

Control of Rigid Robots Equipped with Brushed DC-Motors as Actuators

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Abstract: We extend the application of an adaptive controller previously introduced in the literature under the assumption that no actuator dynamics exists to the case when the dynamics of the brushed DC-motors used as actuators is not neglected. Convergence to the desired positions is ensured without requiring any feedback to cope with the additional electric dynamics. The proposed control scheme does not require the exact knowledge of neither robot nor actuator parameters to select controller gains.

Keywords: Adaptive control, brushed DC-motor actuators, Lyapunov stability, position regulation, robot control.

1. INTRODUCTION

Although most robots are equipped with brushed DC-motors as actuators at present, most control theory developed for robots assumes that the actuator dynamics can be neglected and that torque can be directly used as the control input. The main reason for this is that the introduction of an electrical system between the control input and the torque actually applied to the robot links complicates the controller design in robotics [1]. However, as pointed out in [2], neglecting the actuator dynamics may result in degradation of the closed loop performance. Similar observations have motivated lots of work on robot control taking into account the dynamics of the brushed DC-motors used as actuators (see [1,3-5] and references therein). Most of these results rely on either electric current measurements or the exact knowledge of several of the actuator parameters. Other works avoiding some of these drawbacks rely, however, on torque measurements [6].

In this note we extend the application of a controller previously introduced under the assumption that no actuator dynamics exists [7] to the case when the dynamics of the brushed DC-motors used as actuators is taken into account. It is important to stress

that, contrary to previous works on the subject [8], this is done without requiring any additional loop designed to cope with the additional electrical subsystem dynamics, i.e., both electric current and torque measurements are avoided. Further, contrary to the common assumption in the literature we do not require the actuator electric dynamics to be “fast enough”. We stress that the exact knowledge of neither robot nor actuator parameters is not required. These features represent the main contribution of our work.

Our results rely on the following facts: the torque constant equals the back-electromotive constant in a brushed DC-motor, we obtain a linear version of controller introduced in [7] whose stability analysis is similar to the one presented in [9] and we introduce a novel error variable to describe the electric subsystem.

Along this paper we use the following notation: $\|\cdot\|$ represents both the Euclidean norm of a vector and the spectral norm of a matrix whereas $\lambda_m(\cdot)$ and $\lambda_M(\cdot)$ stand, respectively, for the minimum and maximum eigenvalues of a symmetric matrix.

This note is organized as follows. In Section 2, we present the dynamic model that we consider. In Section 3, we present our main result. Some simulation results are presented in Section 4, whereas some concluding remarks are given in Section 5.

2. THE DYNAMIC MODEL CONSIDERED

The dynamic model of an n degrees of freedom rigid robot equipped only with revolute joints and with n brushed DC-motors as actuators is given as [1]:

$$L \frac{di}{dt} + Ri + K_b \dot{q} = u, \quad (1)$$

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = K_m i, \quad (2)$$

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where $q \in R^n$ represents the link positions, $C(q, \dot{q})\dot{q}$ is the Coriolis and centrifugal effects term and $g(q)$ is the gravitational effect term, given as $g(q) = \frac{\partial U_g(q)}{\partial q}$, where $U_g(q)$ is the potential energy. We define the gear ratio positive definite constant diagonal matrix as $\theta = Nq$, where $\theta \in R^n$ represents the actuator positions, and $D(q) = M(q) + NJN$ where $M(q)$ and J are $n \times n$ symmetric and positive definite matrices representing the links and actuators inertia, respectively. Furthermore, J is constant and diagonal. Variables $i, u \in R^n$ represent, respectively, the electric current and voltage in the brushed DC-motors armature circuits, while L, R, K_e and K_a are $n \times n$ diagonal and positive definite matrices representing the inductance, resistance, back-electromotive constant and torque constant, respectively. Finally, we define $K_b = K_e N$ and $K_m = NK_a$ whereas the applied torques are defined as $\tau = K_m i$. As it is by now well known, four important properties of this model are [9,10,Ch. 4]:

$$\dot{q}^T \left(\frac{1}{2} \dot{D}(q) - C(q, \dot{q}) \right) \dot{q} = 0, \quad \forall \dot{q} \in R^n, \quad (3)$$

$$\dot{D}(q) = C(q, \dot{q}) + C^T(q, \dot{q}), \quad (4)$$

$$\|C(q, y)s\| \leq k_c \|y\| \|s\|, \quad \forall q, y, s \in R^n, \quad (5)$$

$$\left\| \frac{\partial g(q)}{\partial q} \right\| \leq k_g, \quad \forall q \in R^n, \quad (6)$$

where k_c and k_g are finite positive constants. An important property of brushed DC-motors is related to power conservation. The electric power transformed into mechanical power is given in terms of the back-electromotive force, $e_{bef} = K_e \dot{\theta}$, and the electric current through the armature circuits $P_e = e_{bef}^T i$ whereas the resulting mechanical power is given in terms of velocity and the electromagnetic torque $P_m = \dot{\theta}^T \tau_{em}$, where $\tau_{em} = K_a i$. From power conservation $P_e = P_m$, we obtain $K_e = K_a$ which implies, because of the diagonal property of all the involved matrices, that $K_b = K_m$. In this note we consider the following assumptions.

Assumption 1: We can write:

$$RK_m^{-1} g(q) = \Phi(q)\theta^*, \quad (7)$$

where $\Phi(q)$ is a $n \times m$ known matrix whereas θ^* is a $m \times 1$ unknown parameter vector which is assumed to be constant. We also assume that $m \geq n$.

Assumption 2: Given (7) and any constant vector $q_d \in R^n$ there exist two diagonal positive definite $m \times m$ arbitrary matrices Γ and Γ' such that $\Gamma \Phi^T(q_d) = \Gamma' \Phi^T(q_d) K_m R^{-1}$, where Γ can be chosen without requiring the exact knowledge of any of the elements of matrix $K_m R^{-1}$.

Assumption 3: Matrix $\Phi(q)$ and the parameter vector θ^* introduced in (7) are defined in such a way that product $\Phi(q_d) \Gamma \Phi^T(q_d)$ is a $n \times n$ diagonal, positive semidefinite matrix.

Some remarks on these assumptions are in order. First, we stress that Assumption 1 holds even if all actuators have different dynamics, i.e., even if all diagonal entries of matrix RK_m^{-1} are different. Second, the last part of Assumption 2 is possible only if all elements of both matrices Γ and Γ' can be chosen arbitrarily. Third, let $m_i \geq 1, i = 1, \dots, n$, be integers representing the number of components of vector θ^* , given in (7), arising from the i -th component of vector $RK_m^{-1} g(q)$, i.e., $g_i(q) R_i / K_{mi}$. According to this, the number of unknown parameters in θ^* is given as $m = \sum_{i=1}^{i=n} m_i$. Assumption 3 is possible if matrix $\Phi(q)$ is chosen as follows: *first row*: only elements from column 1 to column m_1 are nonzero, *second row*: only elements from column $m_1 + 1$ to column $m_1 + m_2$ are nonzero, ..., *n-th row*: only elements from column $(\sum_{i=1}^{i=n-1} m_i) + 1$ to column $\sum_{i=1}^{i=n} m_i$ are nonzero. Note that this requires $m \geq n$. We give an application example for the three assumptions in Section 4.

3. MAIN RESULT

Proposition 1: Consider the dynamic model (1), (2), together with the following controller:

$$u = -\overline{K_p} \tilde{q} - \overline{K_D} \vartheta - \overline{K_v} \dot{q} + \Phi(q_d) \hat{\theta}, \quad (8)$$

$$\hat{\theta} = \Gamma \Phi^T(q_d) \int_0^t [-\dot{q}(r) + \varepsilon[-\tilde{q}(r) + \vartheta(r)]] dr,$$

$$\vartheta = \text{diag} \left\{ \frac{b_i p}{p + a_i} \right\} q, \quad (9)$$

where $p = (d/dt)$ denotes the differential operator, $q_d \in R^n$ represents the constant desired link positions and $\tilde{q} = q - q_d$. On the other hand, $A = \text{diag}\{a_i\}$, $B = \text{diag}\{b_i\}$ are $n \times n$ positive definite matrices satisfying:

$$\lambda_m(B) > \frac{2\lambda_M(D(q))}{\lambda_m(D(q))}, \quad \lambda_M(LR^{-1}A) < 1, \quad (10)$$

whereas $\Gamma = \text{diag}\{\Gamma_i\}$ is a $m \times m$ arbitrary positive definite matrix. Under Assumptions 1, 2, and 3 we can always find a (small) constant scalar $\varepsilon > 0$, (large enough) diagonal positive definite matrices $\overline{K_P}$, $\overline{K_V}$ and a (possibly non positive definite) diagonal matrix $\overline{K_D}$ ensuring stability of the desired equilibrium point and convergence $q(t) \rightarrow q_d$, as $t \rightarrow \infty$. Further, this convergence is semiglobal in the sense that it stands in a domain which can always be enlarged arbitrarily by choosing suitable controller gains.

Proof: Suppose that we can write:

$$\overline{K_P} = \underline{K_P} + \varepsilon K_b + \varepsilon K_v + \beta_2, \quad (11)$$

$$\overline{K_D} = \underline{K_D} - \varepsilon K_b - \varepsilon K_v - \beta_3, \quad (12)$$

$$\overline{K_V} = K_v + \beta_1, \quad (13)$$

$$\beta_1 = LR^{-1}(\underline{K_P} + \underline{K_D}B + \Phi(q_d)\Gamma\Phi^T(q_d)), \quad (14)$$

$$\beta_2 = \varepsilon LR^{-1}\Phi(q_d)\Gamma\Phi^T(q_d), \quad (15)$$

$$\beta_3 = LR^{-1}(\underline{K_D}A + \varepsilon\Phi(q_d)\Gamma\Phi^T(q_d)), \quad (16)$$

where K_v is an arbitrary diagonal positive definite matrix whereas $\underline{K_D}$ and $\underline{K_P}$ are also diagonal positive definite matrices which have to satisfy some relations to be defined latter. Note that although $\overline{K_D}$ is not required to be positive definite, however a large positive definite value ensures a large positive definite $\underline{K_D}$. Also note that β_1 , β_3 are diagonal positive definite matrices whereas β_2 is a diagonal positive semidefinite matrix under Assumption 3. Replacing control law (8) in (1), using (11)-(16) as well as the realization $\dot{\vartheta} = -A\vartheta + B\dot{q}$ of (9) and defining $\xi = i - R^{-1}u_c$, where $u_c = -\underline{K_P}\tilde{q} - \underline{K_D}\vartheta + \Phi(q_d)\hat{\theta}$, we obtain:

$$L\dot{\xi} = -R\xi - K_b(\dot{q} + \varepsilon(\tilde{q} - \vartheta)) - K_v(\dot{q} + \varepsilon(\tilde{q} - \vartheta)). \quad (17)$$

Further, if we define $\xi = \rho + \sigma$ it is not difficult to realize that (17) can be replaced by the following expressions:

$$L\dot{\rho} = -R\rho - K_b[\dot{q} + \varepsilon(\tilde{q} - \vartheta)], \quad (18)$$

$$L\dot{\sigma} = -R\sigma - K_v[\dot{q} + \varepsilon(\tilde{q} - \vartheta)]. \quad (19)$$

On the other hand, note that using definitions of ξ and u_c above as well as Assumption 1 we can write

(2) as:

$$\begin{aligned} D(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) + K'_P(q - \delta) + K_D\vartheta \\ = K_m\xi + K_mR^{-1}\Phi(q_d)\tilde{\theta}, \end{aligned} \quad (20)$$

where we define:

$$\begin{aligned} K'_P = K_mR^{-1}\underline{K_P}, \quad K_D = K_mR^{-1}\underline{K_D}, \\ \delta = q_d + (K'_P)^{-1}g(q_d), \quad \tilde{\theta} = \hat{\theta} - \theta^*. \end{aligned} \quad (21)$$

Finally, the closed loop dynamics is represented by (18)-(20) and:

$$\dot{\vartheta} = -A\vartheta + B\dot{q}, \quad (22)$$

$$\dot{\tilde{\theta}} = \Gamma\Phi^T(q_d)[- \dot{q} - \varepsilon\tilde{q} + \varepsilon\vartheta]. \quad (23)$$

It is not difficult to verify that $(\tilde{q}, \dot{q}, \vartheta, \tilde{\theta}, \rho, \sigma) = (0, 0, 0, 0, 0, 0)$ is an equilibrium point of this closed loop dynamics. Stability of this equilibrium point is analyzed using the following Lyapunov function candidate:

$$\begin{aligned} W(\tilde{q}, \dot{q}, \vartheta, \tilde{\theta}, \rho, \sigma) &= V(\tilde{q}, \dot{q}, \vartheta) + V_2(\tilde{\theta}) + V_5(\rho, \sigma), \\ V(\tilde{q}, \dot{q}, \vartheta) &= \varepsilon(V_3(\tilde{q}, \dot{q}) + V_4(\tilde{q}, \dot{q}, \vartheta)) + V_1(\tilde{q}, \dot{q}, \vartheta), \\ V_1(\tilde{q}, \dot{q}, \vartheta) &= \frac{1}{2}\dot{q}^T D(q)\dot{q} + \frac{1}{2}\vartheta^T K_D B^{-1}\vartheta + d_1 \\ &\quad + \frac{1}{2}(q - \delta)^T K'_P(q - \delta) + U_g(q), \\ V_2(\tilde{\theta}) &= \frac{1}{2}\tilde{\theta}^T (\Gamma')^{-1}\tilde{\theta}, \\ V_3(\tilde{q}, \dot{q}) &= \tilde{q}^T D(q)\dot{q}, \\ V_4(\tilde{q}, \dot{q}, \vartheta) &= -\vartheta^T D(q)\dot{q}, \\ V_5(\rho, \sigma) &= \frac{1}{2}\rho^T L\rho + \frac{1}{2}\sigma^T K_m K_v^{-1}L\sigma, \end{aligned} \quad (24)$$

where Γ' is an arbitrary $m \times m$ diagonal positive definite matrix and $d_1 = -\frac{1}{2}g(q_d)^T (K'_P)^{-1}g(q_d) - U_g(q_d)$. Function $V(\tilde{q}, \dot{q}, \vartheta)$ has been proposed in [9] where it was shown to be positive definite and radially unbounded if:

$$\lambda_m(K'_P) > 4k_g, \quad (25)$$

$$\frac{1}{2}\sqrt{\frac{\lambda_m(K'_P)}{d_M}} > \varepsilon, \quad \frac{1}{2}\sqrt{\frac{\lambda_m(K_D B^{-1})}{d_M}} > \varepsilon. \quad (26)$$

Note that we form W just by adding V_2 and V_5 to the Lyapunov function proposed in [9], i.e., V . This represents a simple manner to embed the adaptive dynamics and the actuator electric dynamics, both introduced in the present work, in the stability analysis presented in [9]. Using Assumption 2, the

fact that $K_m = K_b$, properties (3)-(6) and following the procedure presented in [9] we find that the time derivative of W along the trajectories of the closed loop system (18)-(20), (22)-(23) can be bounded as:

$$\begin{aligned} \dot{W} \leq & -\frac{\varepsilon}{2} w^T \bar{Q}_1 w - \frac{\varepsilon}{2} z^T \bar{Q}_2 z \\ & - \varepsilon \left[\frac{1}{2} \lambda_m(B) d_m - d_M - k_c \|\vartheta\| - k_c \|\tilde{q}\| \right] \|\dot{q}\|^2 \\ & - \left[\frac{1}{2} \lambda_m(K_D B^{-1} A) - \varepsilon \lambda_M(K_D) \right] \|\vartheta\|^2, \end{aligned} \quad (27)$$

where $w = (\|\tilde{q}\|, \|\vartheta\|, \|\rho\|)^T$, $z = (\|\vartheta\|, \|\dot{q}\|, \|\sigma\|)^T$, $d_m = \lambda_m(D(q))$, $d_M = \lambda_M(D(q))$ and:

$$\begin{aligned} \bar{Q}_1 &= \begin{bmatrix} \bar{Q}_{1(1,1)} & \bar{Q}_{1(1,2)} & 0 \\ \bar{Q}_{1(2,1)} & \bar{Q}_{1(2,2)} & 0 \\ 0 & 0 & \frac{2}{\varepsilon} \lambda_m(R) \end{bmatrix}, \\ \bar{Q}_{1(1,1)} &= 2\lambda_m(K'P) - 2k_g, \\ \bar{Q}_{1(1,2)} &= \bar{Q}_{1(2,1)} = -\lambda_M(K'P) - \lambda_M(K_D) - k_g, \\ \bar{Q}_{1(2,2)} &= \frac{1}{2\varepsilon} \lambda_m(K_D B^{-1} A), \\ \bar{Q}_2 &= \begin{bmatrix} \bar{Q}_{2(1,1)} & \bar{Q}_{2(1,2)} & 0 \\ \bar{Q}_{2(2,1)} & \bar{Q}_{2(2,2)} & 0 \\ 0 & 0 & \frac{2}{\varepsilon} \lambda_m(K_m K_v^{-1} R) \end{bmatrix}, \\ \bar{Q}_{2(1,1)} &= \frac{1}{2\varepsilon} \lambda_m(K_D B^{-1} A), \quad \bar{Q}_{2(2,2)} = \lambda_m(B) d_m, \\ \bar{Q}_{2(1,2)} &= \bar{Q}_{2(2,1)} = -\lambda_M(A) d_M. \end{aligned}$$

Remark 1: Instrumental to obtain (27) are the fact that $K_m = K_b$ and Assumption 2: both of them allow to cancel some terms appearing in \dot{V} , \dot{V}_5 and \dot{V}_2 . On the other hand, we stress that it is common in literature to use, for the electric subsystem, an error variable defined as the difference between the actual applied torque and its desired value. An important drawback of such an approach is that either the exact knowledge of the torque constant K_m or torque measurement is required. We avoid this inconvenience by introducing the error variable $\xi = i - R^{-1}u_c$. Finally, we stress that the linearity of u_c allows to express $-LR^{-1}\dot{u}_c$ as a linear combination of the state variables \tilde{q} , \dot{q} and ϑ . This is important to complete the error dynamics (17): controller (8) contributes with the required terms to complete such an error equation.

Recall that matrices R and $K_m K_v^{-1} R$ are diagonal positive definite, hence conditions for positive definiteness of \bar{Q}_1 are:

$$\lambda_m(K'P) > k_g, \quad \frac{[\lambda_m(K'P) - k_g] \lambda_m(K_D B^{-1} A)}{[\lambda_M(K'P) + \lambda_M(K_D) + k_g]^2} > \varepsilon, \quad (28)$$

whereas \bar{Q}_2 is positive definite if:

$$\lambda_m(K_D B^{-1} A) > 0, \quad \frac{\lambda_m(K_D A) d_m}{2[\lambda_M(A) d_M]^2} > \varepsilon. \quad (29)$$

Finally, by defining $x = (\tilde{q}^T, \dot{q}^T, \vartheta^T, \tilde{\theta}^T, \rho^T, \sigma^T)^T$ we realize that the third right hand term of (27) is negative if:

$$\frac{1}{2k_c} \left[\frac{1}{2} \lambda_m(B) d_m - d_M \right] > \|x\|, \quad (30)$$

whereas the last term in (27) is negative if:

$$\frac{\lambda_m(K_D B^{-1} A)}{2\lambda_M(K_D)} > \varepsilon. \quad (31)$$

Conditions (28)-(31), (25) and (26) are used in [9] to show that in the case when the dynamics of the actuators is not taken into account, i.e., when $\rho = \sigma = 0$, controller gains always exist such that both the right hand of (27) is negative and V , given in (24), is positive definite and radially unbounded in a domain which can be arbitrarily enlarged. This is shown to be possible by choosing a sufficiently small $\varepsilon > 0$ and, simultaneously, $\lambda_m(B)$ and $\lambda_M(B)$ sufficiently large. Further, first expression in (10) is a sufficient condition for the existence of such a domain [9] (see (30) above). We realize that in the case when the electric dynamics of actuators is taken into account, i.e., $\rho \neq 0$, $\sigma \neq 0$, the fore mentioned conditions also ensure stability of the equilibrium point $(\tilde{q}, \dot{q}, \vartheta, \tilde{\theta}, \rho, \sigma) = (0, 0, 0, 0, 0, 0)$. In Remark 2 we explain how to compute controller gains satisfying all of the fore mentioned conditions without requiring the exact knowledge of neither robot nor actuator parameters. The proposed procedure requires that *a*) second expression in (10) be satisfied and *b*) F_2 , given as follows, be a positive definite matrix:

$$F_2 = \left[I - \varepsilon L R^{-1} + \varepsilon L R^{-1} (I - L R^{-1} A)^{-1} B \right]^{-1}, \quad (32)$$

where I stands for the $n \times n$ identity matrix. Hence, *a*) and *b*) are additional conditions which have to be satisfied by the controller gains. Note that *b*) is

always satisfied by choosing a small enough ε . On the other hand, a small A satisfying (10) does not interfere with the existence of controller gains claimed above because, we recall, such existence is basically established by a large matrix B and a small ε . Finally, convergence $\tilde{q}(t) \rightarrow 0$ as $t \rightarrow \infty$ is proven invoking standard adaptive control arguments. It is not difficult to show from (27) that \tilde{q} , \dot{q} , ϑ , ρ , and σ are square integrable. Being \tilde{q} a bounded square integrable function whose time derivative ($\dot{\tilde{q}} = \dot{q}$) is also bounded we ensure convergence $\tilde{q} \rightarrow 0$ as $t \rightarrow \infty$ in a domain which can arbitrarily be enlarged. This completes the proof of Proposition 1.

Remark 2: Using (11)-(16) we can write:

$$\begin{aligned} K_v &= \{\overline{K}_v - LR^{-1}F_1\}F_2, \\ \underline{K}_P &= \overline{K}_P - \varepsilon K_b - \varepsilon K_v - \varepsilon LR^{-1}\Psi, \\ \underline{K}_D &= [\overline{K}_D + \varepsilon K_b + \varepsilon K_v + \varepsilon LR^{-1}\Psi] \cdot [I - LR^{-1}A]^{-1}, \\ F_1 &= \overline{K}_P - \varepsilon K_b - \varepsilon LR^{-1}\Psi + F_3F_4 + \Psi, \\ F_3 &= \overline{K}_D + \varepsilon K_b + \varepsilon LR^{-1}\Psi, \\ F_4 &= (I - LR^{-1}A)^{-1}B, \\ \Psi &= \Phi(q_d)\Gamma\Phi^T(q_d), \end{aligned}$$

where F_2 is given in (32). Note that using upper and lower bounds on the values of matrices LR^{-1} , K_b and proposing known values for the scalar ε and all other involved matrices we can find upper and lower bounds for matrix K_v . This computation is simplified by the fact that all the involved matrices are diagonal. Hence, bounds of these matrices are simply their largest and their smallest diagonal elements. In order to avoid use of the exact values of matrices LR^{-1} , K_b , K_mR^{-1} we propose to use, for instance, 0.90 times their smallest diagonal element and 1.10 times their largest diagonal elements to consider a 10% uncertainty in their nominal values. Using upper and lower bounds for K_v we can proceed similarly to obtain upper and lower bounds for matrices \underline{K}_P and \underline{K}_D . Then, use of (21) as well as upper and lower bounds on the values of matrix K_mR^{-1} allow to compute upper and lower bounds for K'_P and K_D which can be used to verify conditions (28)-(31), (25) and (26). Thus, this procedure allows to select all gains in controller of Proposition 1 without requiring the exact knowledge of neither robot nor actuator parameters. Although this may require a try-error search of the controller gains, however use of a digital

computer program to make computations and to verify all conditions simplifies the search of controller gains. Finally, note that we have to choose a small enough A , to satisfy (10), and a small ε , to render matrix F_2 positive definite, in order to ensure that K_v and \underline{K}_D are also positive definite, which makes easier to ensure that \underline{K}_P is also positive definite. Thus, we also ensure that K'_P and K_D are positive definite.

4. SIMULATION RESULTS

We consider the robot model presented in [11]. This is a two degrees of freedom ($n=2$) experimental robot which, however, is not equipped with brushed DC-motors. Hence, we use the following data which correspond to the brushed DC-motors model numbers MT-4060-ALYBE (joint 1) and MT-4060-BLYBE (joint 2) [12,pp.G-11],

$$\begin{aligned} K_a &= \text{diag}\{0.573, 0.382\} \text{ [Nm/Amp]}, \\ K_e &= \text{diag}\{0.573, 0.382\} \text{ [Volts/(rad/sec)]}, \\ R &= \text{diag}\{2.3, 1.2\} \text{ [\Omega]}, \\ J &= \text{diag}\{12.43 \times 10^{-4}, 12.43 \times 10^{-4}\} \text{ [Kg-m}^2\text{]}, \\ L &= \text{diag}\{9.6 \times 10^{-3}, 4.6 \times 10^{-3}\} \text{ [Hy]} \end{aligned}$$

and a gear ratio matrix $N = \text{diag}\{15, 15\}$. The gravity effect term is given as:

$$g(q) = \begin{bmatrix} (M_1l_{c1} + M_2l_1)g \sin(q_1) + M_2l_{c2}g \sin(q_1 + q_2) \\ M_2l_{c2}g \sin(q_1 + q_2) \end{bmatrix},$$

where M_j, l_j, l_{cj} , $j=1,2$, represent, respectively, mass, length and center of mass of link j whereas $g = 9.81 \text{ [m/s}^2\text{]}$ is gravity acceleration. From this and (7) we define:

$$\begin{aligned} \Phi(q) &= \begin{bmatrix} \sin(q_1) & \sin(q_1 + q_2) & 0 \\ 0 & 0 & \sin(q_1 + q_2) \end{bmatrix}, \\ \theta^* &= \begin{bmatrix} \frac{R_1}{K_{m1}}(M_1l_{c1} + M_2l_1)g \\ \frac{R_1}{K_{m1}}M_2l_{c2}g \\ \frac{R_2}{K_{m2}}M_2l_{c2}g \end{bmatrix}, \end{aligned} \quad (33)$$

i.e., Assumption 1 is satisfied. We stress that, according to the numerical values presented in [11], we have $k_g = 80.578 \text{ [Kg-m}^2\text{/s}^2\text{]}$ and $k_c = 0.336 \text{ [Kg-m}^2\text{]}$. On the other hand, we can write

$\Gamma\Phi^T(q_d) = \Gamma'\Phi^T(q_d)K_m R^{-1}$, given in Assumption 2, as:

$$\begin{bmatrix} \Gamma_{11} \sin(q_{1d}) & 0 \\ \Gamma_{22} \sin(q_{1d} + q_{2d}) & 0 \\ 0 & \Gamma_{33} \sin(q_{1d} + q_{2d}) \end{bmatrix} = \begin{bmatrix} \frac{K_{m1}}{R_1} \Gamma'_{11} \sin(q_{1d}) & 0 \\ \frac{K_{m1}}{R_1} \Gamma'_{22} \sin(q_{1d} + q_{2d}) & 0 \\ 0 & \frac{K_{m2}}{R_2} \Gamma'_{33} \sin(q_{1d} + q_{2d}) \end{bmatrix},$$

where we have used the assumption that Γ and Γ' are 3×3 diagonal matrices. Note, for instance, that the previous expression implies $\Gamma_{11} = \frac{K_{m1}}{R_1} \Gamma'_{11}$. We recall that Γ_{11} and Γ'_{11} are positive arbitrary numbers. Hence, for any given Γ_{11} there exists a Γ'_{11} such that this expression is satisfied without requiring the value of the unknown constant $\frac{K_{m1}}{R_1}$. This shows the validity of Assumption 2. Further, it is not difficult to verify, from (33), that Assumption 3 also stands and that matrix $\Phi(q)$ has the form explained in the last paragraph of Section 2 with $m_1 = 2$, $m_2 = 1$, $m = m_1 + m_2 = 3$. We used the following controller gains $\overline{K_P} = \text{diag}\{200, 200\}$, $\overline{K_D} = \text{diag}\{250, 250\}$, $\overline{K_V} = \text{diag}\{90, 90\}$, $\Gamma = \text{diag}\{500, 500, 500\}$, $A = \text{diag}\{110, 110\}$, $B = \text{diag}\{19, 19\}$, $\varepsilon = 0.27$. These values were selected such that conditions (28)-(31), (25), (26), (10) are all satisfied using $\|x\| < 1$ and ensuring that F_2 is positive

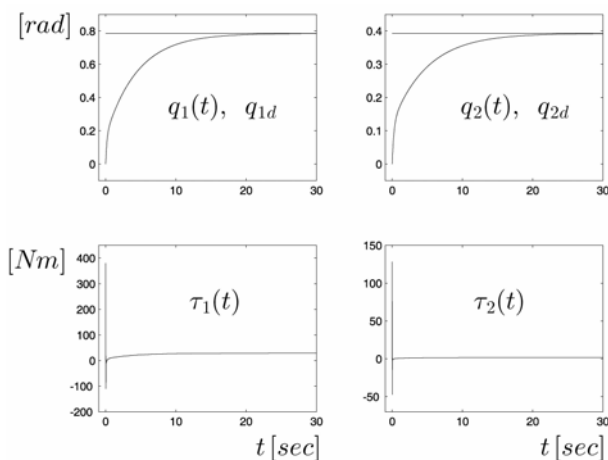


Fig. 1. Simulation results when the controller of Proposition 1 is used.

definite. This was verified by using the procedure described in Remark 2, i.e., only estimated values of robot and actuator parameters are required. In Fig. 1 we show the performance obtained with controller of Proposition 1. The desired values are $q_{1d} = \pi/4$ and $q_{2d} = \pi/8$ whereas all initial conditions are set to zero. Note that convergence to the desired values is obtained as expected and this is achieved in approximately 30 seconds. Comparing with time responses reported in the literature (more than 50 sec. in [10, pp.215] and more than 200 sec. in [13]) for linear robot controllers whose gains are selected in such a way that stability is ensured, we find 30 seconds to be an acceptable time response.

5. CONCLUSIONS

In this note we have presented a control scheme for position regulation in robot manipulators whose design takes into account the dynamics of the brushed DC-motors used as actuators. Measurements of torque and electric current are avoided. Further, contrary to the common assumption in the literature we do not require the actuators electric dynamics time constant to be small. We also present a design procedure to select the controller gains without requiring the exact knowledge of neither robot nor actuator parameters. As we explain, this can always be done using a try-error search based on a digital computer program to verify all the conditions presented in this note.

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