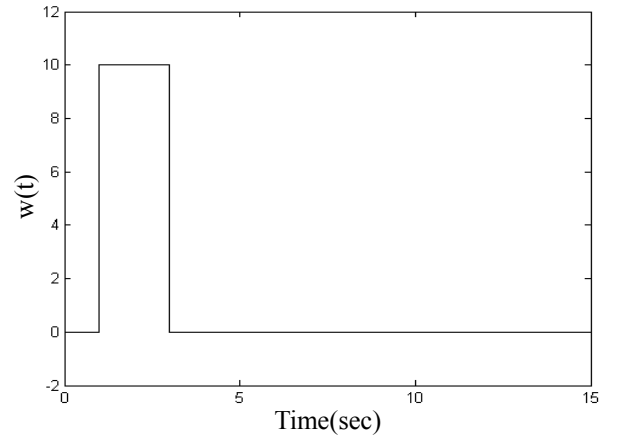
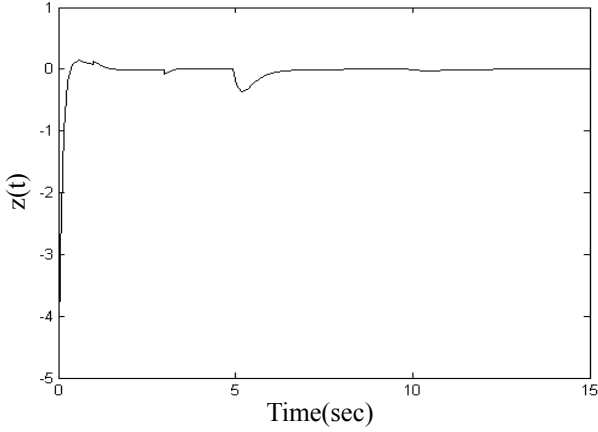
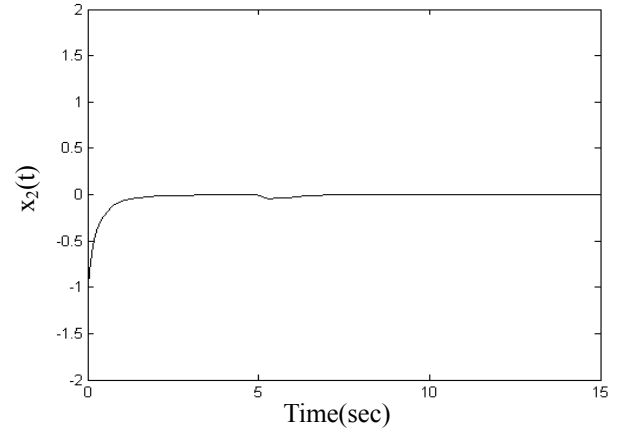
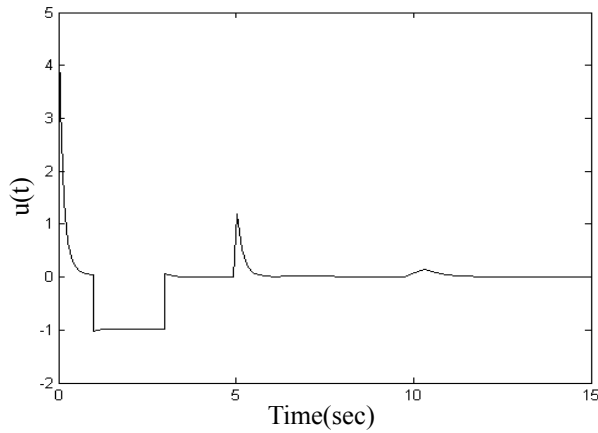
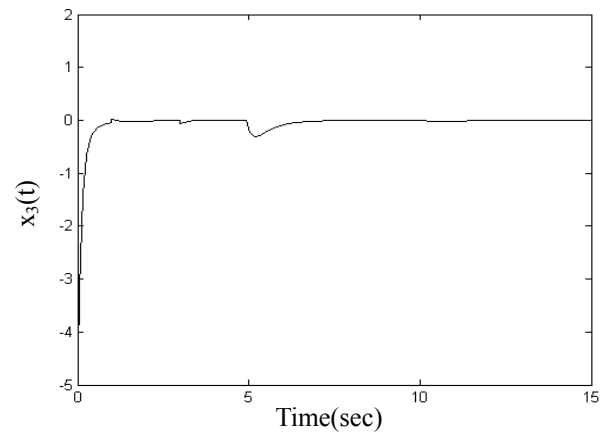
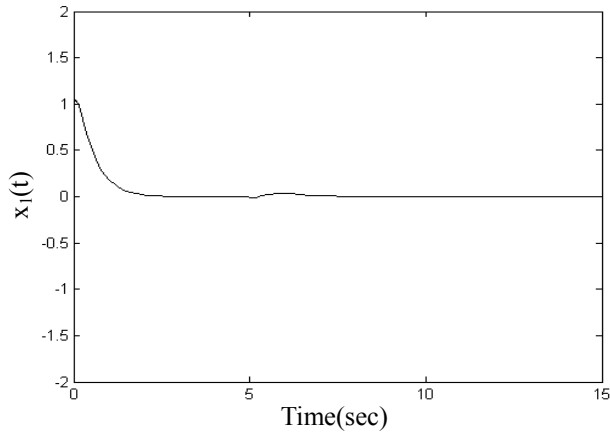


Fig. 1. Disturbance input  $w(t)$ .

Fig. 2. Controlled output  $z(t)$ .Fig. 5. State  $x_2(t)$ .Fig. 3. Control input  $u(t)$ .Fig. 6. State  $x_3(t)$ .Fig. 4. State  $x_1(t)$ .

control input, controlled output, and states are shown in Figs. 2~6. From the simulation results of Figs. 4~6, the obtained controller stabilizes the descriptor system with parameter uncertainties, time delay, and controller gain variations because the values of states converge to zero as time goes to infinity. Also,  $H_\infty$  norm bound ( $\gamma < 4.0649$ ) can be checked from the  $L_2$  induced norm property between  $w(t)$  of Fig. 1 and

$z(t)$  of Fig. 2. Thus, the obtained robust and non-fragile  $H_\infty$  controller guarantees not only asymptotic stability but also  $H_\infty$  norm bound of the closed loop system.

#### 4. CONCLUSIONS

In this paper, we considered the robust and non-fragile  $H_\infty$  controller design method for descriptor systems with parameter uncertainties and time delay, and static feedback controller with multiplicative uncertainty. The condition for the existence of controller and the controller design algorithm were proposed via LMI approach. The proposed robust and non-fragile  $H_\infty$  state feedback controller guaranteed asymptotic stability but also  $H_\infty$  norm bound in spite of time delay, parameter uncertainties and controller gain variations. Moreover, the measure of non-fragility was presented.

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**Jong-Hae Kim** received the B.S., M.S., and Ph.D. degrees in Electronics from Kyungpook National University in 1993, 1995, and 1998, respectively. He was with STRC at Kyungpook National University from Nov. 1998 to Feb. 2002. Also, he was with Osaka University as a Research Fellow for one year from March, 2000. He has been with Sun Moon University since Mar. 2002. His research interests focus on robust control, time delay systems, non-fragile control, reliable control, and industrial application control.



**Do-Chang Oh** received the B.S., M.S., and Ph.D. degrees in Electronics from Kyungpook National University in 1991, 1993, and 1997, respectively. He is currently an Associate Professor at Konyang University. His research interests include model and controller reduction, robust control, time delay systems, and industrial application control.