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On *D*-admissibility Conditions of Singular Systems

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Abstract: In this paper, we first establish D_L-admissibility and D_R-admissibility conditions for singular systems. The admissibility conditions expressed as Lyapunov type inequalities extend the existed results of normal systems to singular systems. As special cases the admissibility conditions of the continuous-time and the discrete-time singular systems can be obtained directly. The results established in this paper can be applied to solve the problems of eigenvalue assignment, regional pole-placement and robust control etc.

Keywords: Admissibility, LMI region, Lyapunov approach, singular systems.

1. INTRODUCTION

In recent years, there has been a growing interest in the system theoretic problem of singular systems because of the extensive applications of descriptor systems to many engineering systems. The singular state-space systems are also referred to as generalized systems, descriptor systems or implicit systems. The essence of its simultaneous description of dynamic and algebraic relationships between state variables makes such models especially suitable for robotic systems, singularly perturbed systems, and highly interconnected large-scale systems, thus it present a much wider class of systems than normal systems [1]. A great number of fundamental notions and results based on state-space systems have been successfully extended to singular systems, such as controllability and observability, pole assignment, stability and stabilization [1-4] and so on. The descriptor system approach is a useful tool to solve the stability and H^{∞} control problems of linear time-delay systems [5].

It is well known that Lyapunov equation plays an important role in the stable analysis of systems. Lyapunov approach has become a powerful tool in solving different problems for standard systems. There are many attempts to generate the Lyapunov approach to generalized systems, and some useful and interesting results are also established [3,4,6]. Stability is a minimum requirement for control systems. In most practical situations, however, a good controller should also deliver sufficiently fast and well-damped time responses. A customary way to guarantee satisfactory transients is to place the closed-loop poles in a suitable region of the complex plane. Therefore, the pole clustering analysis and pole placement design play an important role in practical engineering applications. For examples, fast decay, good damping and reasonable controller dynamics can be imposed by confining the poles in the intersection of a shifted half-plane, a sector and a disk etc [7-13].

[7-9] introduced some special pole regions expressed by linear matrix inequalities (LMI) and established the Lyapunov type results of pole clustering analysis and design for normal systems case. The LMI region covers a large variety of useful clustering regions, including half-planes, disks, sectors, vertical/horizontal strips, and any intersection thereof. This class of regions can be applied successfully in solving some robust control problems based on LMI structure of regions. To reduce the conservatism, [7,14,15] use parameter-dependent Lyapunov matrices to solve the robust *D*-stability problems by using Lyapunov inequalities for some particular structure of uncertainties. From a practical standpoint, LMI approaches are appealing for applications because there are effective and powerful algorithms such as interior-point method for the solution of LMI problems and there are also a number of software packages such as MATLAB to be available for solving LMI problems [16].

The purpose of this paper is to establish Lyapunov type admissibility conditions of descriptor systems for LMI *D* regions. The generalized Lyapunov equation proposed by this paper can be used to solve the pole clustering analysis and pole placement design problems. Based on the results of this paper, we can establish robust LMI *D*-admissibility conditions of descriptor systems for polytopic uncertainty structure or other uncertainty structure by using the similar ideas given in [7,14-16].

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The remaining part of the paper is organized as follows. In Section 2, we give some basic concepts and preliminary results used in this paper. In Section 3, we establish new Lyapunov type admissibility conditions of descriptor systems for LMI *D* regions, which are main results of this paper.

The notation of this paper is standard. $\bar{z}$ represents the conjugate of $z \in C$. A^T is denoted as transpose of a matrix A. For symmetric matrices A, B, $A >$ $(\geq) B$ means $A - B$ is positive (semi-) definite. δ is the derivation operator for continuous-time systems ($\delta x(t) = \dot{x}(t)$), and the delay operator for discrete-time case ($\delta x(t) = x(t+1)$). $\otimes$ is the Kronecker product of two matrices, and it is well-known that $(A \otimes B)$ $(C \otimes D) = (AC) \otimes (BD)$.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider a time-invariant finite-dimensional singular system described by

$$E\dot{x}(t) = Ax(t) \quad \text{or} \quad E\dot{x}(k+1) = Ax(k), \tag{1}$$

where $x \in R^n$ is the state vector, $E, A \in R^{n\times n}$, and $rank(E) = r \leq n$, thus may be singular. For brevity, we will use $E\delta x = Ax$ or the pair (E, A) to represent a singular systems in (1). When $\text{rank}(E) =$ 0, the system (1) degenerates to algebraic equations, so we always assume that $r = \text{rank}(E) \neq 0$ in this paper.

Some important features concerning the study of singular systems are recalled [1]. The singular system (1) or the pair (E, A) is said to be regular if and only if $\det(sE - A)$ is not identically zeros, which implies the solution of system (1) exists and is unique for any specified initial condition. The singular system (1) is said to be impulse-free, if $\deg(\det(sE - A)) = r$. Finally, a pair (E, A) is said to be stable if all finite generalized eigenvalues of the pair lie in the stable region, i.e., the open left-half plane for continuous systems or the inside unit disk for discrete systems. For short, the system (1) is said to be admissible if and only if it is regular, impulse-free and stable.

From a practical standpoint, a singular system should be stable and impulse-free. The system will deliver sufficiently fast and well-damped time responses if we assign all finite generalized eigenvalues of the pair (E, A) in some special regions. Thus, we give the definition of *D*-admissible as follows.

Definition 1: The singular system (1) is said to be *D*-admissible if it is regular, impulse-free and all its finite eigenvalues of pair (E, A) are inside region *D*.

The matrix A is said to be *D*-stable if all its eigenvalues lie in the region *D*. In [8], a class of convex LMI-base regions was characterized by

$$D_L = \{z \in C : R_{11} + R_{12}z + R_{12}^T\bar{z} < 0\}, \tag{2}$$

where $R_{11} \in R^{d\times d}$ is a symmetric matrix and $R_{12} \in$ $R^{d\times d}$. Some special regions such as conic sectors, vertical half planes, and vertical strips are D_L regions.

Let $R \in R^{2d\times 2d}$ be a symmetric matrix partitioned as

$$R = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix}.$$

The regions given by

$$D_R = \{z \in C : R_{11} + R_{12}z + R_{12}^T\bar{z} + R_{22}z\bar{z} < 0\} \tag{3}$$

were introduced in [7] as an extension of the so-called LMI regions. It includes most notably the left-plane (continuous-time stability) and the unit disk (discrete-time stability) for the respective particular choices:

$$R = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \qquad R = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{4}$$

Lemma 1 [7]: $A \in C^{n\times n}$ is D_R-stable if and only if there exists a symmetric positive definite matrix $P \in R^{n\times n}$ such that

$$R_{11} \otimes P + R_{12} \otimes (PA) + R_{12}^T \otimes (A^T P) + R_{22} \otimes (A^T PA) < 0. \tag{5}$$

For convenience, we also set

$$F_1(A, P) = R_{11} \otimes P + R_{12} \otimes (PA) + R_{12}^T \otimes (A^T P) + R_{22} \otimes (A^T PA)$$

and

$$F_2(A, P) = R_{11} \otimes P + R_{12} \otimes (PA) + R_{12}^T \otimes (A^T P).$$

The following lemma which relates system matrices with its algebraic characterization can be found in [1].

Lemma 2: For any regular pair (E, A), there exist nonsingular matrices M and N such that

$$MEN = \begin{bmatrix} I_r & 0 \\ 0 & J \end{bmatrix}, \quad MAN = \begin{bmatrix} A_1 & 0 \\ 0 & I_{n-r} \end{bmatrix}, \tag{6}$$

where J is a nilpotent matrix.

Lemma 2 shows that $\Lambda(E, A) = \Lambda(I_r, A_1)$ and (E, A) is impulse-free if and only if $J = 0$, where

$\Lambda(E,A)$ is the set of all finite generalized eigenvalues of the matrix pair (E,A). Therefore, the system (1) is admissible if and only if $J=0$ and $r\neq 0$. If all finite eigenvalues of pair (E,A) are local in the region *D*, we say that (E,A) is *D*-stable. The singular system (1) is *D*-admissible if and only if A_1 is *D*-stable, $J=0$ and $r\neq 0$.

3. MAIN RESULTS

In this section, we establish D_L-admissibility and D_R-admissibility conditions for singular systems, which are the main results of this paper.

Theorem 1: For a given D_L region: $D_L=\{z\in C : R_{11}+R_{12}z+R_{12}^T\bar{z}<0\}$, the following two states are equivalent:

(a) $D_1=\{z\in C : R_{12}z+R_{12}^T\bar{z}<0\}$ is non-empty, and the singular system (1) is D_L-admissible.

(b) There exists a matrix X such that

$$E^TX=X^TE\geq 0, \tag{7}$$

$$R_{11}\otimes(E^TX)+R_{12}\otimes(X^TA)+R_{12}^T\otimes(A^TX)<0. \tag{8}$$

Proof: (b$\Rightarrow$a): Let $M_1,N_1\in R^{n\times n}$ be nonsingular matrices such that

$$M_1EN_1=\begin{bmatrix} I_r & 0\\ 0 & 0\end{bmatrix} \tag{9}$$

then partition $M_1^{-T}XN_1$, M_1AN_1 conforming to the partition form of (9), that is

$$M_1^{-T}XN_1=\begin{bmatrix} X_1 & X_2\\ X_3 & X_4\end{bmatrix},\ M_1AN_1=\begin{bmatrix} A_1 & A_2\\ A_3 & A_4\end{bmatrix}. \tag{10}$$

From $E^TX=X^TE\geq 0$, it is obtained that $X_2=0$. Pre- and post-multiplying (8) by $I_d\otimes N_1^T$ and its transpose, we obtain that

$$R_{11}\otimes\begin{bmatrix} X_1 & 0\\ 0 & 0\end{bmatrix}+R_{12}\otimes\begin{bmatrix} * & *\\ * & X_4^TA_4\end{bmatrix}+R_{12}^T\otimes\begin{bmatrix} * & *\\ * & A_4^TX_4\end{bmatrix}<0.$$

Pre- and post-multiplying the above inequality by $I_d\otimes\begin{bmatrix}0 & I_{n-r}\end{bmatrix}$ and its transpose, we have

$$R_{12}\otimes(X_4^TA_4)+R_{12}^T\otimes(A_4^TX_4)<0, \tag{11}$$

which implies that A_4 is nonsingular. Otherwise, if A_4 is singular, there exists $0\neq x\in R^{n-r}$ such that $A_4x=0$. Then, we can obtain that

$$(I_d\otimes x^T)\left[R_{12}\otimes(X_4^TA_4)+R_{12}^T\otimes(A_4^TX_4)\right](I_d\otimes x)=0,$$

which conflicts with (11). Hence, we have that the singular system (1) is regular and impulse-free.

Since the singular system (1) is regular and impulse-free, according to the result of Lemma 2, there exist nonsingular matrices $M,N\in R^{n\times n}$ such that

$$MEN=\begin{bmatrix} I_r & 0\\ 0 & 0\end{bmatrix},\quad MAN=\begin{bmatrix} A_1 & 0\\ 0 & I_{n-r}\end{bmatrix}. \tag{12}$$

Partition $M^{-T}XN$ conforming to (12), that is

$$M^{-T}XN=\begin{bmatrix} X_1 & X_2\\ X_3 & X_4\end{bmatrix}.$$

From (7) and (12), it can be obtained that $X_1\geq 0$ and $X_2=0$. Pre- and post-multiplying (8) by $I_d\otimes N^T$ and its transpose, we have

$$R_{11}\otimes\begin{bmatrix} X_1 & 0\\ 0 & 0\end{bmatrix}+R_{12}\otimes\begin{bmatrix} X_1A_1 & *\\ * & X_4^T\end{bmatrix}+R_{12}^T\otimes\begin{bmatrix} A_1^TX_1 & *\\ * & X_4\end{bmatrix}<0. \tag{13}$$

Pre- and post-multiplying (13) by $I_d\otimes[I_r\ \ 0]$ and its transpose, we can get

$$R_{11}\otimes X_1+R_{12}\otimes(X_1A_1)+R_{12}^T\otimes(A_1^TX_1)<0,$$

which also implies X_1 is nonsingular. Using Lemma 1, all the finite eigenvalues of (E,A) are inside D_L. It follows immediately that the singular system (1) is D_L-admissible.

By using similar process, we can obtain that

$$R_{12}\otimes(X_4^T)+R_{12}^T\otimes(X_4)<0.$$

If λ is a eigenvalue of X_4, so there exists eigenvalue vector $x\neq 0$ such that $X_4x=\lambda x$ and $x^*x=1$. Then

$$(I_d\otimes x^*)\left[R_{12}\otimes(X_4^T)+R_{12}^T\otimes(X_4)\right](I_d\otimes x)$$
$$=R_{12}\bar{\lambda}+R_{12}^T\lambda<0,$$

which means $\lambda^*\in D_1$, so D_1 is non-empty.

(a$\Rightarrow$b): Suppose that the singular system (1) is D_L-

admissible, then the pair of system matrices (E, A) has the partition form of (12) and $\lambda(A_1) \subset D_L$. By using Lemma 1, it follows that there exists a positive-definite symmetric matrix $Q_1 \in R^{r\times r}$ such that

$$R_{11} \otimes Q_1 + R_{12} \otimes (Q_1 A_1) + R_{12}^T \otimes (A_1^T Q_1) < 0.$$

Since D_1 is non-empty, there exists $\lambda \in D_1$, then we also have $\bar{\lambda} \in D_1$. Then, it can be obtained that $\alpha = \lambda + \bar{\lambda} \in D_1$, so $\alpha I_{n-r} \in R^{n-r\times n-r}$ is D_1-stable. By using Lemma 1, it follows that there exists a positive-definite symmetric matrix $Q_2 \in R^{n-r\times n-r}$ such that

$$R_{12} \otimes (\alpha Q_2) + R_{12}^T \otimes (\alpha Q_2) < 0.$$

Choose matrices X, T as

$$X = M^T \begin{bmatrix} Q_1 & 0 \\ 0 & \alpha Q_2 \end{bmatrix} N^{-1}, \quad T = \begin{bmatrix} I_d \otimes [I_r \quad 0] \\ I_d \otimes [0 \quad I_{n-r}] \end{bmatrix} \left[I_d \otimes N^T \right]. \tag{14}$$

Then, we can get that

$$T\left(R_{11} \otimes (E^T X) + R_{12} \otimes (X^T A) + R_{12}^T \otimes (A^T X)\right)T^T = \begin{bmatrix} F_2(A_1, Q_1) & 0 \\ 0 & R_{12} \otimes (\alpha Q_2) + R_{12}^T \otimes (\alpha Q_2) \end{bmatrix} < 0.$$

So the above matrix X satisfies (7, 8). □

Remark 1: If a matrix X satisfies $R_{11} \otimes (E^T X) + R_{12} \otimes (X^T A) + R_{12}^T \otimes (A^T X) < 0$, it is obvious that the matrix X is nonsingular. Let $Y = X^{-1}$. Pre- and post-multiplying (7) by Y^T and its transpose, and pre- and post-multiplying (8) by $I_d \otimes Y^T$ and its transpose, we get the equivalent condition of (b) in Theorem 1 as follows: There exists a matrix Y such that

$$Y^T E^T = EY \geq 0, \quad R_{11} \otimes (Y^T E^T) + R_{12} \otimes (AY) + R_{12}^T \otimes (Y^T A^T) < 0. \tag{15}$$

Remark 2: From Theorem 1, we can easily obtain that the singular system (1) is regular, impulse-free and each of its finite eigenvalues $\lambda < -\alpha/2$ if and only if there exists a nonsingular matrix X such that

$$E^T X = X^T E \geq 0, \quad A^T X + X^T A + \alpha E^T X < 0. \tag{16}$$

This result can also be found in [17], so Theorem 1 extends the results of [17].

Remark 3: Consider now the conic sector $S(0,\theta)$ defined as $x \tan\theta < -|y|$. $S(0,\theta)$ is an LMI region characterized by

$$\begin{bmatrix} \sin\theta(z+\bar{z}) & \cos\theta(z-\bar{z}) \\ \cos\theta(-z+\bar{z}) & \sin\theta(z+\bar{z}) \end{bmatrix} < 0. \tag{17}$$

From Theorem 1, a singular system (1) is regular, impulse-free and each of its finite poles in $S(0,\theta)$ if and only if there exists a nonsingular matrix X such that

$$E^T X = X^T E \geq 0, \quad \begin{bmatrix} \sin\theta(X^T A + A^T X) & \cos\theta(X^T A - A^T X) \\ \cos\theta(-X^T A + A^T X) & \sin\theta(X^T A + A^T X) \end{bmatrix} < 0. \tag{18}$$

In the most practical applications, the pole regions lie in the open left-half plane for continuous-time case because of stability of the systems. Some D_L regions such as vertical strips don't satisfy the condition (a) of Theorem 1. Now, we establish Lyapunov type condition for D_L regions located in the open left-half plane.

Theorem 2: For a given D_L region: $D_L = \{z \in C : R_{11} + R_{12} z + R_{12}^T \bar{z} < 0\}$ lies in open left-half plane, the singular system (1) is D_L-admissible if and only if there exist matrices X, P such that

$$\begin{aligned} &E^T X = X^T E \geq 0, \\ &E^T P = P^T E \geq 0, \\ &A^T P + P^T A < 0, \\ &R_{11} \otimes (E^T X) + R_{12} \otimes (X^T A) \\ &\quad + R_{12}^T \otimes (A^T X) + I_d \otimes (E^T P) \leq 0. \end{aligned} \tag{19}$$

Proof: *Sufficiency:* According to Remark 2, the singular system (1) is admissible from

$$E^T P = P^T E \geq 0, \quad A^T P + P^T A < 0.$$

Since the singular system (1) is regular and impulse-free, thus there exist nonsingular matrices $M, N \in R^{n\times n}$ such that the pair of system matrices (E, A) has the partition form of (12). Partition $M^{-T} PN$ and $M^{-T} XN$ conforming to (12) as follows

$$M^{-T} PN = \begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix}, \quad M^{-T} XN = \begin{bmatrix} X_1 & X_2 \\ X_3 & X_4 \end{bmatrix}.$$

From $E^T P = P^T E \ge 0,$ it is obtained that $P_2 = 0$ and $P_1 > 0.$ Pre- and post-multiplying the last equation of (19) by $I_d \otimes [I \quad 0]N^T$ and its transpose, we can obtain that

$$R_{11} \otimes X_1 + R_{12} \otimes (X_1 A_1) + R_{12}^T \otimes (A_1^T X_1) + P_1 \le 0,$$

by noticing that $P_1 > 0,$ we have

$$R_{11} \otimes X_1 + R_{12} \otimes (X_1 A_1) + R_{12}^T \otimes (A_1^T X_1) < 0,$$

which implies all the finite eigenvalues of (E, A) are inside D_L.

Necessity: Suppose that the singular system (1) is D_L-admissible, then the pair of system matrices (E, A) has the partition form of (12) and $\lambda(A_1) \subset D_L.$ By using Lemma 1, it follows that there exist positive-definite symmetric matrices Q_1, Q_2 such that $Q_1 A_1 + A_1^T Q_1 < 0$ and $F_2(A_1, Q_2) < 0.$ Choose matrices X, P as

$$P = M^T \begin{bmatrix} \beta Q_1 & 0 \\ 0 & -I_{n-r} \end{bmatrix} N^{-1}, \quad X = M^T \begin{bmatrix} Q_2 & 0 \\ 0 & 0 \end{bmatrix} N^{-1},$$

and matrix T by (14). Since $F_2(A_1, Q_2) < 0,$ the following inequality satisfies for a small enough constant β

$$T\left(R_{11} \otimes (E^T X) + R_{12} \otimes (X^T A) + R_{12}^T \otimes (A^T X)\right)T^T = \begin{bmatrix} F_2(A_1, Q_2) + \beta Q_1 & 0 \\ 0 & 0 \end{bmatrix} \le 0.$$

The matrices P, X can be verified to satisfy (19) by straightforward calculation for small enough $\beta.$ □

The following theorem concerns the D_R-admissible condition for singular systems.

Theorem 3: For a given D_R region: $D_R = \{z \in C : R_{11} + R_{12} z + R_{12}^T \bar{z} + R_{22} z\bar{z} < 0\},$ suppose that $R_{22} > 0.$ Then, the singular system (1) is D_R-admissible if and only if there exists a symmetric matrix X such that

$$E^T XE \ge 0, \tag{20}$$

$$R_{11} \otimes (E^T XE) + R_{12} \otimes (E^T XA) + R_{12}^T \otimes (A^T XE) + R_{22} \otimes (A^T XA) < 0. \tag{21}$$

Proof: *Sufficiency:* There exist nonsingular matrices $M_1, N_1 \in R^{n\times n}$ such that (9) and (10) are satisfied. From (18), it is obtained that $X_1 \ge 0.$ Pre- and post-multiplying (19) by $I_d \otimes N_1^T$ and its transpose, we obtain that

$$R_{11} \otimes \begin{bmatrix} X_1 & 0 \\ 0 & 0 \end{bmatrix} + R_{12} \otimes \begin{bmatrix} * & * \\ * & 0 \end{bmatrix} + R_{12}^T \otimes \begin{bmatrix} * & * \\ * & 0 \end{bmatrix} + R_{22} \otimes \begin{bmatrix} * & * \\ * & H \end{bmatrix} < 0,$$

where

$$H = A_2^T X_1 A_2 + A_2^T X_2 A_4 + A_4^T X_2^T A_2 + A_4^T X_4 A_4.$$

Pre- and post-multiplying the above inequality by $I_d \otimes \begin{bmatrix} 0 & I_{n-r} \end{bmatrix}$ and its transpose, we obtain that

$$R_{22} \otimes H < 0. \tag{22}$$

If A_4 is singular, there exists $0 \ne x \in R^{n-r}$ such that $A_4 x = 0.$ Then, we can obtain that

$$(I_d \otimes x^T)\left[R_{22} \otimes H\right](I_d \otimes x) = R_{22} \otimes (x^T A_2^T X_1 A_2 x) = (x^T A_2^T X_1 A_2 x) R_{22} \ge 0,$$

which conflicts with (20). Hence, A_4 is nonsingular from which we have that the singular system (1) is regular and impulse-free.

Since the singular system (1) is regular and impulse-free, the pair of system matrices (E, A) has the partition form of (12). Partition $M^{-T} XM^{-1}$ conforming to (12), that is

$$M^{-T} XM^{-1} = \begin{bmatrix} X_1 & X_2 \\ X_2^T & X_4 \end{bmatrix}. \tag{23}$$

From (12, 20) and (23), it can be obtained that $X_1 \ge 0.$ Pre- and post-multiplying (19) by $I_d \otimes N^T$ and its transpose, we obtain that

$$R_{11} \otimes \begin{bmatrix} X_1 & 0 \\ 0 & 0 \end{bmatrix} + R_{12} \otimes \begin{bmatrix} X_1 A_1 & * \\ * & * \end{bmatrix} + R_{12}^T \otimes \begin{bmatrix} A_1^T X_1 & * \\ * & * \end{bmatrix} + R_{22} \otimes \begin{bmatrix} A_1^T X_1 A_1 & * \\ * & * \end{bmatrix} < 0.$$

Pre- and post-multiplying above inequality by $I_d \otimes [I_r \quad 0]$ and its transpose, we obtain that

$$F_1(A_1, X_1) < 0,$$

which also implies X_1 is nonsingular and all the finite eigenvalues of (E, A) are inside D_R by using Lemma 1 and 2. It follows immediately that the singular system (1) is D_R-admissible.

Necessity: Suppose that the singular system (1) is D_R-admissible, the pair of system matrices (E,A) has the partition form of (12) and $\lambda(A_1)\subset D_R$. By using Lemma 1, it follows that there exists a positive-definite symmetric matrix $Q_1\in R^{r\times r}$ such that

$$F_1(A_1,Q_1)<0.$$

Now, define a matrix X,T by

$$X=M^T\begin{bmatrix}Q_1 & 0\\ 0 & -I_{n-r}\end{bmatrix}M,\qquad T=\begin{bmatrix}I_d\otimes\begin{bmatrix}I_r & 0\end{bmatrix}\\ I_d\otimes\begin{bmatrix}0 & I_{n-r}\end{bmatrix}\end{bmatrix}\begin{bmatrix}I_d\otimes N^T\end{bmatrix}. \tag{24}$$

Then, we can get that

$$T\begin{bmatrix}I_d\otimes N^T\end{bmatrix}\begin{bmatrix}F_4(X)\end{bmatrix}T^T=\begin{bmatrix}F_1(A_1,Q_1) & 0\\ 0 & R_{22}\otimes(-I_{n-r})\end{bmatrix}<0.$$

So the above symmetric matrix X satisfies (20, 21). □

Remark 4: From Theorem 3, we can easily obtain that the singular systems (1) is regular, impulse-free and each of its finite eigenvalues $|\lambda|<d$ if and only if there exists a symmetric matrix X such that

$$E^TXE\geq 0,\qquad A^TXA-dE^TXE<0. \tag{25}$$

In case $d=1$, the condition of (25) represents admissibility condition of the discrete-time singular systems.

Example 1: To demonstrate the feasibility of the approach presented in this paper, we give a simple example by using LMI solver of MATLAB. We consider the singular systems $E\dot{x}=Ax$ with the following system matrices:

$$E=\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix},\quad A=\begin{bmatrix}-1&1&0\\-2&-3&0\\1&2&3\end{bmatrix}$$

taking $\theta=\frac{\pi}{4}$ and using results of Remark 3, the condition (18) has a solution matrix

$$X=\begin{bmatrix}0.5482&0.0751&0\\0.0751&0.2076&0\\-0.0553&-0.0049&-0.1773\end{bmatrix},$$

which implies that the singular systems is regular, impulse-free and all its finite eigenvalues of pair (E,A) are inside region $S\left(0,\frac{\pi}{4}\right)$ In this case, the above results can also be obtained by direct calculation. If the system matrix A is replaced as

$$A=\begin{bmatrix}-1&2&0\\-2&-1&0\\1&2&3\end{bmatrix}$$

the condition (18) has no solution by taking $\theta=\frac{\pi}{4}$. If we take $\theta>\arctan(\frac{1}{2})$, the condition (18) has solution matrix by using LMI solver of MATLAB. By direct calculation, the pair (E,A) has eigenvalues $-1\pm 2j$ which are not inside $S(0,\frac{\pi}{4})$.

4. CONCLUSION

In this paper, we first establish an LMI's D_L-admissibility condition for descriptor systems, which extends the results of [8] to descriptor systems. We also establish an D_R-admissible condition which extends the results of [7] to generalized systems. The Lyapunov equation established in this paper can be applied to solve the pole-clustering problems and robust control problems. By using the similar ideas given in [7,14,15] for normal systems, it is not difficult to establish robust LMI *D*-admissibility conditions of descriptor systems for polytopic uncertainty based on the results obtained in this paper.

REFERENCES

[1] L. Dai, *Singular Control Systems, Lecture Notes in Control and Information Sciences*, vol. 118, Springer-Verlag, New York, 1989.

[2] C. Lin, J. Lam, J. Wang, and G.-H. Yang, "Analysis on robust stability for interval descriptor systems," *Systems & Control Letters*, vol. 42, pp. 267-278, 2001.

[3] S. Xue and C. Yang, "Stabilization of discrete-time singular systems: A matrix inequalities approach," *Automatica*, vol. 35, pp. 1613-1617, 1999.

[4] Q. Zhang, W. Liu, and D. Hill, "A Lyapunov approach to analysis of discrete singular systems," *Systems & Control Letters*, vol. 45, pp. 237-247, 2002.

[5] E. Fridman and U. Shaked, "A Descriptor system approach to H^∞ control of linear time-delay systems," *IEEE Trans. on Automatic Control*, vol. 47, no. 2, pp. 253-270, 2002.

[6] S. Xue, C. Yang, Y. Niu, and J. Lam, "Robust stabilization for uncertain discrete singular

systems," *Automatica*, vol. 37, pp.769-774, 2001.
[7] D. Peaucelle, D. Arzelier, O. Bachelier, and J. Bernussou, "A new robust *D*-stability condition for real convex polytopic uncertainty," *Systems & Control Letters*, vol. 40, pp. 21-30, 2000.
[8] M. Chilali, P. Gahinet, and P. Apkarian, "Robust pole placement in LMI regions," *IEEE Trans. on Automatic Control*, vol. 44, pp. 2257-2270, 1999.
[9] S. Gutman and E. I. Jury, "A general theory for matrix root clustering in subregions of the complex plan," *IEEE Trans. on Automatic Control*, vol. 26, pp. 853-863, 1981.
[10] X. Chen and K. Zhou, "Fast parallel- frequency-sweeping algorithms for robust D-stability margin," *IEEE Trans. on Circuits and Systems-I: Fundamental Theory and Applications*, vol. 50, no. 3, pp. 418-428, 2003.
[11] R. Yua and D. Wang, "Structural properties and poles assignability of LTI singular systems under output feedback," *Automatica*, vol. 39, pp. 685-692, 2003.
[12] T. Stykel, "Stability and inertia theorems for generalized Lyapunov equations," *Linear Algebra and its Applications*, vol. 355, no. 1-3, pp. 297-314, 2002.
[13] J. Y. Ishihara and M. H. Terra, "On the Lyapunov theorem for singular systems," *IEEE Trans. on Automatic Control*, vol. 47, no. 11, pp. 1926-1930, November 2000.
[14] J. C. Geromel, M. C. De Oliveira, and L. Hsu, "LMI characterization of structural and robust stability," *Linear Algebra and its Applications*, vol. 285, pp. 68-80, 1998.
[15] M. C. De Oliveira, J. C. Geromel, and L. Hsu, "LMI characterization of structural and robust stability: The discrete-time case," *Linear Algebra and its Applications*, vol. 296, pp. 27-38, 1999.
[16] S. P. Boyd, L. EI Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, SIAM, Philadelphia, 1994.
[17] X. Huang and B. Huang, "An BMI approach towards single and simultaneous admissible stabilization of continuous-time descriptor systems via static output feedback," *Dynamics of Continuous, Discrete and Impulse Systems Series B: Applications & Algorithms*, vol. 9, pp. 71-83, 2002.

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