Accommodation Rule Based on Navigation Accuracy for Double Faults in Redundant Inertial Sensor Systems

Cheol-Kwan Yang and Duk-Sun Shim*

Abstract: This paper considers a fault accommodation problem for inertial navigation systems (INS) that have redundant inertial sensors such as gyroscopes and accelerometers. It is well-known that the more sensors are used, the smaller the navigation error of INS is, which means that the error covariance of the position estimate becomes less. Thus, when it is decided that double faults occur in the inertial sensors due to fault detection and isolation (FDI), it is necessary to decide whether the faulty sensors should be excluded or not. A new accommodation rule for double faults is proposed based on the error covariance of triad-solution of redundant inertial sensors, which is related to the navigation accuracy of INS. The proposed accommodation rule provides decision rules to determine which sensors should be excluded among faulty sensors. Monte Carlo simulation is performed for dodecahedron configuration, in which case the proposed accommodation rule can be drawn in the decision space of the two-dimensional Cartesian coordinate system.

Keywords: Accommodation rule, fault accommodation, fault detection and isolation, inertial sensors, parity equation.

1. INTRODUCTION

Today all sorts of control, navigation and communication systems consist of various subsystems and thus the hardware and software structure of those systems are complicated. Therefore the importance of reliability of the whole system has been increased. The reliability of the whole system can be obtained by the fault detection and isolation (FDI) method and fault accommodation after FDI. FDI methods have been studied from the 1960's in various engineering problem areas. As reported in literature such as survey papers [1,2] and books [3,4], various methods of FDI have been studied and applied in diverse applications.

FDI algorithms are designed to use all redundant information of the plant and sensors. Redundancy is broadly classified as hardware redundancy [5-12] and analytic redundancy [4,13-16]. With hardware redundancy, more than the minimum number of sensors is used. For example, two or more sensors are used for scalar variables, and four or more for vector variables. Inertial navigation systems (INS) use basically three accelerometers and gyroscopes to calculate navigation information such as position, velocity and attitude. To obtain reliability and to enhance navigation accuracy, INS may use redundant sensors. Numerous studies on FDI for redundant sensors have been performed so far. There are many papers for FDI such as look-up table [5], squared error (SE) method [5], generalized likelihood test (GLT) method [6] and optimal parity test (OPT) method [7] for hardware redundancy.

With analytical redundancy, additional information is obtained from a system's mathematical model. This type of redundancy is based on the idea that inherent redundancy exists in the dynamic relationship between inputs and outputs of the system model. Analytical redundancy has been studied in many applications such as aerospace systems, public transportation vehicles, and nuclear power plants. Frank has reviewed the state-of-the-art FDI in automatic processes by using analytical redundancy [15], and Betta *et al*. reviewed several analytical redundancy-based techniques [2].

One way of detecting a fault is to fix a fault threshold, and if the fault estimate is greater than the threshold, it is determined that a fault has occurred. One typical way to obtain a fault threshold is by using the probability of false alarm. Measurement noise is usually assumed as white Gaussian. If there is no fault, then the probability density function of the fault

Manuscript received January 11, 2006; revised November 6, 2006; accepted February 13, 2007. Recommended by Editorial Board member Hyo-Choong Bang under the direction of Editor Jae Weon Choi. This work was supported by Grant (R01-2003-000-10430-0) from the Basic Research Program of the Korea Science & Engineering Foundation.

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estimate has Gaussian distribution. Thus if the probability of false alarm is given, the corresponding fault threshold can be determined. [17] determines a fault threshold using both false alarm in the absence of fault and miss detection in the presence of fault. [18] suggests a new accommodation threshold based on the error covariance of an estimated variable, which is related to the navigation accuracy of INS. The more sensors are used, the better the navigation performance of INS is, which means that the error covariance of the position estimate becomes less. Thus when INS uses redundant inertial sensors it may happen that even though there is a fault, the faulty sensor should be used so as not to lose the navigation accuracy of INS. The accommodation threshold gives a decision rule to determine whether a faulty sensor should be excluded or not. If a fault is less than the accommodation threshold, it is said to be a tolerable fault and should not be excluded. If the fault is greater than the accommodation threshold, it is said to be a non-tolerable fault and should be excluded.

This paper suggests a new accommodation rule for double faults in redundant inertial sensors, while [18] suggests an accommodation threshold for a single fault in redundant inertial sensors. So far there are no results for the accommodation rule for the double fault case in the literature. For the double fault case, the accommodation rule is given as a region in the two-dimensional space, while the accommodation rule is given as a threshold for the single fault case.

2. FAULT DETECTION, ISOLATION, AND ACCOMMODATION (FDIA)

Consider a typical measurement equation for redundant inertial sensors.

$$m(t) = Hx(t) + f(t) + \varepsilon(t), \tag{1}$$

where

$m(t) = [m_1\ m_2 \cdots m_n]^T \in R^n$: inertial sensor measurement,

$H = [h_1\ \cdots\ h_n]^T$: n×3 measurement matrix with rank (H^T)=3,

$x(t) \in R^3$: triad-solution(acceleration or angular rate),

$f(t) = [f_1\ f_2\ \cdots\ f_n]^T \in R^n$: fault vector,

$\varepsilon(t) \sim N(0_n, \sigma I_n)$: a measurement noise vector, normal distribution (white noise),

$N(x,y)$: Gaussian probability density function with mean x and standard deviation y.

A parity vector $p(t)$ is obtained using a matrix V as follows:

$$p(t) = Vm(t) = Vf(t) + V\varepsilon(t), \tag{2}$$

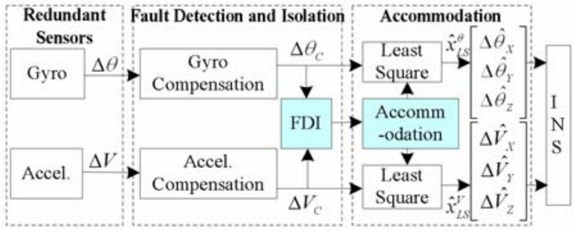

Fig. 1. FDI and Accommodation for INS with redundant sensors.

where the matrix V satisfies

$$VH = 0\,(V \in R^{(n-3)\times n}) \text{ and } VV^T = I, V = [v_1\ v_2 \cdots v_n]. \tag{3}$$

Algorithms to obtain the matrix V above can be found in the literature [5-7].

Terminology definition is given as follows [14].

Fault detection: the indication that something is going wrong in the system.

Fault isolation: the determination of the exact location of the fault.

Fault identification: the determination of the size and type or nature of the fault.

Fault accommodation: the reconfiguration of the system using healthy components.

Fig. 1 shows the block diagram of the FDIA (fault detection, isolation and accommodation) procedure in inertial navigation systems. From the sensor measurement, a parity equation is generated, and FDIA is performed. Triad solutions are calculated by the least square method and entered into the navigation equations. The navigation accuracy depends on the estimation error of the triad solutions, i.e., acceleration or angular rate.

In this paper only fault accommodation is considered.

Three assumptions are made as follows.

Assumption 1: Any three sensors are not on the same plane.

Assumption 2: All sensors have the same noise characteristics, i.e., same standard deviation σ of white Gaussian noise.

Assumption 3: Fault detection, isolation, and identification are performed in advance.

3. ACCOMMODATION RULE FOR SINGLE FAULT BASED ON SYSTEM PERFORMANCE

Consider measurement equation (4) the same as (1).

$$m = Hx + f + \varepsilon, \quad \varepsilon \sim N(0_n, \sigma I_n) \tag{4}$$

Triad solution $\hat{x} = [\hat{x}_x\ \hat{x}_y\ \hat{x}_z]^T$ in Fig. 1, which is acceleration or angular rate, can be obtained by the least square method as follows:

$$\hat{x}(t)=(H^T H)^{-1}H^T m(t). \tag{5}$$

Define estimation error of $x(t)$ as $e(t)=\hat{x}(t)-x(t)$. Navigation solution such as position, velocity, and attitude is calculated from $\hat{x}(t)$. Thus the navigation accuracy of INS depends on the error covariance $C(t)=E[e(t)e(t)^T]$.

Consider two matrices $C_{+i}(t)$ and $C_{-i}(t)$ given below

$$C_{+i}(t)=f(t)^2(H^T H)^{-1}h_i h_i^T(H^T H)^{-1} + \sigma^2(H^T H)^{-1}, \tag{6}$$

$$C_{-i}(t)\equiv E[(\hat{x}_{-i}(t)-x(t))(\hat{x}_{-i}(t)-x(t))^T] = \sigma^2(H^T W_i H)^{-1}, \tag{7}$$

where matrices C_{+i} and C_{-i} denote the covariance of $\hat{x}$ including and excluding the i-th sensor respectively and W_i is a $n\times n$ diagonal matrix with (i,i) component set to 0 and the other components set to 1.

Lemma 1 [18]**:** Suppose that i-th sensor has fault. For (6) and (7), the following two inequalities are equivalent:

i) $|f(t)|\le \dfrac{\sigma}{\|v_i\|_2}$

ii) $C_{+i}(t)-C_{-i}(t)\le 0$,

where σ and v_i are standard deviation of sensor noise and i-th column of V matrix, which satisfies (3).

And $|f(t)|=\dfrac{\sigma}{\|v_i\|_2}\Leftrightarrow C_{+i}(t)=C_{-i}(t)$. □

Lemma 1 implies that when the magnitude of i-th fault is less than $\sigma/\|v_i\|_2$, the error covariance of estimate $\hat{x}$ including i-th sensor is less than the error covariance of estimate $\hat{x}$ excluding it, thus the i-th faulty sensor should be used despite its fault to improve the navigation accuracy. From Lemma 1, we have the exclusion threshold $\sigma/\|v_i\|_2$ as an accommodation rule.

4. ACCOMMODATION RULE FOR DOUBLE FAULTS BASED ON SYSTEM PERFORMANCE

In this section we propose a new accommodation rule for double faults in redundant sensors.

4.1. Navigation performance analysis

For (1), suppose that double faults f_i and f_j occur, which means that $f(t)=[0\cdots f_i\ \ 0\cdots f_j\ \ 0\cdots]^T$.

To analyze the navigation performance, the error covariance of triad solution $\hat{x}(t)$ needs to be calculated. The covariance matrices are defined as follows. Matrix C_{+i+j} denotes the error covariance of $\hat{x}(t)$ including i-th and j-th sensor outputs, and C_{-i-j} the error covariance of $\hat{x}(t)$ excluding i-th and j-th sensors, and so on for C_{-i+j} and C_{+i-j}.

4.1.1 Covariance matrix C_{+i+j}

The error for $\hat{x}(t)$ can be calculated as follows

$$\hat{x}_{+i+j}-x=(H^T H)^{-1}\{\, f_i h_i + f_j h_j + H^T\varepsilon\,\}, \tag{8}$$

where $\hat{x}_{+i+j}=[\,\hat{x}_{++x}\ \ \hat{x}_{++y}\ \ \hat{x}_{++z}]^T$.

Then the estimation error of x can be described as the error covariance matrix C_{+i+j} in (9)

$$C_{+i+j}=E\left[(\hat{x}_{+i+j}-x)(\hat{x}_{+i+j}-x)^T\right] = \sigma^2(H^T H)^{-1}+(H^T H)^{-1}\left[h_i\, h_j\right] \cdot\begin{bmatrix} f_i^2 & f_i f_j \\ f_i f_j & f_j^2\end{bmatrix}\begin{bmatrix} h_i^T \\ h_j^T\end{bmatrix}(H^T H)^{-1}. \tag{9}$$

4.1.2 Covariance matrix C_{-i-j}

The error for $\hat{x}$ can be calculated as follows

$$\hat{x}_{-i\,-j}-x=(H^T W_{ij}\, H)^{-1}\ H^T W_{ij}\,\varepsilon, \tag{10}$$

where $\hat{x}_{-i-j}=[\,\hat{x}_{--x}\ \ \hat{x}_{--y}\ \ \hat{x}_{--z}]^T$ and W_{ij} is a diagonal matrix with diagonal elements of 1 except (i,i) component and (j,j) component which components are 0.

Then the estimation error of $\hat{x}$ can be described as the error covariance matrix C_{-i-j} in (11).

$$C_{-i-j}=E\left[(\hat{x}_{-i-j}-x)\,(\hat{x}_{-i-j}-x)^T\right] = \sigma^2\,(H^T H)^{-1}+\frac{\sigma^2}{D_{ij}}(H^T H)^{-1}\left[\,h_i h_j\right] \cdot\begin{bmatrix} \|v_j\|_2^2 & -v_j^T v_i \\ -v_i^T v_j & \|v_i\|_2^2\end{bmatrix}\begin{bmatrix} h_i^T \\ h_j^T\end{bmatrix}(H^T H)^{-1}, \tag{11}$$

where $D_{ij}=\|v_i\|_2^2\|v_j\|_2^2-\left\langle v_i,v_j\right\rangle^2=\|v_i\|_2^2\|v_j\|_2^2\sin^2\theta_{ij}$ and θ_{ij} is the angle between two vectors v_i and v_j, which are column vectors of matrix V defined in (3).

4.1.3 Covariance matrix C_{-i+j}

The error for $\hat{x}$ can be calculated as follows

$$\hat{x}_{-i+j} - x = (H^T W_i H)^{-1} H^T W_i (V_{Fj} f_j + \varepsilon), \quad (12)$$

where $\hat{x}_{-i+j} = [\hat{x}_{-+x} \; \hat{x}_{-+y} \; \hat{x}_{-+z}]^T$ and $V_{Fj} = [0 \cdots 0 \; 1 \; 0 \cdots 0]^T \in \mathrm{R}^{n\times 1}$ with j-th component of 1, which results in $H^T W_i V_{Fj} = h_j$.

Then the estimation error of $\hat{x}$ can be described as the error covariance matrix C_{-i+j} in (13)

$$\begin{aligned} C_{-i+j} &= f_j^2 (H^T W_i H)^{-1} h_j h_j^T (H^T W_i H)^{-1} \\ &\quad + \sigma^2 (H^T W_i H)^{-1} \\ &= f_j^2 (H^T W_i H)^{-1} h_j h_j^T (H^T W_i H)^{-1} \\ &\quad + \sigma^2 (H^T H)^{-1} + \frac{\sigma^2}{\|v_i\|_2^2} (H^T H)^{-1} h_i h_i^T (H^T H)^{-1}. \end{aligned} \quad (13)$$

4.2. Accommodation rule for double faults

In this section three theorems can be obtained from the results of Section 4.1, which provide accommodation rules for double faults.

Theorem 2: Consider the measurement equation (1) and the triad solution (5), and suppose that i-th and j-th sensors have faults. For the two estimation error covariance matrices (9) and (11), the following two inequalities are equivalent:

i) $tr(C_{+i+j}) < tr(C_{-i-j})$

where tr(•) denotes the trace of a matrix.

ii) $f_i^2 \|(H^T H)^{-1} h_i\|_2^2 + f_j^2 \|(H^T H)^{-1} h_j\|_2^2 + 2 f_i f_j < (H^T H)^{-1} h_i, (H^T H)^{-1} h_j > < \zeta_1,$ (14)

where < , > denotes an inner product and

$$\zeta_1 = \sigma^2 \frac{\|(H^T H)^{-1} h_i\|_2^2 \, \|v_j\|_2^2 + \|(H^T H)^{-1} h_j\|_2^2 \, \|v_i\|_2^2 - \gamma}{D_{ij}},$$

$$\gamma = 2 \langle (H^T H)^{-1} h_i, (H^T H)^{-1} h_j \rangle \langle v_i, v_j \rangle.$$

Proof: First we have

$$\begin{aligned} tr(C_{+i+j}) &= E[(\hat{x}_{++x} - x_x)^2] + E[(\hat{x}_{++y} - x_y)^2] \\ &\quad + E[(\hat{x}_{++z} - x_z)^2] \end{aligned}$$

and

$$\begin{aligned} tr(C_{-i-j}) &= E[(\hat{x}_{--x} - x_x)^2] + E[(\hat{x}_{--y} - x_y)^2] \\ &\quad + E[(\hat{x}_{--z} - x_z)^2]. \end{aligned}$$

Define **A** and **B** as follows

$$\begin{aligned} A &= \begin{bmatrix} h_i^T \\ h_j^T \end{bmatrix} (H^T H)^{-1} (H^T H)^{-1} \begin{bmatrix} h_i & h_j \end{bmatrix} \\ &= \begin{bmatrix} \|(H^T H)^{-1} h_i\|_2^2 & < (H^T H)^{-1} h_i, (H^T H)^{-1} h_j > \\ < (H^T H)^{-1} h_i, (H^T H)^{-1} h_j > & \|(H^T H)^{-1} h_j\|_2^2 \end{bmatrix} \end{aligned}$$

and

$$B = \begin{bmatrix} f_i^2 - \dfrac{\sigma^2 \|v_j\|_2^2}{D_{ij}} & f_i f_j + \dfrac{\sigma^2 v_j^T v_i}{D_{ij}} \\ f_i f_j + \dfrac{\sigma^2 v_i^T v_j}{D_{ij}} & f_j^2 - \dfrac{\sigma^2 \|v_i\|_2^2}{D_{ij}} \end{bmatrix}.$$

Then $tr(C_{+i+j} - C_{-i-j}) = tr(\mathrm{AB}) < 0$ gives the following inequality with long manipulation.

$$\begin{aligned} & f_i^2 \|(H^T H)^{-1} h_i\|_2^2 + f_j^2 \|(H^T H)^{-1} h_j\|_2^2 \\ &\quad + 2 f_i f_j < (H^T H)^{-1} h_i, (H^T H)^{-1} h_j > \\ &\langle \sigma^2 \frac{\|(H^T H)^{-1} h_i\|_2^2 \|v_j\|_2^2 + \|(H^T H)^{-1} h_j\|_2^2 \|v_i\|_2^2 - \gamma}{D_{ij}} \end{aligned}$$

□

Remark 1: Theorem 2 means that if faults f_i and f_j occur, and the magnitudes of the two faults satisfy (14) located inside an ellipse, then the corresponding faulty sensors should not be excluded to obtain less estimation error by using them.

Theorem 3: Consider the measurement equation (1) and the triad solution (5), and suppose that i-th and j-th sensors have faults. For the two estimation error covariance matrices (11) and (13), the following two inequalities are equivalent:

i) $tr(C_{-i+j}) < tr(C_{-i-j})$

ii) $\mathrm{f}_\mathrm{j}^2 < \zeta_2,$ (15)

where $\zeta_2 = \dfrac{tr(A)}{tr(B)}$ and

$$A = \sigma^2 (H^T H)^{-1} \left\{ \frac{1}{D_{ij}} [h_i h_j] \begin{bmatrix} \|v_j\|_2^2 & -v_j^T v_i \\ -v_i^T v_j & \|v_i\|_2^2 \end{bmatrix} \begin{bmatrix} h_i^T \\ h_j^T \end{bmatrix} - \frac{1}{\|v_i\|_2^2} h_i h_i^T \right\} (H^T H)^{-1},$$

$$B = (H^T W_i H)^{-1} h_j h_j^T (H^T W_i H)^{-1}.$$

Proof: The proof has the same procedure as Theorem 2. □

Remark 2: Theorem 3 means that even though faults f_i and f_j are located outside the ellipse in (14) and $|f_j|<|f_i|$, if (15) is satisfied, then the j-th sensor should not be excluded since less estimation error can be obtained by using j-th sensor.

Theorem 4: Consider the measurement equation (1) and the triad solution (5), and suppose that i-th and j-th sensors have faults. For the two estimation error covariance matrices (9) and (13), the following two inequalities are equivalent:

i) $tr(C_{-i+j}) < \mathrm{tr}(C_{+i+j})$

ii) $$f_i^2 + f_j^2 \frac{\{\|(H^T H)^{-1}h_j\|_2^2 - \|(H^T W_i H)^{-1}h_j\|_2^2\}}{\|(H^T H)^{-1}h_i\|_2^2} + \frac{2f_i f_j <(H^T H)^{-1}h_i,(H^T H)^{-1}h_j>}{\|(H^T H)^{-1}h_i\|_2^2} > \frac{\sigma^2}{\|v_i\|_2^2}. \quad (16)$$

Proof: From (9) and (13), $tr(C_{-i+j})$ and $tr(C_{+i+j})$ can be calculated easily as follows

$$tr(C_{-i+j}) = f_j^2\|(H^T W_i H)^{-1}h_j\|_2^2 + \sigma^2 tr((H^T H)^{-1}) + \frac{\sigma^2}{\|v_i\|_2^2}\|(H^T H)^{-1}h_i\|_2^2,$$

$$tr(C_{+i+j}) = f_i^2\|(H^T H)^{-1}h_i\|_2^2 + f_j^2\|(H^T H)^{-1}h_j\|_2^2 + 2f_i f_j <(H^T H)^{-1}h_i,(H^T H)^{-1}h_j> + \sigma^2 tr((H^T H)^{-1}).$$

By calculating $tr(C_{-i+j}) - tr(C_{+i+j}) < 0$, inequality (16) can be obtained. □

Remark 3: Theorem 4 means that even though faults f_i and f_j satisfy (14), located inside the ellipse, and $|f_j|<|f_i|$, if (16) is satisfied, then i-th sensor should be excluded since less estimation error can be obtained by its exclusion.

According to the results of Theorem 2 through Theorem 4, double faults can be categorized into four groups.

Category I: When double faults satisfy the following three inequalities

i) $$f_i^2\|(H^T H)^{-1}h_i\|_2^2 + f_j^2\|(H^T H)^{-1}h_j\|_2^2 + 2f_i f_j <(H^T H)^{-1}h_i,(H^T H)^{-1}h_j> < \zeta_1$$

ii) $$f_i^2 + f_j^2 \frac{\{\|(H^T H)^{-1}h_j\|_2^2 - \|(H^T W_i H)^{-1}h_j\|_2^2\}}{\|(H^T H)^{-1}h_i\|_2^2} + \frac{2f_i f_j <(H^T H)^{-1}h_i,(H^T H)^{-1}h_j>}{\|(H^T H)^{-1}h_i\|_2^2} < \frac{\sigma^2}{\|v_i\|_2^2}$$

iii) $|f_j|<|f_i|$.

The two faulty sensors should not be excluded.

Category II: When double faults satisfy the following three inequalities

i) $$f_i^2\|(H^T H)^{-1}h_i\|_2^2 + f_j^2\|(H^T H)^{-1}h_j\|_2^2 + 2f_i f_j <(H^T H)^{-1}h_i,(H^T H)^{-1}h_j> < \zeta_1$$

ii) $$f_i^2 + f_j^2 \frac{\{\|(H^T H)^{-1}h_j\|_2^2 - \|(H^T W_i H)^{-1}h_j\|_2^2\}}{\|(H^T H)^{-1}h_i\|_2^2} + \frac{2f_i f_j <(H^T H)^{-1}h_i,(H^T H)^{-1}h_j>}{\|(H^T H)^{-1}h_i\|_2^2} \geq \frac{\sigma^2}{\|v_i\|_2^2}$$

iii) $|f_j|<|f_i|$.

The i-th sensor should be excluded, but not for the j-th sensor.

Category III: When double faults satisfy the following three inequalities

i) $$f_i^2\|(H^T H)^{-1}h_i\|_2^2 + f_j^2\|(H^T H)^{-1}h_j\|_2^2 + 2f_i f_j <(H^T H)^{-1}h_i,(H^T H)^{-1}h_j> \geq \zeta_1$$

ii) $f_j^2 < \zeta_2$

iii) $|f_j|<|f_i|$.

The i-th sensor should be excluded, but not for the j-th sensor.

Category IV: When double faults satisfy the following three inequalities

i) $$f_i^2\|(H^T H)^{-1}h_i\|_2^2 + f_j^2\|(H^T H)^{-1}h_j\|_2^2 + 2f_i f_j <(H^T H)^{-1}h_i,(H^T H)^{-1}h_j> \geq \zeta_1$$

ii) $f_j^2 \geq \zeta_2$

iii) $|f_j|<|f_i|$.

The two faulty sensors should be excluded.

Remark 4: For the 4 categories above, we consider only half of the first quadrant in two dimensional space. i.e., $0 \leq \theta \leq \pi/4$. It can be noticed that Category II and Category III give the same result.

4.3 Accommodation rule for double faults with dodecahedron configuration

In order to show the decision rule for a real configuration for redundant inertial sensors, we use the symmetric dodecahedron configuration as shown in Fig. 2, which uses 6 sensors. In this case the

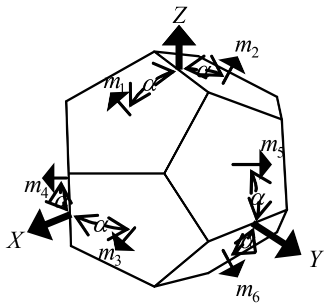

Fig. 2. Dodecahedron configuration with 6 identical sensors.

measurement matrix and parity matrix have the following relations.

$$H^T H = 2I_3,\ \|h_i\|_2 = 1,\ \ \|v_i\|_2 = \frac{1}{\sqrt{2}}(\mathrm{i} = 1,2,\cdots,6)$$

The angles $\hat{\theta}_{ij}$ between direction cosine vectors h_i of 6 sensors are 63.4° and 116.6°. So are angles of parity vectors. Thus, $\sin^2\theta_{ij} = \sin^2\hat{\theta}_{ij} = 0.8$ and $\cos^2\theta_{ij} = \cos^2\hat{\theta}_{ij} = 0.2$ always hold.

Inequalities (14)-(16) become (17)-(19) for the dodecahedron configuration.

Table 1. Four categories of double faults with dodecahedron configuration ($0 \le \theta \le \pi/4$ region only, ○: use, x : exclusion).

Cate gory	Conditions	i-th faulty sensor	j-th faulty sensor
I	$f_i^2 + 0.8944 f_i f_j + f_j^2 < 6\sigma^2$, $f_i^2 - \frac{3}{5} f_j^2 + 0.8944 f_i f_j < 2\sigma^2$, $\lvert f_j \rvert < \lvert f_i \rvert$	○	○
II	$f_i^2 + 0.8944 f_i f_j + f_j^2 < 6\sigma^2$, $f_i^2 - \frac{3}{5} f_j^2 + 0.8944 f_i f_j \ge 2\sigma^2$, $\lvert f_j \rvert < \lvert f_i \rvert$	x	○
III	$f_i^2 + 0.8944 f_i f_j + f_j^2 \ge 6\sigma^2$, $\lvert f_j \rvert < 1.5811\sigma$, $\lvert f_j \rvert < \lvert f_i \rvert$	x	○
IV	$f_i^2 + 0.8944 f_i f_j + f_j^2 \ge 6\sigma^2$, $\lvert f_j \rvert \ge 1.5811\sigma$, $\lvert f_j \rvert < \lvert f_i \rvert$	x	x

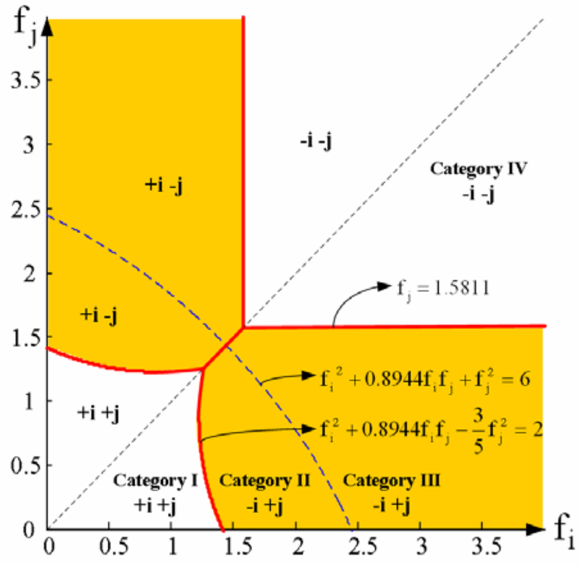

Fig. 3. Decision rule for exclusion of faulty sensors for the dodecahedron configuration (For $\pi/4 \le \theta \le \pi/2$ region, i and j should be interchanged in Table 1).

$$f_i^2 + 0.8944 f_i f_j + f_j^2 < 6\sigma^2, \tag{17}$$

$$\lvert f_j \rvert \ge 1.5811\sigma, \tag{18}$$

$$f_i^2 - \frac{3}{5} f_j^2 + 0.8944 f_i f_j \ge 2\sigma^2. \tag{19}$$

We can obtain Table 1 for the symmetric dodecahedron configuration summarizing the above observation. Table 1 can be plotted in a two-dimensional plane as in Fig. 3.

Remark 5: Table 1 is considered only for $0 \le \theta \le \pi/4$ region in the first quadrant. We need to interchange i and j for $\pi/4 \le \theta \le \pi/2$ region. In Fig. 3, the ellipse $f_i^2 + 0.8944 f_i f_j + f_j^2 = 6\sigma^2$ does not contribute to the decision rule. However, it does in the second and fourth quadrants.

5. SIMULATIONS

In this section, Monte Carlo simulations are performed 10,000 times for each fault to confirm the accommodation rule for single fault case and double fault case, respectively. Six identical sensors are used with dodecahedron configuration [19] as in Fig. 2.

The measurement matrices H and V satisfying $VH = 0$ and $VV^T = \mathrm{I}$ can be obtained as follows:

$$\mathrm{H} = \begin{bmatrix} 0.5257 & -0.5257 & 0.8507 \\ 0 & 0 & 0.5257 \\ 0.8507 & 0.8507 & 0 \end{bmatrix}$$

$$\left.\begin{matrix} 0.8507 & 0 & 0 \\ -0.5257 & 0.8507 & 0.8507 \\ 0 & 0.5257 & -0.5257 \end{matrix}\right]^T,$$

$$V = \left[\begin{matrix} 0.3717 & 0.3717 & 0 \\ -0.6015 & 0.6015 & 0.3717 \\ 0 & 0 & -0.6015 \end{matrix}\right.$$

$$\left.\begin{matrix} 0 & -0.6015 & 0.6015 \\ 0.3717 & 0 & 0 \\ 0.6015 & 0.3717 & 0.3717 \end{matrix}\right], \quad (20)$$

where $\|v_1\| = \|v_2\| = \cdots = \|v_6\| = 1/\sqrt{2}$.

Consider the simulation for the single fault case. We assume that first sensor has a fault like $f(t) = [f_1(t)\ 0 \cdots 0]^T$, and the fault $f_1(t)$ is a bias, and the measurement noise is white Gaussian with mean 0 and variance $\sigma = 1$. The exclusion threshold stated in Lemma 1 is $\mathrm{Th} = \sqrt{2}\sigma$ for the matrix V obtained above.

Fig. 4 is the Monte Carlo simulation result of the single fault case showing trace$(C_{+1}(t))$ and trace $(C_{-1}(t))$. Horizontal axis denotes the fault size to noise ratio (F/N ratio), and the vertical axis denotes the magnitudes of trace$(C_{+1}(t))$ and trace$(C_{-1}(t))$. When fault signal $f_1(t)$ is greater than $\sqrt{2}\sigma$, the inequality trace$(C_{+1}(t)) >$ trace$(C_{-1}(t))$ holds. This inequality, which is consistent with Lemma 1, means that when a fault size is greater than $\sqrt{2}\sigma$, the faulty sensor should be excluded to provide less error covariance of $\hat{x}(t)$. The exclusion of the faulty sensor will result in superior navigation accuracy.

For the double fault case, we assume that the first and second sensors have fault like $f(t) = [f_1\ f_2\ 0\ 0\ 0\ 0]^T$, and the faults f_1 and f_2 are constants and satisfy the straight line as Fig. 5, and the measurement noise is white Gaussian with mean 0 and variance $\sigma = 1$.

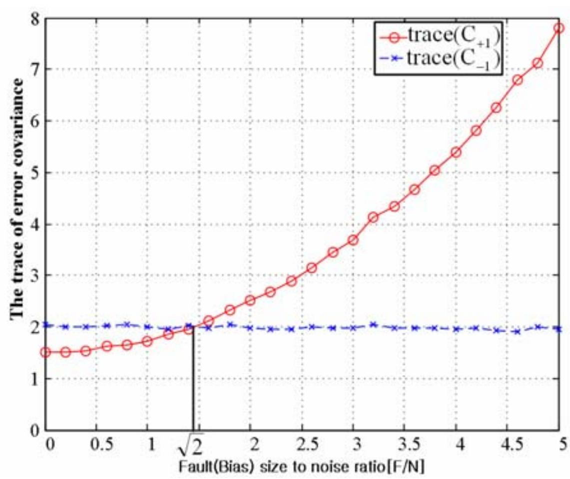

Fig. 4. Trace$(C_{+1}(t))$ and trace$(C_{-1}(t))$ with respect to F/N ratio.

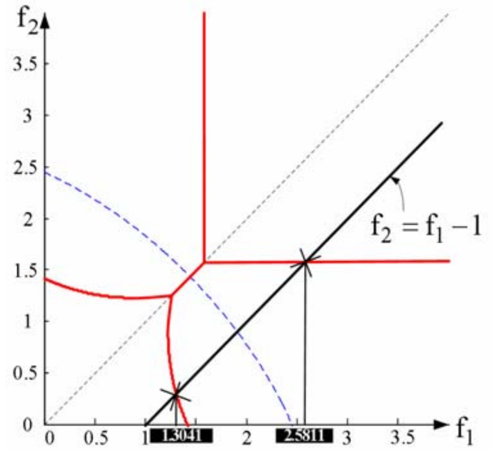

Fig. 5. Decision rule for exclusion of faulty sensors and the relation of two fault magnitudes for simulation.

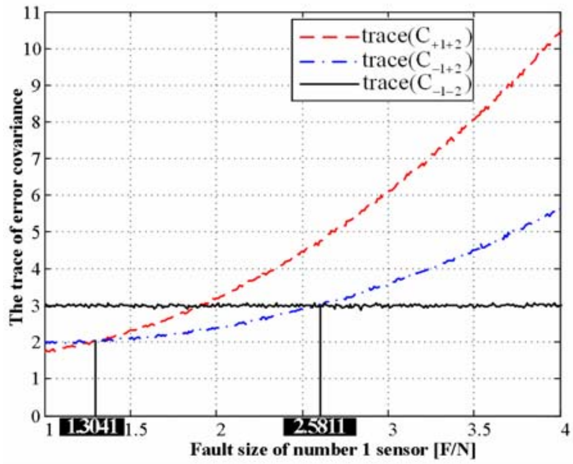

Fig. 6. trace$(C_{+1+2}(t))$, trace$(C_{-1+2}(t))$ and trace $(C_{-1\text{-}2}(t))$ with respect to fault magnitude.

Fig. 6 shows the results of the accommodation rule for double faults according to the fault size in Fig. 5. When fault f_1 and f_2 belong to the region of Category I, the trace of C_{+1+2} is the minimum among three traces. When fault f_1 and f_2 belong to that of Category II and III, the trace of C_{-1+2} is the minimum, and to the Category IV, the trace of C_{-1-2} is the minimum.

6. CONCLUSIONS

We consider a fault accommodation problem for redundant inertial sensor systems depending on navigation accuracy and propose a new accommodation rule for the double fault case. The accommodation rule can be drawn in two-dimensional decision space. Monte-Carlo simulation has been performed for dodecahedron configuration to confirm the improvement of the navigation accuracy for the single and double fault cases. The accommodation rule can be applied to any configurations and any number of sensors.

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