

# Design of PD Observers in Descriptor Linear Systems

Ai-Guo Wu and Guang-Ren Duan

**Abstract:** A class of new observers in descriptor linear systems, proportional-derivative (PD) observers, are proposed. A parametric design approach for such observers is proposed based on a complete parametric solution to the generalized Sylvester matrix equation. The approach provides complete parameterizations for all the observer gains, gives the parametric expression for the corresponding left eigenvector matrix of the observer system matrix, realizes elimination of impulsive behaviors, and guarantees the regularity of the observer system.

**Keywords:** Degrees of freedom, descriptor systems, parametric approach, PD observers.

## 1. INTRODUCTION

Descriptor linear systems appear in many fields, such as power systems, electrical networks, social economic systems, and so on, and therefore have been dealt with intensively [1-4]. Parallel to the conventional linear system theory, the problem of designing observers for descriptor linear systems is investigated by a number of scholars. In [4], the design of full-order observers for descriptor linear systems is considered. In [5], the design of reduced-order observers for descriptor linear systems is developed. While many approaches for designing Luenberger observers for descriptor linear systems are also investigated in [6]. From the classic control theory, it is well known that the integral action is useful to achieve steady-state accuracy. Based on such ideas, recently, the PI observers for descriptor linear systems have attracted the attention of many researches [7,8]. In this paper, a new type of observers, which is termed as proportional derivative observer, is introduced. This observer includes the derivative of the estimation error.

This paper is a revised version of [9]. In this note, the type of proportional-derivative (PD) observers for descriptor linear systems is proposed by introducing a derivative term in full-order state observers. Based on a complete parametric solution to a type of generalized Sylvester matrix equations in [10], a parametric design approach for the proposed PD

observers is presented. The proposed method guarantees the regularity of the observer system and realizes the elimination of impulsive responses. In terms of two parameter sets which represent the degrees of the design freedom, i.e., the set of the finite eigenvalues of the observer system  $\{s_i, i=1,2,\dots,n\}$ , a group of parameter vectors  $\{v_i, g_i, i=1,2,\dots,n\}$ , parameterizations for all the gain matrices of the observer system are established. The existence conditions for the proposed type of PD observers are given by some constraints on these design parameters. The proposed method utilizes the right coprime factorizations of the original system. The proposed approach offers all the design degrees of freedom which can be utilized to achieve additional various system performances. In addition, we show that the proposed PD observer possesses separation property, thus the type of PD observers is qualified as an effective observer.

## 2. PROBLEM FORMULATION

Consider the following descriptor linear system

$$\begin{cases} E\dot{x}(t) = Ax(t) + Bu(t), \\ y_p(t) = C_p x(t), \\ y_d(t) = C_d \dot{x}(t), \end{cases} \quad (1)$$

where  $x \in \mathbb{R}^n$ ,  $u \in \mathbb{R}^r$ ,  $y_p \in \mathbb{R}^m$ , and  $y_d \in \mathbb{R}^l$  are respectively the state vector, the input vector, the output vector and the derivative output vector.  $E$ ,  $A \in \mathbb{R}^{n \times n}$ ,  $B \in \mathbb{R}^{n \times r}$ ,  $C_p \in \mathbb{R}^{m \times n}$ , and  $C_d \in \mathbb{R}^{l \times n}$  are the known real matrices. Let

$$C = \begin{bmatrix} C_p^T & C_d^T \end{bmatrix}^T, \quad (2)$$

for the system (1), and assume that the following

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assumptions are met:

**Assumption A1:** Matrix  $C$  is of full-row rank;

**Assumption A2:** The matrix triple  $(E, A, C)$  is R-observable, that is, for any  $s \in \mathbb{C}$ ,

$$\text{rank} \begin{bmatrix} sE - A \\ C \end{bmatrix} = n; \quad (3)$$

**Assumption A3:**

$$\text{rank} \begin{bmatrix} E \\ C_d \end{bmatrix} = n. \quad (4)$$

For system (1), a PD observer in the following form is introduced

$$E\dot{\hat{x}} = A\hat{x} + Bu + L_p(y_p - C_p\hat{x}) + L_d(y_d - C_d\hat{x}), \quad (5)$$

where  $\hat{x} \in \mathbb{R}^n$  is the estimated state vector, and  $L_p \in \mathbb{R}^{n \times m}$  and  $L_d \in \mathbb{R}^{n \times l}$  are the observer gains. Specifically, the matrices  $L_p$  and  $L_d$  are called proportional and derivative gains, respectively.

**Definition 1:** The system (5) is called a PD observer for the system (1), if for any initial conditions  $x(0)$  and  $\hat{x}(0)$  and any input  $u(t)$ , the following relation holds:

$$\lim_{t \rightarrow \infty} (x(t) - \hat{x}(t)) = 0. \quad (6)$$

Let  $e(t) = \hat{x}(t) - x(t)$ , then combining (1) and (5) gives

$$E_o \dot{e} = A_o e \quad (7)$$

with

$$E_o = E + L_d C_d, \quad A_o = A - L_p C_p. \quad (8)$$

Thus the system (5) is a PD observer for the system (1) if and only if the matrix pair  $(E_o, A_o)$  is Hurwitz stable. Since nondefective matrix pairs have lower eigenvalue sensitivities than defective ones, we require that the Jordan form of the matrix pair  $(E_o, A_o)$  is diagonal. Furthermore, we demand that the matrix pair  $(E_o, A_o)$  has  $n$  finite eigenvalues, which ensure that the observer system (5) is regular, then elimination of all possible initial time impulsive responses is realized in system (5). Based on the above reasoning, we can state our problem of designing PD observers for descriptor linear system (1) as follows:

**Problem PDO:** Given matrices  $E, A \in \mathbb{R}^{n \times n}$ ,  $B \in \mathbb{R}^{n \times r}$ ,  $C_p \in \mathbb{R}^{m \times n}$ , and  $C_d \in \mathbb{R}^{l \times n}$  satisfying

Assumptions A1, A2, and A3, find the complete parameterizations for the observer gain matrices  $L_p$  and  $L_d$  such that the matrix pair  $(E_o, A_o)$  in (8) satisfying the following requirements:

- 1) it is Hurwitz stable and nondefective;
- 2) it is regular and possesses  $n$  finite eigenvalues.

### 3. SOLUTION TO PROBLEM PDO

In this section, we will provide a general solution to Problem PDO proposed in Section 2.

#### 3.1. Some basic relations

Since the matrix pair  $(E_o, A_o)$  is required to be nondefective, it has a diagonal Jordan form

$$\Lambda = \text{diag}(s_1, s_2, \dots, s_n), \quad (9)$$

where  $s_i, i=1, 2, \dots, n$ , are not necessary distinct. Obviously,  $s_i, i=1, 2, \dots, n$ , are the finite eigenvalues of  $(E_o, A_o)$ . To ensure that the matrix pair  $(E_o, A_o)$  is Hurwitz stable and real, the following constraint is satisfied:

**Constraint 1:**  $\{s_i, i=1, 2, \dots, n\}$  is self-conjugate and  $\text{Re } s_i < 0, i=1, 2, \dots, n$ .

Denote the left eigenvector associated with the finite eigenvalue  $s_i$  of  $(E_o, A_o)$  by  $t_i \in \mathbb{C}^n, i=1, 2, \dots, n$ . Construct the left eigenvector matrix by

$$T = [t_1 \quad t_2 \quad \dots \quad t_n], \quad (10)$$

then by definition we have  $\text{rank } T = n$ , and

$$T^T (A - L_p C_p) = \Lambda T^T (E + L_d C_d), \quad (11)$$

which can be written equivalently as

$$T^T A - [T^T L_p \quad \Lambda T^T L_d] C = \Lambda T^T E. \quad (12)$$

Let

$$Z^T = -[T^T L_p \quad \Lambda T^T L_d], \quad (13)$$

(12) can be equivalently written as

$$T^T A + Z^T C = \Lambda T^T E. \quad (14)$$

#### 3.2. Solution to matrices $T$ and $Z$

Taking transpose of (14) gives

$$A^T T + C^T Z = E^T T \Lambda, \quad (15)$$

which is the generalized Sylvester matrix equation investigated in [10]. According to the result in [10],

the following lemma is obtained.

**Lemma 1:** Under Assumption A2, all the matrices  $T$  and  $Z$  satisfying (14) are represented by the following parametric expressions

$$T = [N(s_1)g_1 \quad N(s_2)g_2 \quad \cdots \quad N(s_n)g_n], \quad (16)$$

$$Z = [D(s_1)g_1 \quad D(s_2)g_2 \quad \cdots \quad D(s_n)g_n], \quad (17)$$

where  $g_i \in \mathbf{C}^{m+l}$ ,  $i=1,2,\dots,n$ , are a group of arbitrary parameter vectors and  $N(s) \in \mathbf{R}^{n \times (m+l)}[s]$  and  $D(s) \in \mathbf{R}^{(m+l) \times (m+l)}[s]$  are the right coprime polynomial matrices satisfying the following right coprime factorization:

$$(sE^T - A^T)^{-1}C^T = N(s)D^{-1}(s). \quad (18)$$

To ensure that the gain matrices  $L_p$  and  $L_d$  are both real, the following constraint must be satisfied:

**Constraint C2:**  $g_i = \bar{g}_l$ , if  $s_i = \bar{s}_l$ .

Having the parameterization of the matrix  $T$ , condition  $\text{rank } T = n$  can be converted into the following constraint:

**Constraint C3:**

$$\det[N(s_1)g_1 \quad N(s_2)g_2 \quad \cdots \quad N(s_n)g_n] \neq 0.$$

### 3.3. Solution to matrices $L_p$ and $L_d$

Partition the matrix  $Z$  in (13) into

$$Z = \begin{bmatrix} Z_p \\ Z_d \end{bmatrix}, Z_p \in \mathbf{C}^{m \times n}, Z_d \in \mathbf{C}^{l \times n}. \quad (19)$$

Then we have

$$T^T L_p = -Z_p^T, \quad (20)$$

$$\Lambda T^T L_d = -Z_d^T. \quad (21)$$

From Constraints C1 and C2, it follows that the matrices  $\Lambda$  and  $T$  are both nonsingular, the observer gain matrices  $L_p$  and  $L_d$  are obtained as

$$L_p = -T^{-T} Z_p^T, \quad (22)$$

$$L_d = -T^{-T} \Lambda^{-1} Z_d^T. \quad (23)$$

For consideration of regularity of the system (7), the following lemma is given.

**Lemma 2:** Given the matrices  $E, A \in \mathbf{R}^{n \times n}$ ,  $B \in \mathbf{R}^{n \times r}$ ,  $C_d \in \mathbf{R}^{m \times n}$ , and  $C_d \in \mathbf{R}^{l \times n}$ , suppose that the observer system (7) possesses  $n$  eigenvalues, then the matrix pair  $(E_o, A_o)$  is regular if and only if

$$\det(E + L_d C_d) \neq 0. \quad (24)$$

**Proof:** It follows from (11) that  $T^T A_o = \Lambda T^T E_o$ . Due to the above relation and (8), it is easily obtained that

$$\begin{aligned} T^T (sE_o - A_o) &= (sT^T E_o - \Lambda T^T E_o) \\ &= (sI - \Lambda) T^T (E + L_d C_d). \end{aligned} \quad (25)$$

Thus we have

$$\begin{aligned} \det T^T \det(sE_o - A_o) \\ = \det(sI - \Lambda) \det T^T \det(E + L_d C_d), \end{aligned} \quad (26)$$

Since  $\det T^T$  is non-zero in view of Constraint C3, this it is easily known that  $\det(sE_o - A_o)$  is not identically zero if and only if (24) is satisfied since  $\det(sI - \Lambda)$  is not identically zero.  $\square$

In view of (23), we have

$$\Lambda T^T (E + L_d C_d) = \Lambda T^T E - Z_d^T C_d. \quad (27)$$

Since  $\Lambda T^T$  is nonsingular, from the above it is obvious that (24) is equivalent to

$$\det(\Lambda T^T E - Z_d^T C_d) \neq 0. \quad (28)$$

Partition the polynomial matrix  $D(s)$  satisfying (18) into the following form:

$$D(s) = \begin{bmatrix} D_p(s) \\ D_d(s) \end{bmatrix}, D_p(s) \in \mathbf{R}^{(m+l) \times (m+l)}[s], \quad (29)$$

then  $Z_d$  can be expressed by

$$Z_d = [D_d(s_1)g_1 \quad D_d(s_2)g_2 \quad \cdots \quad D_d(s_n)g_n]. \quad (30)$$

With the above preparation, the regularity condition (24) is converted into the following constraint on the design parameters  $s_i, g_i, i=1,2,\dots,n$ :

**Constraint C4:**  $\det[h_1 \quad h_2 \quad \cdots \quad h_n] \neq 0$  with

$$h_i = (s_i E^T N(s_i) - C_d^T D_d(s_i)) g_i.$$

Summarizing the results in the above subsections, we have the following theorem for solution to Problem PDO.

**Theorem 1:** Given the system (1) satisfying the Assumptions A1, A2 and A3, Problem PDO has a solution if and only if there exist a group of parameter  $s_i, g_i, i=1,2,\dots,n$ , satisfying Constraints C1-C4. If such parameters exist, then the observer gain matrices  $L_p$  and  $L_d$  are given by (22) and (23) through (16)-(19).

For the proposed parametric approach for solving

Problem PDO, we provide the following remarks.

**Remark 1:** A method to solve the coprime factorization (18) has been given in [10].

**Remark 2:** It has been shown that under the condition of Assumption A3, there always exists a real matrix  $L_d$  satisfying (24). Moreover, all the matrices  $L_d$  satisfying (24) form a Zariski open set, thus Constraint C4 can be neglected if only one special solution is of interest.

**Remark 3:** The presented approach can provide all the degrees of design freedom, which is represented by the free parameters  $s_i, g_i, i=1,2,\dots,n$ . They can be further utilized to achieve additional various desired system specifications and performances such as disturbance decoupling, LTR/LQR and robustness against parameter uncertainty.

#### 4. SEPARATION PROPERTY

##### 4.1. The closed-loop system

The PD observer-based state plus output derivative feedback

$$\begin{cases} E\dot{\hat{x}} = A\hat{x} + Bu + L_p(y_p - C_p\hat{x}) + L_d(y_d - C_d\dot{\hat{x}}) \\ u = K\hat{x} - K_d y_d + v, \end{cases} \quad (31)$$

with  $v \in \mathbb{R}^r$  being an external input, is applied to the system (1), the closed-loop system is obtained as

$$\begin{cases} E_c \dot{x}_c = A_c x_c + B_c v \\ y = C_c x_c \end{cases} \quad (32)$$

with

$$\begin{aligned} x_c &= \begin{bmatrix} x \\ \hat{x} \end{bmatrix}, \quad E_c = \begin{bmatrix} E + BK_d C_d & 0 \\ BK_d C_d - L_d C_d & E + L_d C_d \end{bmatrix}, \\ A_c &= \begin{bmatrix} A & BK \\ L_p C_p & A - L_p C_p + BK \end{bmatrix}, \\ B_c &= \begin{bmatrix} B \\ B \end{bmatrix}, \quad C_c = \begin{bmatrix} C_p & 0 \\ sC_d & 0 \end{bmatrix}, \end{aligned}$$

where  $s$  is the Laplace operator.

##### 4.2. Separation property

**Theorem 2:** Given system (1), the closed-loop transfer function under the PD observer-based state plus output derivative feedback (31) is equal to the one under state feedback plus output derivative feedback control law

$$u = Kx - K_d y_d + v. \quad (33)$$

**Proof:** The closed system of (1) under control law (33) is

$$\begin{cases} (E + BK_d C_d)\dot{x} = (A + BK)x + Bv, \\ y = \begin{bmatrix} C_p \\ sC_d \end{bmatrix} x, \end{cases}$$

whose transfer function is easily found to be

$$\begin{bmatrix} C_p \\ sC_d \end{bmatrix} [s(E + BK_d C_d) - (A + BK)]^{-1} B. \quad (34)$$

Let

$$Q = \begin{bmatrix} I_n & 0 \\ -I_n & I_n \end{bmatrix}, \quad P = \begin{bmatrix} I_n & 0 \\ I_n & I_n \end{bmatrix}, \quad (35)$$

then we have

$$\begin{aligned} QE_c P &= \begin{bmatrix} E + BK_d C_d & 0 \\ 0 & E_o \end{bmatrix}, \\ QA_c P &= \begin{bmatrix} A + BK & BK \\ 0 & A_o \end{bmatrix}, \\ QB_c &= \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad C_c P = \begin{bmatrix} C_p & 0 \\ sC_d & 0 \end{bmatrix}. \end{aligned} \quad (36)$$

Therefore, the closed-loop system (32) is restricted system equivalent to the following system:

$$\left( \begin{bmatrix} E + BK_d C_d & 0 \\ 0 & E_o \end{bmatrix}, \begin{bmatrix} A + BK & BK \\ 0 & A_o \end{bmatrix}, \begin{bmatrix} B \\ 0 \end{bmatrix}, \begin{bmatrix} C_p & 0 \\ sC_d & 0 \end{bmatrix} \right)$$

under transformation  $(P, Q)$  with  $P$  and  $Q$  being defined by (35). By some simple computations, it is easily found that the transfer function of the system (32) is equal to the expression in (34) which is the transfer function under the state plus output derivative feedback (33). In view of the fact that two restricted equivalent systems have the same transfer functions, the conclusion is true.  $\square$

Based on the proof of the above theorem, it is easily seen that the proposed PD observer satisfies the separation principle, which is given by the following theorem.

**Theorem 3:** Given the system (1), the eigenvalues of the closed-loop system under the PD observer-based state plus output derivative feedback (31) are the union of those of the PD observer and the state plus derivative output feedback.

**Proof:** It follows from (36) that the closed-loop system matrix pair  $(E_c, A_c)$  is restricted system equivalent to the following matrix pair under transformation  $(P, Q)$  with  $P$  and  $Q$  being defined by (36):

$$\left( \begin{bmatrix} E + BK_d C_d & 0 \\ 0 & E_o \end{bmatrix}, \begin{bmatrix} A + BK & BK \\ 0 & A_o \end{bmatrix} \right),$$

therefore there holds

$$\sigma(E_c, A_c) = \sigma(E_o, A_o) \cup \sigma(E + BK_d C_d, A + BK).$$

Thus the conclusion is proved.  $\square$

From the above two theorems, the proposed PD observer satisfies the essential features of observers and hence is qualified as an effective observer.

## 5. AN ILLUSTRATIVE EXAMPLE

Consider a system in the form of (1) with the following parameters

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$C_p = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \quad C_d = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

For this system,  $n=5$ ,  $m=l=2$ . It is easy to verify that the Assumptions A1, A2 and A3 are met. In the following, we will design a PD observer in the form of (5) for this system.

**Step 1:** Solving the right coprime factorization (18) gives

$$N(s) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & s \end{bmatrix}, \quad D(s) = \begin{bmatrix} 0 & s & -1 & 0 \\ -1 & 0 & s & 0 \\ 0 & -1 & 0 & -1 \\ s & -1 & 0 & s^2 \end{bmatrix}.$$

**Step 2:** For simplicity, we restrict the eigenvalues  $s_i$ ,  $i=1, 2, 3, 4, 5$ , of the observer system to be negative real, thus Constraint C1 is automatically satisfied. In this case, the parameter vectors  $g_i$ ,  $i=1, 2, 3, 4, 5$ , are restricted to be real, Constraint C2 is therefore met. Denote

$$g_i = [g_{i1} \ g_{i2} \ g_{i3} \ g_{i4}]^T, \quad i=1,2,3,4,5,$$

then from (16) and (17), Constraints C3 and C4 are respectively derived as

$$\det \begin{bmatrix} g_1 & g_2 & g_3 & g_4 & g_5 \\ s_1 g_{14} & s_2 g_{24} & s_3 g_{34} & s_4 g_{44} & s_5 g_{54} \end{bmatrix} \neq 0,$$

$$\det \begin{bmatrix} g_{12} + s_1 g_{14} & g_{22} + s_2 g_{24} & \cdots & g_{52} + s_5 g_{54} \\ s_1 g_{12} & s_2 g_{22} & \cdots & s_5 g_{52} \\ s_1 g_{13} & s_2 g_{23} & \cdots & s_5 g_{53} \\ g_{12} + g_{14} & g_{22} + g_{24} & \cdots & g_{52} + g_{54} \\ s_1 g_{14} & s_2 g_{24} & \cdots & s_5 g_{54} \end{bmatrix} \neq 0.$$

**Step 3:** Specially, we choose the parameters satisfying the foregoing constraints as

$$s_i = -i, \quad i=1,2,3,4,5;$$

$$g_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad g_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad g_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad g_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad g_5 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}.$$

**Step 4:** Based on the parameters obtained in Step 3, we obtain

$$T = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ -1 & -2 & -3 & 0 & 0 \end{bmatrix},$$

$$Z_p = \begin{bmatrix} -1 & -1 & 0 & -5 & -1 \\ 0 & -2 & -1 & -5 & -6 \end{bmatrix},$$

$$Z_d = \begin{bmatrix} -2 & -1 & -1 & -1 & 0 \\ 0 & 4 & 6 & -5 & -5 \end{bmatrix}.$$

**Step 5:** According to (22) and (23), the observer gain matrices are

$$L_p = \begin{bmatrix} -1 & \frac{10}{3} \\ 4 & -1 \\ 2 & \frac{8}{3} \\ -5 & \frac{8}{3} \\ -2 & \frac{5}{3} \end{bmatrix}, \quad L_d = \begin{bmatrix} -\frac{13}{36} & -\frac{5}{4} \\ -\frac{1}{4} & -\frac{1}{4} \\ \frac{13}{36} & \frac{1}{4} \\ -\frac{95}{36} & -\frac{5}{4} \\ -\frac{8}{9} & -\frac{3}{2} \end{bmatrix}.$$

## 6. CONCLUSION

A type of PD observers for descriptor linear systems are proposed. Based on a general parametric solution to a type of generalized Sylvester matrix equations, a parametric approach to the design of the proposed PD observers is presented. The proposed approach offers all the degrees of design freedom which can be utilized to achieve additional various system specifications. In addition, it is shown that the proposed PD observer possesses separation property which is the essential feature of observers. Therefore, it is qualified as an effective observer.

## REFERENCES

- [1] G. R. Duan, "Eigenstructure assignment in descriptor linear systems via output feedback," *International Journal of Control*, vol. 72, no. 4, pp. 345-364, April 1999.
- [2] J. D. Cobb, "Controllability, observability and duality in singular systems," *IEEE Trans. on Automatic Control*, vol. 29, no. 12, pp. 1076-1082, December 1984.
- [3] F. L. Lewis and K. Ozcaldiran, "Geometric structure and feedback in singular systems," *IEEE Trans. on Automatic Control*, vol. 34, no. 4, pp. 459-455, April 1989.
- [4] L. Dai, "Observers for discrete singular systems," *IEEE Trans. on Automatic Control*, vol. 33, no. 2, pp. 187-191, February 1988.
- [5] M. El-Tohami, V. Lovass-Nagy, and R. Mukundan, "On the design of the observers for generalized state space systems using singular value decomposition," *International Journal of Control*, vol. 38, no. 4, pp. 673-683, March 1983.
- [6] M. Hou and P. C. Müller, "Design of a class of Luenberger observers for descriptor linear systems," *IEEE Trans. on Automatic Control*, vol. 40, no. 1, pp. 133-136, January 1995.
- [7] D. Koenig and S. Mammar, "Design of proportional-integral observer for unknown input descriptor systems," *IEEE Trans. on Automatic Control*, vol. 47, no. 12, pp. 2057-2062, December 2002.
- [8] G. R. Duan and A. G. Wu, "Design of PI observers in descriptor linear systems," *Proc. of the 22nd Chinese Control Conference* (in Chinese), pp. 99-103, 2003.
- [9] G. R. Duan and A. G. Wu, "Design of PD observers in descriptor linear systems," *Proc. of the 22nd Chinese Control Conference* (in Chinese), pp. 104-108, 2003.
- [10] G. R. Duan, "On the solution to sylvester matrix equation  $AV+BW=EVF$ ," *IEEE Trans. on Automatic Control*, vol. 41, no. 4, pp. 2490-2494, April 1996.



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