

# Nonlinear Optimal Control of an Input-Constrained and Enclosed Thermal Processing System

Kwan-Woong Gwak and Glenn Y. Masada

**Abstract:** Temperature control of an enclosed thermal system which has many applications including Rapid Thermal Processing (RTP) of semiconductor wafers showed an input-constraint violation for nonlinear controllers due to inherent strong coupling between the elements [1]. In this paper, a constrained nonlinear optimal control design is developed, which accommodates input constraints using the linear algebraic equivalence of the nonlinear controllers, for the temperature control of an enclosed thermal process. First, it will be shown that design of nonlinear controllers is equivalent to solving a set of linear algebraic equations—the linear algebraic equivalence of nonlinear controllers (LAENC). Then an input-constrained nonlinear optimal controller is designed based on that LAENC using the constrained linear least squares method. Through numerical simulations, it is demonstrated that the proposed controller achieves the equivalent performances to the classical nonlinear controllers with less total energy consumption. Moreover, it generates the practical control solution, in other words, control solutions do not violate the input-constraints.

**Keywords:** Constrained linear least squares, input constraint, LAENC, nonlinear optimal control.

## 1. INTRODUCTION

Estimating and controlling energy inputs from distributed energy sources to generate desired heating profiles (temperature or heat flux) for material processing is a classic optimal design problem. In many applications, such as the rapid thermal processing (RTP) of semiconductor wafers, infrared paint dryers, annealing furnaces, and directed energy manufacturing systems such as selective laser sintering, precise thermal control is necessary for quality assurance.

Control engineers approached to this energy input estimation/control problem using mathematical modeling and feedback control, and good examples are found in the RTP temperature control of semiconductor wafers [2-13]. Typical models of RTP systems are described by PDEs. Solving these equations are computationally expensive and it is

practically difficult to implement PDEs into the controller design. Therefore, many control engineers used lumped parameter models [3-6] and empirical models [5,7]. Several feedback control algorithms have been applied on those models (linear programming [3], linear-quadratic [7], linear-quadratic-Gaussian [5], internal model control [4], and neural network [8]) and promising results have been reported, however, most of the designs are based on low-order (dimensional) linear dynamic models which are valid only for small RTP chambers. Strong coupling between the elements in the system that cause ill-conditioned behavior did not appear in their work because of using the low order models. Low order models cannot guarantee high performance for large dimensional systems such as infrared paint dryers and annealing furnaces. Also, variations in DC gain of a linearized model by temperature changes [11] can be a significant problem for precision control. Hence, in this research, a generalized high-order, nonlinear, and highly coupled ill-conditioned thermal system model is considered. For such a system model, application of nonlinear control system is natural as the dominant heat transfer mode is radiation.

Feedback linearization (FBL) and sliding mode control (SMC) are two widely used control schemes for nonlinear systems. FBL is popular because well-developed design techniques for linear systems can be applied to synthesize the appropriate controller for the linearized systems. SMC is attractive because it provides high speed response, good transient

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performance and is insensitive to certain parameter variations and external disturbances. Despite these desirable features, the major drawback of using FBL/SMC is their inability to handle explicit input constraints while the thermal system treated in this research clearly has input constraints—heater capacity and non-negativity of the heater input. If a design does not take into account the bounds on the control input and if the control signals saturate during operation, the stability and performance of the closed loop system are diminished. Hence, control solutions for highly nonlinear systems with input constraints are important research areas. However, few results have been reported for constrained FBL/SMC nonlinear control systems while several design techniques are available for constrained linear systems.

Hence, in this paper, we propose a constrained FBL/SMC nonlinear optimal controller that can be represented in a simple constrained linear least square problem for the temperature control of an input-constrained and ill-conditioned thermal process. The fundamental approach is to use the linear algebraic equivalence of the nonlinear controller (LAENC)—design of FBL/SMC is equivalent to solving a set of linear algebraic equations. Input constraints and optimization issues are addressed based on that LAENC.

The proposed controller is structurally very similar to model predictive control (MPC) since control actions are obtained through an optimal control strategy that minimizes a performance function. However it has the distinct advantage over MPC in that it preserves the characteristics of FBL/SMC—one can design the output/state behavior with FBL, and constrain the states to remain on the desired manifold with SMC. This is not possible with MPC. Also, the proposed controller is easy to design and is computationally efficient since it solves the linear least squares problem.

## 2. BACKGROUND

A common approach to designing nonlinear FBL control with constraints uses changes in the reference commands. Pappas et al. [14] calculated the regions of attraction of the controllers and characterized the space of feasible trajectories that do not violate the input constraints. Similarly, Aguilar et al. [15] detuned the FBL to avoid input constraints. But these approaches [14,15] generated unnecessarily poor performance [28]. Yip and Hedrick [16] devised a dynamic reference governor which changes the reference command to the feedback linearized system using a filtered version of the reference command. However, it is difficult to extend this approach to multi-input multi-output (MIMO) systems.

Valluri and Soroush [17] derived two nonlinear

control laws satisfying input constraints by minimizing the difference between the closed loop output response and the nominal linear output response that the same control law induces when there are no constraints.

Another approach to accommodate input constraints is anti-windup schemes that modify the controllers to minimize the adverse effects of windup. It is based on an observer-based structure in which the difference between the computed input and the actual input to the process is fed back to the controller dynamics in an attempt to minimize the difference. Kendi and Doyle [18] proposed an instantaneous optimization method to minimize the performance loss associated with enforcing the actuator constraints. Within this framework, the nonlinear controller was represented as an optimal linear controller with an auxiliary feedback loop which cancels the effects of nonlinear dynamics and measured disturbances. However, since the nonlinear corrective action is implemented by feedback, the difficulty arises in implementing a nonlinear internal-model-based antiwindup controller. Kapoor and Daoutidis [19] devised a nonlinear observer-based anti-windup algorithm, in which the anti-windup gain is a nonlinear function of the states of the system and with which it is possible to attenuate the effect of windup arbitrarily fast. However, in these researches ([17-19]), input constraints are not considered explicitly as part of the controller design. Instead, the controller is combined with an anti-windup compensator designed to minimize performance degradation caused by constraints [28]. Moreover, it is not possible to restrict the output behavior to the desired dynamics.

For SMC, Shyu and Lin [20] handled input constraints by computing a bound for the existence of sliding motion and used a switching surface with an integrator; but it is applicable only to linear systems. Lu and Chen [21] obtained the range of allowable reference inputs by estimating the maximum and minimum values of the control action to ensure sliding behavior throughout the response—but the approach is only applicable to linear time-varying systems. Okabayashi and Furuta [22] showed the effectiveness of using a nonlinear hypersurface (switching surface) instead of a conventional hyperplane as the switching surface; but this approach is applicable only to linear systems.

Popular approaches for the control of nonlinear systems with input constraints uses nonlinear optimization of a model predictive control (MPC). MPC is a class of control algorithms that utilize an explicit process model to predict the effects of future control actions on the output of the process [23]. The control sequence is computed using an optimal control strategy that minimizes a performance function which includes the differences between the desired and

predicted process variables, and a penalty on the control effort. MPC is usually divided into nonlinear model predictive control (NMPC) and linear model predictive control (LMPC).

NMPC uses the nonlinear process model to predict the effects of future manipulated inputs on future values of the controlled outputs. NMPC handles input constraints by solving a nonlinear programming problem on-line for each sampling period. This approach is computationally expensive and potentially unreliable as the nonlinear program may converge to a local minimum or even diverge [24].

LMPC minimizes a quadratic performance function, and the optimal solution, when subjected to linear constraints, is found by using quadratic programming (QP) routines. Using the convex nature of the problem, QP produces numerically efficient results by relying on fast gradient descent methods [25]. Despite the nice features of LMPC for linear systems, LMPC can not be applied directly to nonlinear processes. To exploit LMPC for nonlinear processes, a hybrid scheme is used by several researchers [24-28]—a combination of FBL and MPC. Using the resulting linearized system by FBL, the NMPC control problem is transformed into an optimization problem that minimizes a quadratic function, whose solutions can be found using reliable and fast quadratic programming routines. However, FBL maps the original input linear constraints into nonlinear and state dependent constraints on the controller output [25], which invalidates the direct use of QP routines.

Also, it requires the knowledge of future values of the input and state variables, which is not possible to determine until the constraints are specified. Instead of the exact mapping of future input constraints, Botto et al. [25] used iterative approximate mapping techniques, Henson and Krtz [29] extended the linear constraint relations of the first prediction over the entire control horizon and others also proposed approximate mapping methods [24,26,27]. Therefore, MPC requires an additional complex approximation computation to utilize LMPC, and it is only applicable to feedback linearizable or linear systems.

All the above-mentioned methods of dealing with input constraints have disadvantages, such as complex designs, limitation to linear systems only, and the need to change reference signals. In this paper, we propose a nonlinear optimal input-constraint-satisfying controller that is simple to design and is computationally efficient.

### 3. SYSTEM MODELING AND NONLINEAR CONTROL DESIGN

The problem considered in this paper is to estimate the necessary thermal conditions on the heater surfaces to achieve a desired temperature distribution

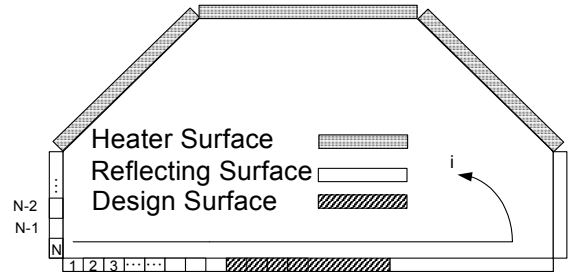


Fig. 1. Geometry of the thermal system.

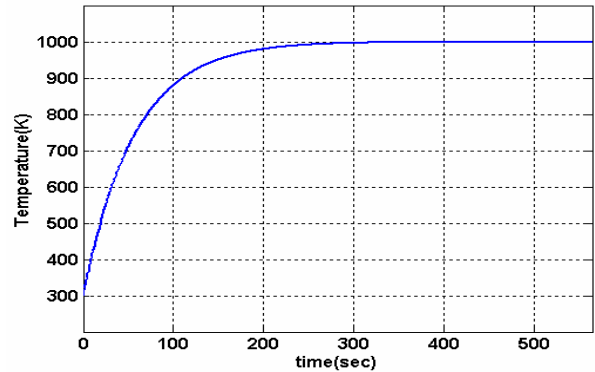


Fig. 2. Desired temperature profile of the design surface.

on a design surface. Consider the two-dimensional enclosure shown in Fig. 1, made up of the design surface (bottom center), reflective surfaces (sides and bottom sides), and heater surfaces (top). The design surface, heated by the heater surfaces, is controlled to a prescribed profile shown in Fig. 2 and to be spatially uniform in temperature. The dimensions, assumptions, exchange factors, material properties, and desired thermal objectives are adopted from [30].

The system is divided into 64 elements; ten for the design surface ( $i=9-18$ ), 30 for the heater surfaces ( $i=31-60$ ) and 24 for the reflective surfaces ( $i=1-8, 19-30, 61-64$ ), and the state equations for each element  $i$  are derived as follows using the energy equation

$$\dot{x}_i = \frac{1}{\rho_i c_{p,i} \delta_i \Delta l_i} \left[ -\varepsilon_i \sigma x_i^4 \Delta l_i + \sum_{k=1}^N \varepsilon_i E F_{k-i} \sigma x_k^4 \Delta l_k + Q_i \right], \quad (1)$$

where state  $x_i$  represents the surface temperature of each surface  $i$ , and  $Q$  stands for the power input for heater strips (W/m),  $\Delta l_i$  for the length of  $i$ th element(m),  $c_p$  for specific heat (J/kg K),  $\rho$  for density (Kg/m<sup>3</sup>),  $\delta$  for the plate thickness(m),  $\varepsilon$  for the emissivity of the surface,  $\sigma$  for the Stefan-Boltzmann constant ( $\sigma = 5.67 \times 10^{-8}$  W/m<sup>2</sup>K<sup>4</sup>),  $E F_{i-j}$  for the exchange factor between element  $i$  and  $j$ , and  $N$  for the total number of elements.

Equation (1) can be expressed in a more familiar and general form as:

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u}, \\ \mathbf{y} &= \mathbf{h}(\mathbf{x}),\end{aligned}\quad (2)$$

where  $\mathbf{x} \in \mathbf{R}^{64}$ ,  $\mathbf{u} \in \mathbf{R}^{30}$ ,  $\mathbf{y} \in \mathbf{R}^{10}$ ,  $\mathbf{f} \in \mathbf{R}^{64} \rightarrow \mathbf{R}^{64}$ ,  $\mathbf{g} : \mathbf{R}^{64} \rightarrow \mathbf{R}^{64}$ ,  $\mathbf{h} : \mathbf{R}^{64} \rightarrow \mathbf{R}^{10}$  and  $\mathbf{f}$ ,  $\mathbf{g}$  and  $\mathbf{h}$  are smooth functions of  $\mathbf{x}$ . The output,  $\mathbf{y}$ , is the temperature of the design surface, therefore

$$\mathbf{y}(t) = \mathbf{h}(\mathbf{x}) = \begin{bmatrix} x_9 \\ \vdots \\ x_{18} \end{bmatrix} = [\mathbf{0}_{10 \times 8} \quad \mathbf{I}_{10 \times 10} \quad \mathbf{0}_{10 \times 46}] \mathbf{x} = \mathbf{G}\mathbf{x}.\quad (3)$$

The system equation is non-square; 30 inputs and 10 outputs. For non-autonomous, non-square, affine nonlinear systems described by (2), both FBL and SMC are appropriate control algorithms.

A closer look at the output equation reveals that the relative degree of the system is two, hence the output is differentiated twice:

$$\ddot{\mathbf{y}}(t) = \mathbf{L}_f^2 \mathbf{h}(\mathbf{x}) + \mathbf{L}_g \mathbf{L}_f \mathbf{h}(\mathbf{x})\mathbf{u}.\quad (4)$$

Let the desired dynamics for the FBL be defined as

$$\ddot{\mathbf{e}} + \lambda_1 \dot{\mathbf{e}} + \lambda_2 \mathbf{e} = \mathbf{0} \quad \text{where, } \mathbf{e} = \mathbf{y} - \mathbf{y}_d.\quad (5)$$

Then

$$\ddot{\mathbf{y}} = \ddot{\mathbf{y}}_d - \lambda_1 \dot{\mathbf{e}} - \lambda_2 \mathbf{e}.\quad (6)$$

Substituting (6) into (4) yields

$$(\mathbf{L}_g \mathbf{L}_f \mathbf{h}(\mathbf{x}))\mathbf{u} = \{\ddot{\mathbf{y}}_d - \lambda_1 \dot{\mathbf{e}} - \lambda_2 \mathbf{e} - \mathbf{L}_f^2 \mathbf{h}(\mathbf{x})\}.\quad (7)$$

Finally, the nonlinear controller is

$$\mathbf{u} = (\mathbf{L}_g \mathbf{L}_f \mathbf{h}(\mathbf{x}))^+ \{\ddot{\mathbf{y}}_d - \lambda_1 \dot{\mathbf{e}} - \lambda_2 \mathbf{e} - \mathbf{L}_f^2 \mathbf{h}(\mathbf{x})\},\quad (8)$$

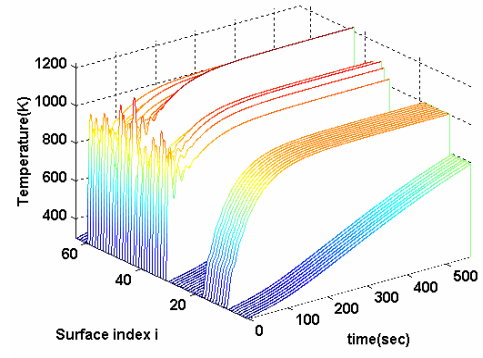
where  $+$  represents the pseudo-inverse since  $\mathbf{L}_g \mathbf{L}_f \mathbf{h}(\mathbf{x})$  is not square.

For SMC, instead of setting the design surface temperature as the output, the sliding surfaces  $\mathbf{S} = \dot{\mathbf{e}} + \lambda_{\text{SMC}} \mathbf{e}$  are selected as the outputs. With only one differentiation

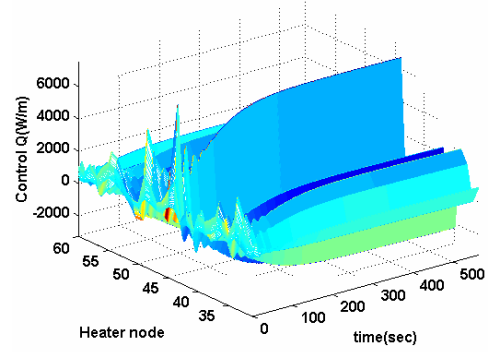
$$\begin{aligned}\dot{\mathbf{S}} &= \dot{\mathbf{e}} + \lambda_{\text{SMC}} \dot{\mathbf{e}} = \ddot{\mathbf{y}} - \ddot{\mathbf{y}}_d + \lambda_{\text{SMC}} \dot{\mathbf{e}} \\ &= \mathbf{L}_f^2 \mathbf{h}(\mathbf{x}) + \mathbf{L}_g \mathbf{L}_f \mathbf{h}(\mathbf{x})\mathbf{u} - \ddot{\mathbf{y}}_d + \lambda_{\text{SMC}} \dot{\mathbf{e}} \\ &= -\mathbf{D} \cdot \text{sgn}(\mathbf{S}).\end{aligned}\quad (9)$$

Rearranging:

$$(\mathbf{L}_g \mathbf{L}_f \mathbf{h}(\mathbf{x}))\mathbf{u} = \{\ddot{\mathbf{y}}_d - \lambda_{\text{SMC}} \dot{\mathbf{e}} - \mathbf{L}_f^2 \mathbf{h}(\mathbf{x}) - \mathbf{D} \cdot \text{sgn}(\mathbf{S})\},\quad (10)$$



(a) Temperature



(b) Control input Q

Fig. 3 (a) Temperature distribution along the surfaces (b) control input Q for FBL with  $\lambda_1 = 0.1$ ,  $\lambda_2 = 0.1$ .

where  $\mathbf{D}$  is a positive definite diagonal gain matrix and  $\text{sgn}(\mathbf{S})$  is the signum function. The nonlinear control using SMC is

$$\mathbf{u} = (\mathbf{L}_g \mathbf{L}_f \mathbf{h}(\mathbf{x}))^+ \{\ddot{\mathbf{y}}_d - \lambda_{\text{SMC}} \dot{\mathbf{e}} - \mathbf{L}_f^2 \mathbf{h}(\mathbf{x}) - \mathbf{D} \cdot \text{sgn}(\mathbf{S})\}.\quad (11)$$

Fig. 3 shows the system response using a classic nonlinear FBL controller—Fig. 3(a) shows excellent tracking of the design surface temperatures to the desired temperature trajectory. However, the heater surface control input distribution (Fig. 3(b)) shows that 14 out of 30 heaters have negative heat inputs—the heaters act as heat sinks rather than heat sources. The control input solutions (heating surface Q's) are not practical solutions. In other words, the solution violates input constraints that all control inputs must be positive since the actuators are heaters. The non-practical and constraint-violating control solutions arise from the ill-conditioned nature of the controller [1].

#### 4. LINEAR ALGEBRAIC EQUIVALENCE OF THE NONLINEAR CONTROLLERS (LAENC)

From the design processes of FBL and SMC, notice

that both controller designs ((8) and (11)) end up in the form

$$\mathbf{u} = \mathbf{x}_c = \mathbf{A}^+ \mathbf{b} \Rightarrow \mathbf{A} \mathbf{x}_c = \mathbf{b}, \quad (12)$$

where  $\mathbf{A} = \mathbf{L}_g \mathbf{L}_f^{r-1} \mathbf{h}(\mathbf{x})$

$$\mathbf{b} = \begin{cases} \mathbf{v} - \mathbf{L}_f^r \mathbf{h}(\mathbf{x}) & \text{for FBL} \\ \mathbf{y}_d^r - \mathbf{L}_f^r \mathbf{h}(\mathbf{x}) - \sum_{k=1}^{r-1} \beta_k \frac{d^k e_i}{dt^k} - \eta \cdot \text{sgn}(\mathbf{s}) & \text{for SMC.} \end{cases}$$

The subscript  $c$  on control input vector  $\mathbf{x}_c$  will be omitted hereafter for the simplicity. All the vector  $\mathbf{x}$ 's appear hereafter represent the control input vector, not a state vector.

Note that (12) is based on the assumption that the system can be represented in normal form, is affine in control, has an invertible decoupling matrix  $\mathbf{D}(\mathbf{x})$  and has stable zero dynamics.

The control input vector  $\mathbf{x}$  is simply obtained by matrix inversion. However, the solution to (12) is neither unique nor does it always exist. Once the dimension of the solution vector (number of inputs, i.e., controllers) is less than the number of equations (outputs/sliding surfaces), and matrix  $\mathbf{A} \in \mathbf{R}^{p \times m}$  has full column rank ( $m$ ), the problem is a least squares problem. On the other hand, if the dimension of the solution vector is larger than the number of equations and matrix  $\mathbf{A}$  has full row rank ( $p$ ) for  $\mathbf{A} \in \mathbf{R}^{p \times m}$ , the problem is redundant and has infinite number of solutions. In selecting an optimal solution, a unique solution can be found by considering certain constraints, like minimizing the control input vector  $\|\mathbf{x}\|_2^2$ . Such an optimal solution can be found by using the pseudo-inverse of  $\mathbf{A}$ , represented as  $\mathbf{A}^+$ . Even if there is no solution, the pseudo-inverse provides the best approximation that minimizes the squared error;

$$\|\mathbf{A} \mathbf{x} - \mathbf{b}\|_2^2. \quad (13)$$

## 5. CONSTRAINED NONLINEAR OPTIMAL CONTROL

Equation (12) is a set of linear algebraic equations in which the solution  $\mathbf{x}$  is the control input vector. This is a significant representation of the nonlinear control design since well developed linear algebraic equation solution techniques can be applied directly to satisfy the nonlinear controller (FBL/SMC) characteristics (performance). Hence many different problems can be formulated—minimization, maximization, optimization (tradeoff) and constraints can be treated for FBL/SMC.

As an example, one can think of the constrained linear least squares problem defined by

$$\min_{\mathbf{x}} \|\mathbf{A} \mathbf{x} - \mathbf{b}\|_2^2 \quad \text{such that} \quad \mathbf{x}_{\min} \leq \mathbf{x} \leq \mathbf{x}_{\max}. \quad (14)$$

For this case, the solution is always guaranteed but a residual arises to satisfy the input bounds. This residual can be considered as a modeling error which diminishes performance.

Once the solution of (14) secures the closed loop stability and performance, another constraints can be considered—minimizing the control effort. Then, a new cost function is defined as

$$\begin{aligned} J &= \sum_{i=1}^p (A_i x - b_i)^2 + \sum_{i=1}^m \gamma^2 m_i^2 x_i^2 \\ &= \|\mathbf{A} \mathbf{x} - \mathbf{b}\|_2^2 + \gamma^2 \|\mathbf{M} \mathbf{x}\|_2^2 \end{aligned} \quad (15)$$

and a new input-constrained nonlinear optimization (tradeoff) problem is posed as;

$$\begin{aligned} \min_{\mathbf{x}} J &= \min_{\mathbf{x}} \left\{ \|\mathbf{A} \mathbf{x} - \mathbf{b}\|_2^2 + \gamma^2 \|\mathbf{M} \mathbf{x}\|_2^2 \right\} \\ &\quad \text{such that} \quad \mathbf{x}_{\min} \leq \mathbf{x} \leq \mathbf{x}_{\max}, \end{aligned} \quad (16)$$

where  $\gamma^2$  is the weighting parameter that balances tradeoff between minimum effort and control performance.

As the existence of a solution vector, that guarantees the performance and the stability of the closed loop system while satisfying the input constraints, is unknown a priori, we let  $\gamma=0$  to begin with and then start to increase its value to some positive number.

The analytical solution to this problem can be obtained by the following equation if there are no constraints on  $\mathbf{x}$ . That is, minimizing (15) with respect to  $\mathbf{x}$  gives:

$$\mathbf{x} = (\mathbf{A}^T \mathbf{A} + \gamma^2 \mathbf{M}^T \mathbf{M})^{-1} \mathbf{A}^T \mathbf{b}. \quad (17)$$

However, with the input constraints, an analytical solution for the optimal control vector can no longer be found. Note here that (16) can also be represented in augmented form as

$$\begin{aligned} \min_{\mathbf{x}} \left\| \begin{pmatrix} \mathbf{A} \\ \gamma \mathbf{M} \end{pmatrix} \mathbf{x} - \begin{pmatrix} \mathbf{b} \\ \mathbf{0} \end{pmatrix} \right\|_2^2 &= \min_{\mathbf{x}} \|\mathbf{A}_{aug} \mathbf{x} - \mathbf{b}_{aug}\|_2^2 \\ \text{such the} \quad \mathbf{x}_{\min} &\leq \mathbf{x} \leq \mathbf{x}_{\max} \\ \text{where} \quad \mathbf{A}_{aug} &= \begin{pmatrix} \mathbf{A} \\ \gamma \mathbf{M} \end{pmatrix} \in \mathbf{R}^{(p+m) \times m}, \\ \mathbf{b}_{aug} &= \begin{pmatrix} \mathbf{b} \\ \mathbf{0} \end{pmatrix} \in \mathbf{R}^{(p+m) \times 1}. \end{aligned} \quad (18)$$

This formulation is in the form of linear least squares—therefore constrained linear least squares method can be applied to solve the constrained nonlinear optimal control problem.

Equation (18) is an optimization problem that tradeoffs the control effort and control performance by using weight  $\gamma$  while satisfying input constraints. As  $\gamma$  increases, more weight is given to reduce the control effort thereby diminishing the system performance (increasing residual).

Notice that this approach is very similar to using linear optimal (LQ) control and MPC, where the basic idea is to find control inputs that minimize a cost function that balances error and control effort. However there is a distinct difference between this problem formulation and that using LQ and MPC—the control input obtained by this solution not only minimizes the cost function to balance error and control effort size, which LQ and MPC can do as well, but it also preserves the characteristics of the nonlinear controller on which the process was based, i.e. FBL/SMC. In other words, satisfying  $\mathbf{Ax} = \mathbf{b}$  guarantees the designer's desired output dynamics/sliding behavior. LQ and MPC minimize the performance index but does not make the system output follow the desired output dynamics/sliding behavior.

Due to the performance index tradeoffs,  $\mathbf{Ax} = \mathbf{b}$  is not perfectly satisfied, but the solution can be close depending on the value of  $\gamma$ . This solution vector,  $\mathbf{x}$ , guarantees the output dynamics/sliding behavior to be as close as it was originally designed.

To summarize, the algorithm for the constrained nonlinear optimal control design is defined as constrained linear least squares problem as

$$\min_{\mathbf{x}} \left\{ \|\mathbf{Ax} - \mathbf{b}\|_2^2 + \gamma^2 \|\mathbf{Mx}\|_2^2 \right\} \quad \text{such that } \mathbf{x}_{\min} \leq \mathbf{x} \leq \mathbf{x}_{\max}. \quad (19)$$

Notice here that this optimization has two terms (residual and control input size), but can be easily extended to additional linear constraints terms as follows depending on the design objectives,

$$\min_{\mathbf{x}} J = \min_{\mathbf{x}} \left\{ \|\mathbf{Ax} - \mathbf{b}\|_2^2 + \gamma^2 \|\mathbf{Mx}\|_2^2 + \beta^2 \|\mathbf{Lx}\|_2^2 + \alpha^2 \|\mathbf{Cx} - \mathbf{d}\|_2^2 + \dots \right\}, \quad (20)$$

where  $\mathbf{L}$ ,  $\mathbf{C}$ ,  $\mathbf{d}$  are arbitrary constraint matrices and  $\beta$ ,  $\alpha$  are the optimization parameters. This means the number of rows of matrix  $\mathbf{A}_{aug}$  in (21) can be extended to any size depending on the size of the additional constraints added. Equation (20) is also equivalent to

$$\min_{\mathbf{x}} \left\| \begin{pmatrix} \mathbf{A} \\ \gamma\mathbf{M} \\ \beta\mathbf{L} \\ \alpha\mathbf{C} \end{pmatrix} \mathbf{x} - \begin{pmatrix} \mathbf{b} \\ \mathbf{0} \\ \mathbf{0} \\ \alpha\mathbf{d} \end{pmatrix} \right\|_2^2 = \min_{\mathbf{x}} \|\mathbf{A}_{aug}\mathbf{x} - \mathbf{b}_{aug}\|_2^2, \quad (21)$$

where  $\mathbf{A}_{aug} = \begin{pmatrix} \mathbf{A} \\ \gamma\mathbf{M} \\ \beta\mathbf{L} \\ \alpha\mathbf{C} \end{pmatrix}$ ,  $\mathbf{b}_{aug} = \begin{pmatrix} \mathbf{b} \\ \mathbf{0} \\ \mathbf{0} \\ \alpha\mathbf{d} \end{pmatrix}$ .

This problem is still in the same form as (18), hence it can be solved by the same solution methods used for (18). Of course, the constraints added could be nonlinear instead of linear constraints, however the problem becomes a nonlinear optimization problem. Therefore in this paper, the focus is restricted to the linear constraints only such that the optimization problem could be solved without nonlinear programming technique which is computationally expensive and potentially unreliable.

In summary, the linear equivalence of the nonlinear controller design is very powerful in solving nonlinear optimization problems with constrained inputs. It is simple to implement, satisfies input constraints and preserves the characteristics of FBL/SMC, all simultaneously. Note that the proposed controller is applicable only to a class of systems that satisfy the assumptions of the LAENC mentioned in the previous section.

## 6. APPLICATION TO THE TEMPERATURE CONTROL OF AN INPUT-CONSTRAINED AND ILL-CONDITIONED THERMAL PROCESS

As the control input solutions for both classical nonlinear controllers in (8) and (11) violate the input constraints, the proposed constrained nonlinear optimal controller is applied to the temperature control of an input-constrained and ill-conditioned thermal process in this section. Note that solution vector  $\mathbf{x}$  represents the control input vector, not a state vector.

As the existence of a solution vector is unknown a priori, we can start with  $\gamma = 0$  to begin with, so the temperature control problem of the thermal system with nonnegative input constraints is defined as follows.

$$\min_{\mathbf{x}} \|\mathbf{Ax} - \mathbf{b}\|_2^2 \quad \text{such that } \mathbf{x} \geq \mathbf{0} \quad (22)$$

Fig. 4(a) shows the temperature distribution of all the surfaces subject to the heater inputs computed by (22). The linear least squares algorithm with nonnegative constraints in MATLAB<sup>®</sup> is used to solve

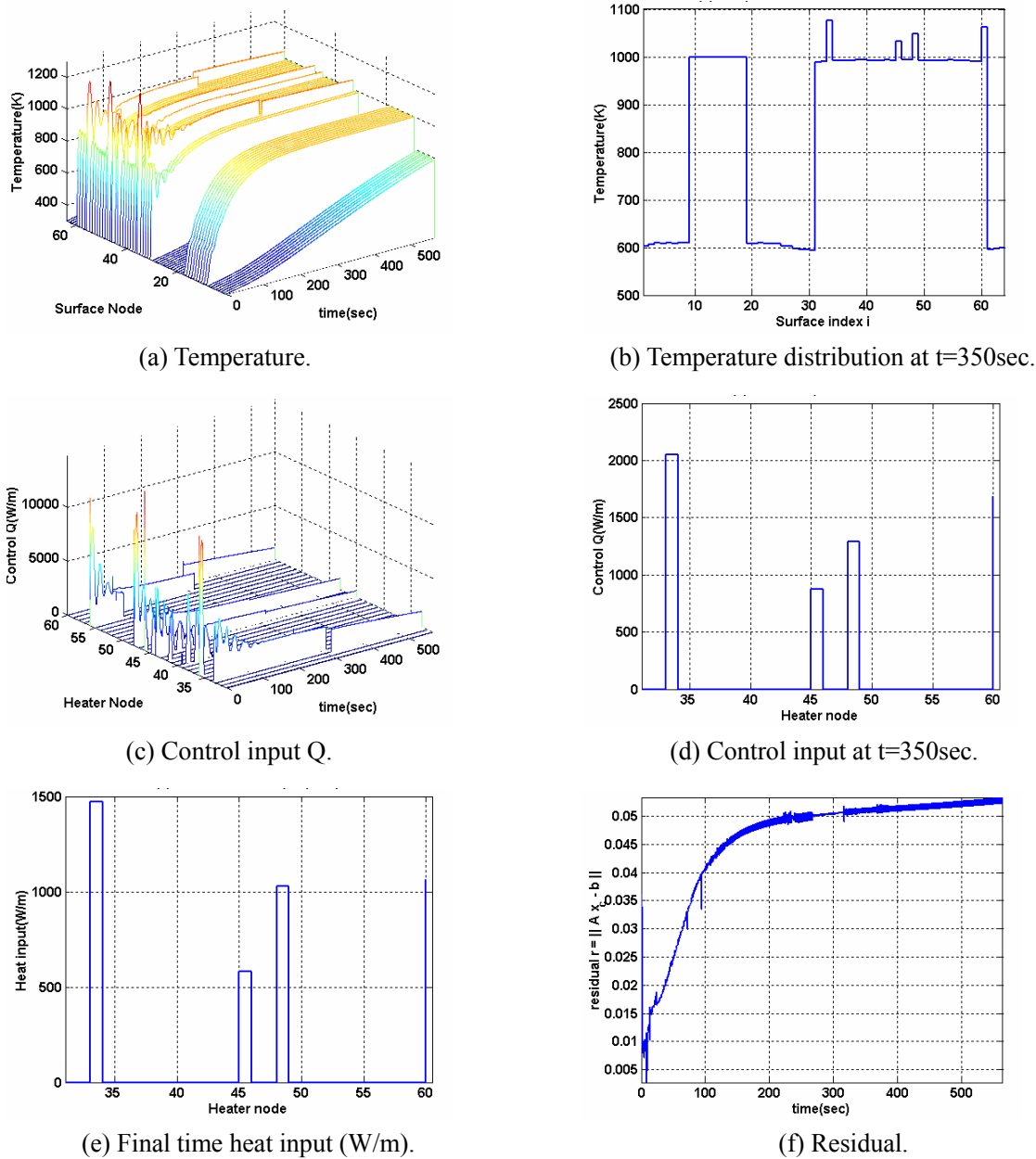


Fig. 4. Responses of the input constrained nonlinear controller in (22).

(22). Fig. 4(a) shows that the proposed controller preserves the characteristics of FBL on which it was originally designed. In other words, the outputs (surfaces 9-18) follow the desired error dynamics. Fig. 4(c) shows that all heater inputs satisfy the input constraints, i.e.,  $Q \geq 0$ . Fig. 4(f) shows the Euclidean norm of the residuals, considered as a modeling error which is inevitably caused by the input constraints. The residuals are relatively small in magnitude; hence, input-constraint-satisfying solutions are obtained with little degradation in performance which is verified in Fig. 4(a).

Positive net energy must be put into the system to control the design surface temperatures to follow the desired temperature profile. However due to the ill-

conditioned nature of the FBL/SMC controllers, the control input distribution was uneven (Fig. 3(b)), resulting in large positive inputs for some heaters and negative inputs for others. However to satisfy the constraint, i.e.,  $\mathbf{x} \geq \mathbf{0}$ , in this case of (22), it is expected that the linear least squares algorithm with nonnegative constraints will result in some inputs to be zero, i.e., some heaters are turned off, while the rest are strictly positive. This is verified in Fig. 4(c) where eighteen (surfaces 31, 32, 35, 37, 38, 39, 41, 43, 47, 49, 50, 51-55, 57, 59) out of 30 heaters are turned off throughout the process. The remaining heaters must provide all the energy needed to satisfy  $\mathbf{Ax}=\mathbf{b}$ , resulting in high peak values in certain heaters—in Fig. 4(c) the peak control input is 14,788W/m, which

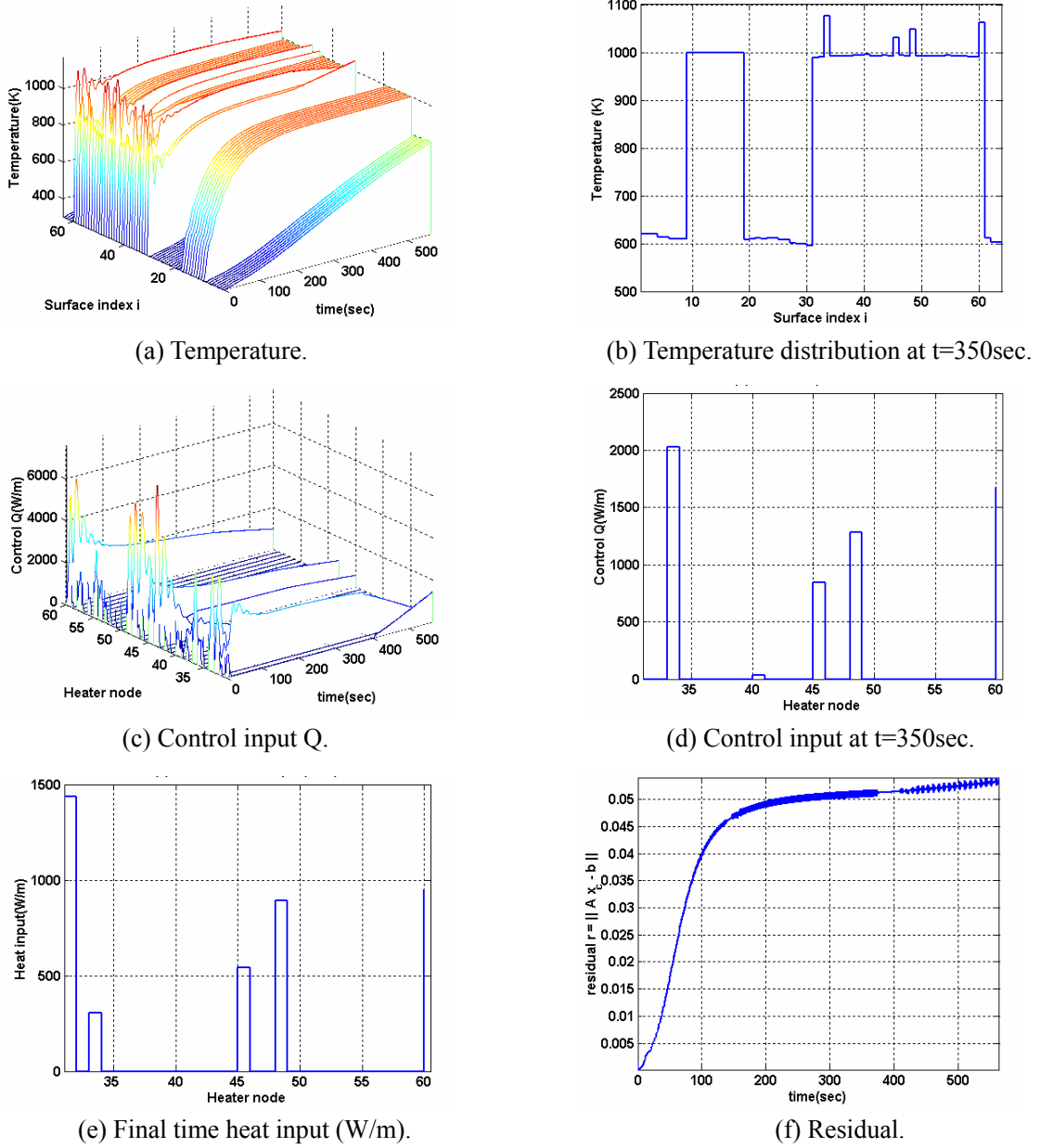


Fig. 5. Responses of the constrained nonlinear optimal controller with  $\gamma^2=1\text{E-}6$  in (23).

may violate the heater capacity. Figs. 4(c) and (d) show that the process is controlled by a few inputs, which implies that a smaller number of heaters can satisfy the design goal—an economic and computation advantage. This observation can lead to a system optimization problem in which different numbers and locations of heaters could be determined.

As the solution to (22) secures the closed loop stability and performance while satisfying the input constraints, tradeoff parameter  $\gamma$  now can be increased.

The identity matrix  $\mathbf{I}_{30 \times 30}$  is used for the weighting matrix  $\mathbf{M}$  in (18), i.e., the optimization problem is between performance (residual) and minimizing control effort. This is a nonlinear optimal

control problem that solves a linear least squares optimization with nonnegative input constraints:

$$\min_{\mathbf{x}} \|\mathbf{A}_{aug} \mathbf{x} - \mathbf{b}_{aug}\|_2^2 \quad \text{such that } \mathbf{x} \geq \mathbf{0}. \quad (23)$$

Figs. 5 and 6 are the result of the linear least squares with nonnegative input constraints in (23) with  $\gamma^2=1\text{E-}6$  and  $\gamma^2=1\text{E-}4$ , respectively. Larger  $\gamma^2$  places more weight on reducing control effort, hence less total  $Q$ , resulting in larger maximum and total error values. However larger  $\gamma^2$  does not always guarantee less total energy use since it tries to minimize control effort by sacrificing residual—increased residual causes larger errors in the

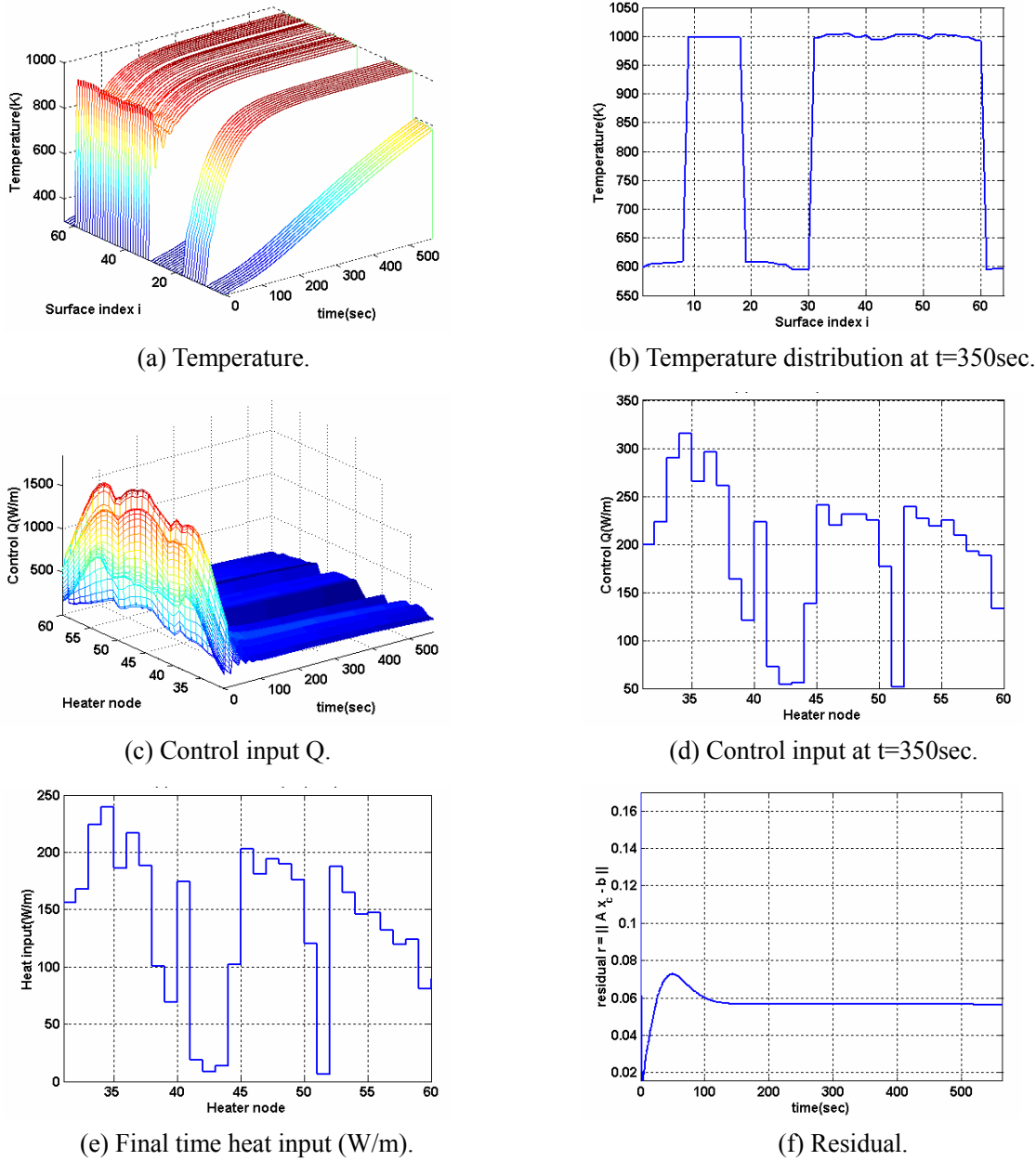


Fig. 6. Responses of the constrained nonlinear optimal controller with  $\gamma^2=1E-4$  in (23).

performance and performance degradation increases the control inputs. Therefore the value of  $\gamma^2$  should be selected carefully.

Notice that Figs. 6(a) and (c) are similar to those of the Tikhonov-FBL results in [1]—the control input distribution is more uniform than in the Fig. 5 case. Note that Eq. (23) is equivalent to

$$\min_{\mathbf{x}} \left\{ \|\mathbf{Ax} - \mathbf{b}\|_2^2 + \gamma^2 \|\mathbf{Mx}\|_2^2 \right\} \text{ such that } \mathbf{x} \geq \mathbf{0},$$

where  $\mathbf{M} = \mathbf{I}_{30 \times 30}$ . Hence larger  $\gamma^2$  implies more weight on minimizing the Euclidean norm,  $\|\mathbf{x}\|_2$ —Euclidean distance of the vector  $\mathbf{x}$  from the origin

which can be written as  $\|\mathbf{x}\|_2^2 = x_1^2 + x_2^2 + \dots + x_m^2$ . To minimize the Euclidean norm while keeping the total sum of all the elements the same (because the total amount of input energy at any moment must be kept as needed), the heater inputs should be more uniform. Hence, larger values of  $\gamma^2$  result in more uniform heater input distributions, thereby  $\gamma^2=1E-4$  is large enough to make the heater inputs to be sufficiently uniform such that all the inputs are strictly positive and no inputs turned off. However, the smoothing effect for  $\gamma^2=1E-6$  is not large enough, resulting in uneven heater input distribution, i.e., some are strictly positive and some are negative which are forced to be

Table 1. Performance index  $\int J dt = \int (\|\mathbf{Ax} - \mathbf{b}\|_2^2 + \gamma^2 \|\mathbf{Mx}\|_2^2) dt$ .

Controller	$\gamma^2 = 1\text{E-}6$	$\gamma^2 = 1\text{E-}4$
Tik-FBL 0.001 [1]	36.64	117.1
Least Squares Problem for Augmented System: Eq. (23)	27.14	116.9

Table 2. Performance measures.

Controller	Max. control input (W/m)	Total energy (J/m)	Max. error in design surface (K)	Max. steady-state error (K)
Eq. (22)	14788	4.4305e+6	31.34	0.42
Eq. (23) with $\gamma^2 = 1\text{E-}6$	7572	4.4518e+6	31.27	0.42
Eq. (23) with $\gamma^2 = 1\text{E-}4$	1857	4.4132e+6	31.42	0.45
FBL	7517	2.6425e+7	-31.1	0.0005

zero to satisfy the constraint as shown in Figs. 5(c), (d) and (e). Therefore a larger number of heaters are used in Figs. 6(d) and (e) compared to Figs. 5(d) and (e).

Table 1 shows the values of the performance index of the proposed and Tikhonov-FBL [1] controllers—the proposed controller with  $\gamma^2 = 1\text{E-}6$  has 25% less cost while the controller with  $\gamma^2 = 1\text{E-}4$  shows no significant difference, but the cost for having more uniform temperatures and more participating heaters (larger  $\gamma^2$ ) increased by a factor of four (27.14 versus 117).

Although the nonlinear optimal controller with  $\gamma^2 = 1\text{E-}6$  has less cost than the case with  $\gamma^2 = 1\text{E-}4$ , it generates relatively high peak control inputs in certain heaters as shown in Fig. 5(c) which may violate heater capacity.

Performance measures for four different controllers are shown in Table 2. It is noticed from the Table 2 that the controller defined by (23) with  $\gamma^2 = 1\text{E-}4$  requires the smallest total energy consumption and maximum control input while generating similar size errors to other controllers in the Table 2.

## 7. CONCLUSION

The design of constrained nonlinear optimal

controllers based on the linear algebraic equivalence of nonlinear controllers are proposed in the simple form of constrained linear least squares problem. The designs are simple, and simultaneously preserve the characteristics of FBL/SMC and satisfy the input constraints. The proposed algorithm can be applied to nonlinear optimal control problem with input constraints using classic FBL/SMC design methods if the system can be represented in the normal form, is affine in control, has invertible decoupling matrix  $\mathbf{D}(\mathbf{x})$ , and has stable zero dynamics. The designs have been applied to an input-constrained and ill-conditioned thermal process, and produced input-constraint-satisfying solutions. Effect of optimization parameter  $\gamma^2$  is discussed.

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