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Incorporating Performance Degradation in Fault Tolerant Control System Design with Multiple Actuator Failures

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Abstract: A fault tolerant control system design technique has been proposed and analyzed for managing performance degradation in the presence of multiple faults in actuators. The method is based on a control structure with a model reference reconfigurable control design in an inner loop and command input adjustment in an outer loop. The reduced dynamic performance requirements in the presence of different actuator faults are accounted for through different performance reduced (degraded) reference models. The degraded steady-state performances are governed by the reduced levels of command input. The reconfigurable controller is designed on-line automatically in an explicit model reference control framework so that the dynamics of the closed-loop system follow that of the performance reduced reference model under each fault condition. The reduced command input level is determined to prevent potential actuator saturation. The proposed method has been evaluated and analyzed using an aircraft example against actuator faults subject to constraints on the magnitude and slew-rate of actuators.

Keywords: Command input adjustment/management, fault detection and diagnosis (FDD), model-following reconfigurable control (MFRC), performance degradation.

1. INTRODUCTION

In view of potential performance limits induced by physical limitations (such as actuator magnitude and slew-rate saturation) in practical control systems, research on performance degradation in a control system design has recently begun to attract considerable attention [3-5,11,18]. Even though fault tolerant control of safety-critical systems is currently a very active research topic and a significant amount of research has been done in this area in the last two decades [1,7,10,13,17], fault tolerant control system (FTCS) designs with explicit consideration of possible performance degradation have not, until recently, received the same level of attention [18]. Most of the earlier work on FTCS design is centered around the philosophy to recover the pre-fault system performance as much as possible [6,8,10,16]. In practice, however, as a result of an actuator fault, the degree of the system redundancy and the available actuator capabilities could be significantly reduced. If the design objective is still to maintain the original system performance, this may force the remaining actuators to work beyond the normal duty to compensate for the handicaps caused by the fault. This situation is highly undesirable in practice due to physical limitations of actuators. The consequence of the so-designed FTCS may lead to actuator saturation, or worse still, to cause further damage. Therefore, trade-off between achievable performance and available actuator capability should be carefully considered in all FTCS designs. Designing a fault tolerant control system against actuator faults to achieve specified degraded performance without violating the actuator limits is therefore the main focus of this paper.

In [18], two reference models are used in conjunction with a model-following control scheme in order to deal with normal operation and system contingencies under actuator failures, respectively. Although a very important concept has been presented therein, it becomes evident that a twin model approach is not comprehensive enough to capture all potential system malfunctions. Different actuator faults in a system can exhibit distinctive characteristics; they cannot and should not be

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modeled only by a single degraded model. In fact, each fault should be handled differently. One of the objectives of this paper is to extend the previous work by incorporating different degraded reference models for achieving a gracefully degraded performance in the presence of different actuator faults. This involves the design of multiple degraded reference models and the selection of different levels of command input associated with each fault condition.

By representing each actuator fault via a degraded reference model synthesized with the consideration of the system performance limitation under the fault condition, the overall fault handling capability of a control system can be enhanced considerably. Since a unity steady-state gain of each reference model is required for the purpose of command tracking, the degradation levels in dynamic performance in the presence of faults are mainly governed by specified degraded reference models. Therefore, adjustments to the system command input levels are also crucial in achieving a gracefully degraded performance in the event of system component failures. In this paper, an alternative dynamic adjustment strategy based on the concept of pre-filter has been examined to provide a way for the dynamic adjustment of command input during the initial period of controller reconfiguration. The relationship between the pre-filter technique and the command governor scheme in [18] is also examined.

The main contribution of this paper is to provide a way for dealing with different actuator faults with different degraded reference models and different levels of dynamically adjusted command input to achieve a graceful performance degradation, within the framework of an on-line integrated FDD and a reconfigurable controller design.

The paper is organized as follows: The overall control structure accounting for achievable performance degradation under multiple actuator faults is presented in Section 2. A scheme for selecting a set of degraded reference models is presented in Section 3. A strategy for dynamic adjustments of the command input is proposed in Section 4. A model-following reconfigurable control design scheme associated with multiple reference models is developed in Section 5. Performance assessment of the designed FTCS for an aircraft example is presented in Section 6 followed by the conclusions in Section 7.

2. THE INNER-OUTER STRUCTURE ACCOUNTING FOR PERFORMANCE DEGRADATION WITH MULTIPLE ACTUATOR FAULTS

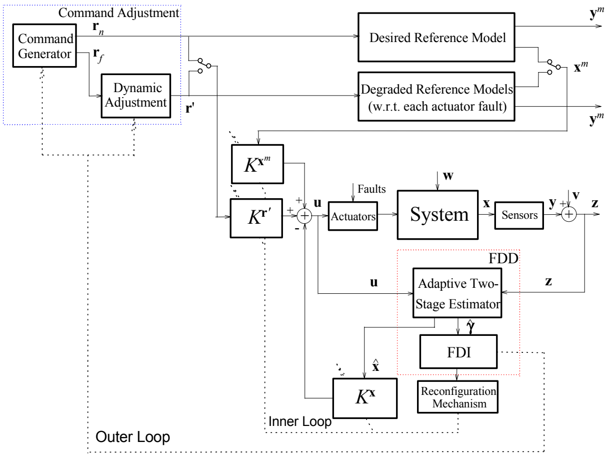

Fig. 1. Overall structure of the proposed FTCS.

The overall structure of the proposed FTCS is depicted in Fig. 1, which includes modules of multiple reference models, fault detection and diagnosis (FDD), a reconfiguration mechanism, and a model-following reconfigurable controller in the inner-loop; and the command adjustment/management module in the outer-loop. Note that in the presence of different faults, different degraded reference models are used with respect to (w.r.t.) different fault conditions.

The inner-outer loop structure enables us to achieve a specified degraded performance for the system at both transient and steady-state periods, with the purpose of avoiding potential slew-rate and magnitude saturation of the actuators. The inner-loop is also responsible for the guarantee of the stability and the desired achievement of dynamic performance through a reconfigurable model reference control strategy. The main function of the outer-loop is to re-adjust the command input levels such that a potential magnitude saturation in the steady-state as well as the transient interval during the control reconfiguration can be avoided.

To implement the above fault-tolerant control design in real-time, the post-fault system model has to be determined on-line and the state variables must be available for feedback. In practice, only part of the state variables may be measurable. To provide required state and fault parameters for feedback control, simultaneous state and parameter estimation techniques need to be used as shown in Fig. 1. A two-stage adaptive Kalman filter [15,16] has been used for such a purpose. Furthermore, the fault detection and isolation (FDI) scheme and the reconfiguration mechanism are also needed. The details on the fault detection and diagnosis (FDD) and reconfiguration mechanism modules in Fig. 1 have been omitted herein. Interested readers can refer to [16] for details. In the following sections, modules relating to multiple degraded reference models, command adjustment, and a reconfigurable model reference controller will be described in detail.

3. DESIGN OF MULTIPLE DEGRADED REFERENCE MODELS

3.1. The role of a reference model in achieving degraded performance

As can be seen in Fig. 1, reference models play an important role in achieving specified/degraded performance under normal and fault conditions. In particular, the dynamic behavior of the post-fault system is governed by the dynamics of the designed reference model. In fact, a reference model specifies an ideal trajectory which the post-fault system should reach a new steady-state condition defined by the system operating set-point. The reference models also affect the magnitude and slew-rate of generated closed-loop control signals through feedback-loop. Therefore, an appropriate selection of degraded reference models is important to achieve a specified degraded performance. If the reference model is selected so that its outputs follow the desired outputs very quickly, a large overshoot and a short rise time may occur. The corresponding control signals needed to track the responses of the reference model may become large, reaching or exceeding the actuator slew-rate or magnitude saturation region. However, if the reference model is selected so that the outputs track the command input rather slowly, then, the tracking accuracy improves without showing overshoots or going into an actuator saturation region. However the responses may become sluggish. Therefore, for the purpose of achieving a specified performance with a degraded level in the event of a system component failure, it is preferable to design appropriate reference models which make a good trade-off between the achievable performance and the physical constraints of the system.

3.2. The design of performance reduced reference model

Assume that a reference model of the system under the normal condition is represented by:

$$\begin{aligned} \dot{\mathbf{x}}_0^m &= A_0^m \mathbf{x}_0^m + B_0^m \mathbf{r}'_0, \\ \mathbf{y}_0^m &= C_0^m \mathbf{x}_0^m, \end{aligned} \tag{1}$$

where $\mathbf{x}_0^m \in \Re^n$ is the state vector of the reference model; $\mathbf{y}_0^m \in \Re^p$ is the output vector; and $\mathbf{r}'_0 \in \Re^l$ is the command input vector. It is assumed that $p = l$ for the purpose of command tracking. The above model, known as the *desired* reference model, specifies the desired dynamic characteristics of the system under a normal condition.

The corresponding transfer function matrix of this model is then:

$$T_0(s) = C_0^m (Is - A_0^m)^{-1} B_0^m, \tag{2}$$

where I is an identity matrix.

Let's assume that the eigenvalues of this system are represented as:

$$\Lambda_0 = diag[\lambda_1^0, \lambda_2^0, \ldots, \lambda_n^0]. \tag{3}$$

In the presence of a fault, it is expected that the system eigenvalues of the degraded reference models will move towards the imaginary axis to reflect the loss of the system dynamic performance.

Suppose that the eigenvalues of the degraded reference models under each actuator fault condition, $j = 1, \ldots, l,$ are represented as:

$$\Lambda_j = \Psi_j^{-1} \Lambda_0, \quad j = 1, \ldots, l \tag{4}$$

where the mode degradation matrices can be selected as:

$$\begin{aligned} \Psi_j = diag[\psi_1^j, \psi_2^j, \ldots, \psi_n^j], \quad \psi_i^j \geq 1, \\ \forall i = 1, \ldots, n; \quad j = 1, \ldots, l. \end{aligned} \tag{5}$$

The transfer function matrix of each reference model for the degraded system can then be obtained as:

$$\begin{aligned} T_j(s) &= C_0^m (Is\Psi_j - A_0^m)^{-1} B_0^m \\ &= C_0^m (Is - \Psi_j^{-1} A_0^m)^{-1} \Psi_j^{-1} B_0^m \\ &= C_j^m (Is - A_j^m)^{-1} B_j^m, \qquad j = 1, \ldots, l. \end{aligned} \tag{6}$$

Hence, a set of degraded reference models becomes:

$$\begin{aligned} \dot{\mathbf{x}}_j^m &= A_j^m \mathbf{x}_j^m + B_j^m \mathbf{r}'_j, \\ \mathbf{y}_j^m &= C_j^m \mathbf{x}_j^m, \qquad j = 1, \ldots, l, \end{aligned} \tag{7}$$

where $A_j^m = \Psi_j^{-1} A_0^m$, $B_j^m = \Psi_j^{-1} B_0^m$, $C_j^m = C_0^m$, $\forall j =$ 1,…, l.

The matrix triplets $\{A_j^m, B_j^m, C_j^m, j = 1, \ldots, l\}$ specify the characteristics of the degraded reference models for various fault conditions, where the same number of reference models as the number of actuators has been selected. By choosing different values of Ψ_j, different dynamic behaviors and different levels of performance degradation can be specified. The selection of each element in Ψ_j is application dependent and should be in conjunction with a time response analysis (represented by performance measures such as settling time, rise time and percentage of overshoot etc) of a designed reference model.

It should be pointed out that the synthesis of reasonable multiple reference models is a non-trivial task. One has to have a clear understanding of system performance requirements, availability of actuator redundancies, and underlying physical limitations of control actuators. These quantities set the ultimate performance limits for systems under system failures. It is crucial to embed these limits into the degraded reference models so that the physical limits of the system are not violated in the reconfigured system, i.e. all variables stay within an allowable space corresponding to the particular actuator fault. Fortunately, the above requirements can be achieved for practical engineering applications since, unlike the on-line design of reconfigurable controller, design of reference models are carried out in an off-line manner. This feature provides users an opportunity to determine expected degraded reference models with a good balance between an expected performance degradation and a potential actuator saturation. Note that the number of degraded reference models is equal to the number of actuators. This will also reduce the work load for designing reference models.

For the convenience to design reference models, the above reference models are firstly synthesized in continuous time domain and then converted to discrete models for the purpose of synthesis of an on-line reconfigurable controller.

4. COMMAND INPUT DYNAMIC ADJUSTMENT VIA A PRE-FILTER SCHEME

To ensure that all of the system variables are within the safe region and that all of the control actuators are free from saturation for the reconfigured system, one has to make appropriate adjustments to the level of a required control command input as well. A scheme proposed in [18] and other techniques developed in [2,9,14] can be used for such a purpose.

Generally speaking, an adjustment of the command input should include two parts: 1) the selection of a set of new command inputs to the system at the steady-state with respect to different fault conditions; 2) the dynamic adjustment of the command inputs during the initial period of control reconfiguration. The first part is to set acceptable new steady-state operating conditions which minimize the performance degradation while simultaneously avoiding a potential actuator magnitude saturation at steady-state. The role of the second part is to reduce the possibility of an actuator slew-rate as well as a magnitude saturation during the transient interval of the control reconfiguration process.

In this paper, a procedure proposed in [18] has been extended to determine a set of new command inputs associated with different actuator faults. However, the more difficult part occurs when avoiding the saturation of an actuator during the initial control system reconfiguration due to significant fault-induced changes in system dynamics and limited control authority of remaining actuators. With the practical constraints on actuator amplitude and slew-rate, there are many more challenges in order to balance the requirements from both degraded performance and saturation avoidance during this short period. As an alternative of the proposed command governor in [18], a pre-filter scheme is examined in this paper. This can be described as follows.

Assuming that a fault is detected at the time instant k_D, then the following modified command input $\mathbf{r}'(k)$ will be generated based on the new set-point $\mathbf{r}_j(k),\ \forall k \geq k_D;\ j \in \{1,\ldots,l\},$ as

$$\mathbf{r}'(k) = (1-\rho)\cdot\mathbf{r}'(k-1) + \rho\cdot\mathbf{r}_j(k), \tag{8}$$

where ρ $(0 \leq \rho \leq 1)$ is a weighting parameter governing the decay rate of switching and the initial value of $\mathbf{r}^m(k-1) = \mathbf{r}^0,\ \forall k < k_D$. Ideally, $\mathbf{r}'(k) = \mathbf{r}'(k-1) = \mathbf{r}^0$ if $\rho = 0$, and $\mathbf{r}'(k) = \mathbf{r}_j(k)$ when $\rho = 1$. The smaller the ρ is, the slower the decay rate of the switching is. As k increases, $\mathbf{r}'(k)$ will approach to $\mathbf{r}_j(k)$.

It should be noted that a pre-filter is located outside the feedback loop, and hence does not affect the stability or robustness properties of the loop, and has no effect on the response to disturbances. The primary effect of the pre-filter is that it defines the ideal system response to set-point changes.

In [18], a dynamic reference governor has been proposed for the dynamic tapering of the command input. It is interesting to note that the scheme in [18] would have been equivalent to the above pre-filter scheme if fixed weighting parameters had been used. The advantage of the above pre-filter scheme is that only one parameter needs to be determined to achieve bumpless command switching. The term ‘bumpless’ here means that no discontinuity (or abrupt change) appears during the command input switching.

Note also that a modification to the original command input starts as soon as a fault is detected at time k_D. This allows the pre-filter to adjust the command input before and during the reconfiguration process in order to prevent the potential actuator saturation from affecting the steady-state performance of the reconfigured system.

5. DESIGN OF A MODEL REFERENCE RECONFIGURABLE CONTROLLER

5.1. Formulation of reconfigurable control with multiple actuator faults

To better illustrate the reconfigurable control design process, the system model under both normal and various actuator fault conditions can be written as:

$$\begin{aligned} &\mathbf{x}(k+1) = F\mathbf{x}(k) + G_j\mathbf{u}(k) + \mathbf{w}(k), \\ &\mathbf{y}(k) = H^{\mathbf{y}}\mathbf{x}(k), \qquad j = 0,\ldots,l \\ &\mathbf{z}(k) = H\mathbf{x}(k) + \mathbf{v}(k), \end{aligned} \tag{9}$$

where $\mathbf{x} \in \mathscr{R}^n$ is the state vector; $\mathbf{z} \in \mathscr{R}^m$ is the measurement vector; $\mathbf{u} \in \mathscr{R}^l$ is the control input vector, $\mathbf{y}$ is the controlled system output vector, and $\mathbf{w} \in \mathscr{R}^n$ and $\mathbf{v} \in \mathscr{R}^m$ are independent random processes with means $\bar{\mathbf{w}}$ and $\bar{\mathbf{v}}$ and covariances Q and R, respectively. They represent the system and measurement noises, respectively. The initial state is assumed to have mean $\bar{\mathbf{x}}_0$ and covariance $\bar{P}_0$, and it is independent from $\mathbf{w}$ and $\mathbf{v}$.

During a normal operation, the system matrices are represented by $\{F, G_0, H\}$. Once an actuator fault occurs, the matrix G becomes $G_j, j \in \{1,\ldots,l\}$, at an unknown time k_F with an unknown change in G_0.

The corresponding multiple reference models with the specified degraded performance for the normal condition and each fault condition can be described by:

$$\begin{aligned} &\mathbf{x}_j^m(k+1) = F_j^m\mathbf{x}_j^m(k) + G_j^m\mathbf{r}'_j(k), \\ &\mathbf{y}_j^m(k) = H_j^m\mathbf{x}_j^m(k), \qquad j{=}0,\ldots,l \end{aligned} \tag{10}$$

where $\mathbf{x}_j^m(k)$ is the state, $\mathbf{r}'_j(k)$ the command input, and $\mathbf{y}_j^m(k)$ the output of each reference model. The constant matrices $\{F_j^m, G_j^m, H_j^m, j = 0,\ldots,l\}$ are of appropriate dimensions, which are set the same as the dimensions of matrices $\{F, G_j, H, j = 0,\ldots,l\}$.

Based on the system representation (9), and the desired reference model ($j = 0$ in (10)), one needs to synthesize the following control gains $\{K_0^{\mathbf{x}}, K_0^{\mathbf{x}^m}, K_0^{\mathbf{r}'}\}$ for generating the desired control signals under the normal system operation:

$$\mathbf{u}_0(k) = \underbrace{-K_0^{\mathbf{x}}\mathbf{x}_0(k)}_{\text{feedback}} + \underbrace{K_0^{\mathbf{x}^m}\mathbf{x}_0^m(k)}_{\text{reference model}} + \underbrace{K_0^{\mathbf{r}'}\mathbf{r}'_0(k)}_{\text{feedforward}}. \tag{11}$$

Once a fault is detected, a new set of controller gains $\{K_j^{\mathbf{x}}, K_j^{\mathbf{x}^m}, K_j^{\mathbf{r}'}, j = 1,\ldots,l\}$ will have to be synthesized based on the degraded reference model corresponding to a specified fault in (10) so that the closed-loop system follows the degraded reference model with the new control signal as:

$$\mathbf{u}_j(k) = -K_j^{\mathbf{x}}\mathbf{x}_j(k) + K_j^{\mathbf{x}^m}\mathbf{x}_j^m(k) + K_j^{\mathbf{r}'}\mathbf{r}'_j(k), \tag{12}$$

where $j = 1,\ldots,l$, $k \geq k_R$, and k_R represents the controller reconfiguration time.

Since different reference models have been assigned to different actuator faults, different reconfigurable controllers need to be designed in an on-line manner. The design of these controller gains is discussed next.

5.2. The design of reconfigurable controller gains

One of the main objectives of the model-following reconfigurable control is to make the selected system variables track the outputs of the degraded reference model corresponding to a particular fault condition, respectively, i.e., to synthesize a control sequence $\mathbf{u}_j(k)$ that forces the command tracking error $\mathbf{e}_j(k)$ to be zero at the steady-state for each given condition

$$\begin{aligned} \mathbf{e}_j(k) &= \mathbf{y}_j(k) - \mathbf{y}_j^m(k) \\ &= H_j^{\mathbf{y}}\mathbf{x}_j(k) - H_j^m\mathbf{x}_j^m(k), \quad j = 1,\ldots,l. \end{aligned} \tag{13}$$

When the tracking is achieved, the following will be true:

$$\mathbf{y}_j^*(k) = H_j^{\mathbf{y}}\mathbf{x}_j^*(k) = H_j^m\mathbf{x}_j^m(k), \quad j = 1,\ldots,l. \tag{14}$$

Under the assumption that the ideal system state $\mathbf{x}_j^*(k)$ and the control trajectories $\mathbf{u}_j^*(k)$ are linear combinations of states and inputs of each reference model, the solution for $\mathbf{x}_j^*(k)$ and $\mathbf{u}_j^*(k)$ can be determined by:

$$\mathbf{x}_j^*(k) = S_j^{11}\mathbf{x}_j^m(k) + S_j^{12}\mathbf{r}'_j(k), \tag{15}$$

$$\mathbf{u}_j^*(k) = S_j^{21}\mathbf{x}_j^m(k) + S_j^{22}\mathbf{r}'_j(k), \quad j = 1,\ldots,l, \tag{16}$$

where $S_j^{mn}, m,n = 1,2; j = 1,\ldots,l$, are constant feed-forward gain matrices, which can be calculated by

$$S_j^{11} = \Phi_j^{11}S_j^{11}(F_j^m - I) + \Phi_j^{12}H_j^m, \tag{17}$$

$$S_j^{12} = \Phi_j^{11}S_j^{11}G_j^m, \tag{18}$$

$$S_j^{21} = \Phi_j^{21}S_j^{11}(F_j^m - I) + \Phi_j^{22}H_j^m, \tag{19}$$

$$S_j^{22} = \Phi_j^{21}S_j^{11}G_j^m, \qquad j = 1,\ldots,l \tag{20}$$

and the matrices Φ_j^{mn}, $m,n = 1,2$; $j = 1,\ldots,l$, are determined by

$$\Phi_j = \begin{bmatrix} \Phi_j^{11} & \Phi_j^{12} \\ \Phi_j^{21} & \Phi_j^{22} \end{bmatrix} = \begin{bmatrix} F - I & G_j \\ H^{\mathbf{y}} & 0 \end{bmatrix}^{-1}, j = 1,\ldots,l. \tag{21}$$

It should be noted that Φ_j^{mn} depends on the system models (9), whereas S_j^{mn} depend on both the system and the reference models (10) at the normal and fault conditions.

To incorporate feedback into the design, let's define

$$\tilde{\mathbf{x}}_j(k) = \mathbf{x}_j(k) - \mathbf{x}_j^*(k), \tilde{\mathbf{u}}_j(k) = \mathbf{u}_j(k) - \mathbf{u}_j^*(k),$$
$$\tilde{\mathbf{y}}_j(k) = \mathbf{y}_j(k) - \mathbf{y}_j^*(k)$$

then,

$$\tilde{\mathbf{x}}_j(k+1) = F\tilde{\mathbf{x}}_j(k) + G_j\tilde{\mathbf{u}}_j(k), \tag{22}$$

$$\tilde{\mathbf{y}}_j(k) = H^{\mathbf{y}}\tilde{\mathbf{x}}_j(k), \quad j = 1, ..., l. \tag{23}$$

For a feedback control signal given by

$$\tilde{\mathbf{u}}_j(k) = -K_j^{\mathbf{x}}\tilde{\mathbf{x}}_j(k) = -K_j^{\mathbf{x}}[\mathbf{x}(k) - \mathbf{x}_j^*(k)], \quad j = 1, ..., l. \tag{24}$$

From the definition of $\tilde{\mathbf{u}}_j(k)$ in (24), we have:

$$\begin{aligned}\mathbf{u}_j(k) &= \mathbf{u}_j^*(k) + \tilde{\mathbf{u}}_j(k)\\ &= \mathbf{u}_j^*(k) - K_j^{\mathbf{x}}[\mathbf{x}_j(k) - \mathbf{x}_j^*(k)].\end{aligned} \tag{25}$$

Substituting (15) and (16) into (25), the overall control signal for each condition can be determined by:

$$\mathbf{u}_j(k) = K_j^{\mathbf{x}}\mathbf{x}_j(k) + K_j^{\mathbf{x}^m}\mathbf{x}_j^m(k) + K_j^{\mathbf{r}'}\mathbf{r}_j'(k), \quad j = 1, ..., l. \tag{26}$$

Note that the control law in (26) consists of a feedback part, $K_j^{\mathbf{x}}$, a reference model part depending on the state of the reference model, $K_j^{\mathbf{x}^m} = S_j^{21} + K_j^{\mathbf{x}}S_j^{11}$, and a feedforward part relating to the command input, $K_j^{\mathbf{r}'} = S_j^{22} + K_j^{\mathbf{x}}S_j^{12}$. It should be pointed out that even though multiple reference models have been used for reconfigurable controller designs, only one set of controllers as specified in (26) needs to be carried out for a particular actuator fault identified by the FDD scheme. This will keep the calculation of a reconfigurable control law small for on-line application.

6. AN ILLUSTRATIVE EXAMPLE

To demonstrate the effectiveness of the proposed approach and for easy comparison with the results in [18], a same F-8 aircraft model used in [12] is adopted.

6.1. Aircraft model

The linearized aircraft model can be described as:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= A\mathbf{x}(t) + B\mathbf{u}(t),\\ \mathbf{y}(t) &= C^{\mathbf{y}}\mathbf{x}(t),\end{aligned} \tag{27}$$

where the state and the input vectors are $\mathbf{x} = [p\ r\ \beta\ \varphi]^T$ and $\mathbf{u} = [\delta_a\ \delta_r]^T$, respectively, with p representing the roll rate, r the yaw rate, β the sideslip angle, φ the bank angle, δ_a the aileron deflection, and δ_r the rudder deflection.

To maintain the desired values for the sideslip and the bank angle during both the normal operation and under fault conditions, the output matrix $H^{\mathbf{y}}$ is chosen as

$$H^{\mathbf{y}} = C^{\mathbf{y}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

6.2. Design of reference models and command inputs

Since there are two control inputs which associate with two actuators in the system, two degraded reference models and two set of new command inputs are needed to be determined.

6.2.1 Design of reference models

Following the design procedure outlined in Section 3.2 and for the selected weighting matrices

$$\Psi = \begin{cases} diag[2,6,3,3], & \text{Aileron fault} \\ diag[3,1,4,4], & \text{Rudder fault} \end{cases}$$

the parameters of the system, the desired and the degraded reference models are given in Table 1.

The desired reference model is modified to achieve unity steady-state gain from a model in [12] which satisfies all necessary performance requirements under the normal operation. In the selection of the degraded reference models, the following two factors

Table 1. The system and the reference models.

	A	B
Open-Loop System Model	$\begin{bmatrix} -3.5980 & 0.1968 & -35.18 & 0 \\ -0.0377 & -0.3576 & 5.884 & 0 \\ 0.0688 & -0.9957 & -0.2163 & 0.0733 \\ 0.9947 & 0.1027 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 14.65 & 6.538 \\ 0.2179 & -3.087 \\ -0.0054 & 0.0516 \\ 0 & 0 \end{bmatrix}$
Desired Reference Model	$\begin{bmatrix} -10.0 & 0 & -10.0 & 0 \\ 0 & -0.7 & 4.5 & 0 \\ 0 & -0.5 & -0.7 & 0 \\ 1 & 0 & 0 & -0.5 \end{bmatrix}$	$\begin{bmatrix} -10.0 & 5.0 \\ -5.48 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$
Degraded Reference Model #1	$\begin{bmatrix} -5.0 & 0 & -5.0 & 0 \\ 0 & -0.1167 & 0.75 & 0 \\ 0 & -0.1667 & -0.2333 & 0 \\ 0.3333 & 0 & 0 & -0.1667 \end{bmatrix}$	$\begin{bmatrix} 5.0 & 2.5 \\ -0.9133 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$
Degraded Reference Model #2	$\begin{bmatrix} -3.3333 & 0 & -3.3333 & 0 \\ 0 & -0.700 & 4.500 & 0 \\ 0 & -0.125 & -0.175 & 0 \\ 0.25 & 0 & 0 & -0.125 \end{bmatrix}$	$\begin{bmatrix} 3.3333 & 1.6667 \\ -5.48 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

have been taken into consideration: 1) to track the output responses specified by the degraded reference model in the presence of any one of the two actuator faults with any level of loss of its control effectiveness, and 2) the closed-loop control signals should not violate the slew-rate and amplitude saturation limits of the actuators under all fault conditions considered.

6.2.2 Selection of command input (set-points)

Dynamic performance, particularly transient performance of a reconfigured system is governed mainly by the selection of the above reference models. To avoid potential actuator saturation and to achieve desired degraded performance in the presence of failure, on-line switching of command input (set-point) may have to be incorporated in the design of fault-tolerant control systems. The rule of selecting each command input is that assigned set-point should not violate the saturation limit of the actuator at the corresponding fault condition, given the synthesized reconfigurable controller. For multi-input and multi-output (MIMO) systems, there are interactive coupling effects among different channels. Therefore, care should be given for selecting set-points so that the relationship (or ratio) between different input channels should be carefully considered. This means that reasonable and realizable set-points need to be set.

Through simple sensitivity analysis, it is observed that sideslip angle is mainly controlled by aileron while bank angle is mainly controlled by rudder. There is also coupling effect from the aileron to the bank angle. Therefore, if there is an aileron fault, demand for sideslip angle should be significantly reduced while the bank angle should be reduced when there is a rudder fault. These facts need to be considered in the determination of the above new command inputs associated to each actuator fault condition. Based on the above consideration, the corresponding command inputs for the normal and the fault conditions are given in Table 2.

To illustrate the behaviors of the synthesized reference models with the new command inputs, the step responses of different degraded models are shown in Fig. 2. The corresponding quantitative measures of the rise time, settling time and percent of overshoot are shown in Table 3 for easy performance comparison among different reference models.

Table 2. Command inputs for the normal and fault conditions.

Setpoints	Normal	Aileron failure	Rudder failure
Sideslip angle	3.0	0.3	0.6
Bank angle	8.0	4.0	1.0

Table 3. Characteristics of reference models.

	T_r		T_s		$\sigma\%$	
	$\delta_a \to \beta$	$\delta_r \to \phi$	$\delta_a \to \beta$	$\delta_r \to \phi$	$\delta_a \to \beta$	$\delta_r \to \phi$
Fault-free	0.91	4.4	5.07	7.93	23.1	0
Aileron fault	3.97	13.2	21.4	23.7	20.7	0
Rudder fault	2.07	17.6	6.99	31.6	14.1	0

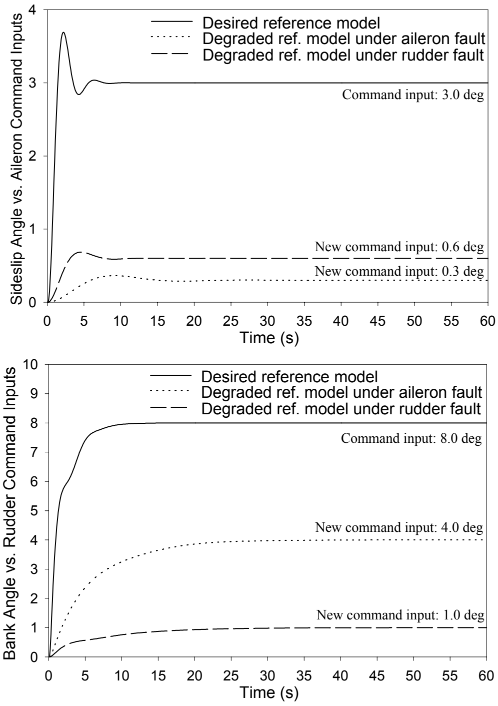

Fig. 2. Step responses of reference models with different input levels.

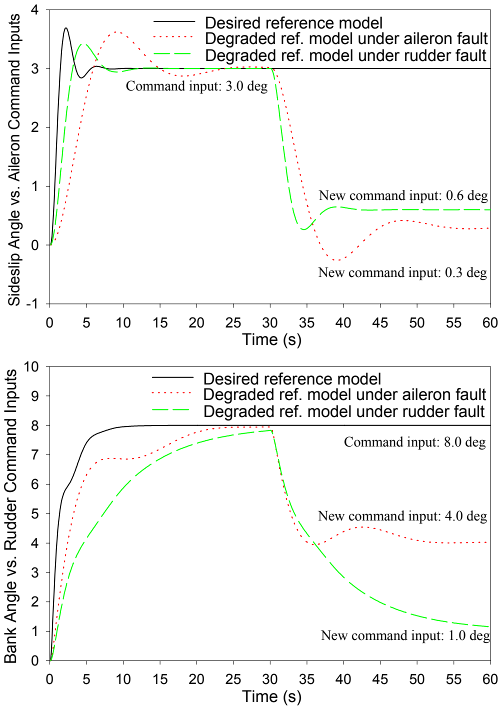

Fig. 3. Responses of reference models with command input changes.

To further illustrate the transient behaviors of designed reference models, step responses with the original command input followed by a step-type command input switching to the new command inputs at the time instant of 30 sec are shown in Fig. 3. As expected, it is evident that degraded reference model associated with the aileron fault has slower response in sideslip while the one associated with the rudder fault has slower response in bank angle. These facts provide us some guidelines for selecting appropriate reference model for each actuator fault condition.

6.3. Simulation results and performance assessment

To evaluate the performance of the proposed method, a loss of 75% of the control effectiveness in the aileron or rudder channel is simulated at time t_F= 8sec. Prior to the occurrence of a fault, a constant input vector, $\mathbf{r} = [3\ \ 8]^T$, is used as the original command input to represent the desired sideslip and the bank angle. Once an actuator fault has been detected, the new command input specified in Table 2 will be used to represent the degraded performance at the steady-state for particular actuator fault.

For the purpose to demonstrate the effects of saturation to the developed FTCS, the amplitude and slew-rate limits for the two control actuators are set as following: $\delta_a^c = \pm 15\,\text{deg}$, $\dot{\delta}_a^c = \pm 50\,\text{deg/sec}$; and $\delta_r^c = \pm 15\,\text{deg}$, $\dot{\delta}_r^c = \pm 50\,\text{deg/sec}$.

6.3.1 System performance under the aileron fault

To compare the performance with and without consideration of performance degradation under the aileron fault, the closed-loop system responses of the two cases are shown in Fig. 4. The corresponding control signals are illustrated in Fig. 5. To illustrate the effect of pre-filter and how the command inputs react to faults, the corresponding command inputs are overlaid on the same graph in Fig. 5.

It can be seen that satisfactory output responses have been obtained with the specified degraded performance. Correspondingly, significantly reduced control demands at the steady-state in both control channels have been required for the aircraft to track degraded reference trajectories. However, if performance degradation had not been considered, meaning that the desired reference model and original command inputs have been used for control reconfiguration, the reconfigured output responses would track neither the original fault-free system output responses nor the expected reference trajectories with the degraded performance. This is because considerably larger control effort in the aileron channel would have been needed if the performance degradation had not been considered. In fact, the required control signal has exceeded the actuator saturation limit immediately after the fault occurrence.

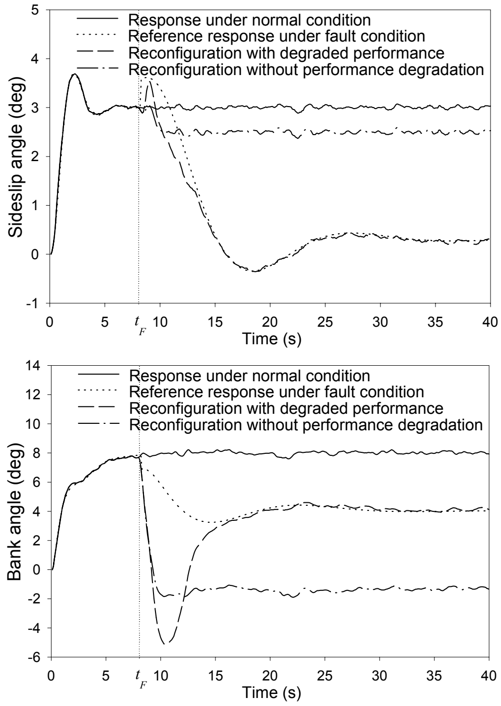

Fig. 4. System outputs with and without degraded performance.

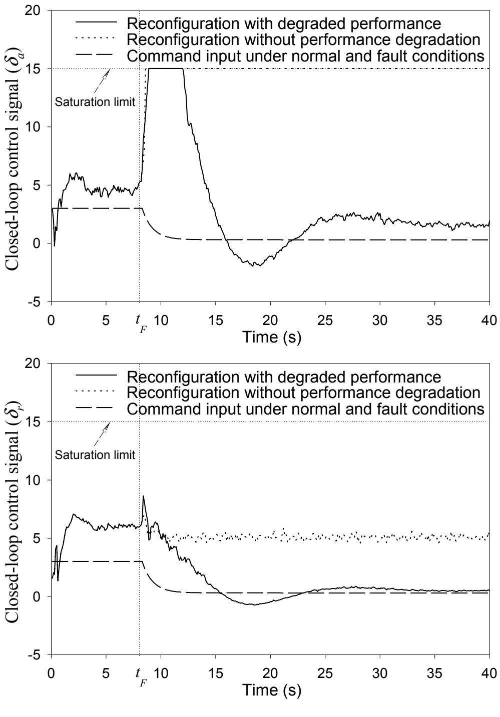

Fig. 5. Control signals with and without degraded performance.

6.3.2 System performance under the rudder fault

The behavior of the system in the presence of the rudder fault has been shown in Figs. 6 and 7. Similar conclusion can be drawn as in the aileron fault case. It is clear that with the consideration of performance degradation, the reconfigured system responses follow the degraded reference responses satisfactorily, with even smaller control effort at the steady-state compared with the one during normal operation. However, without considering performance degrada-

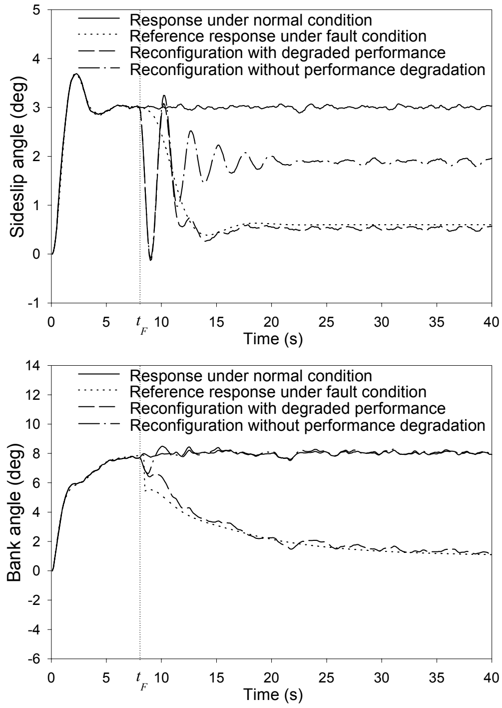

Fig. 6. System outputs.

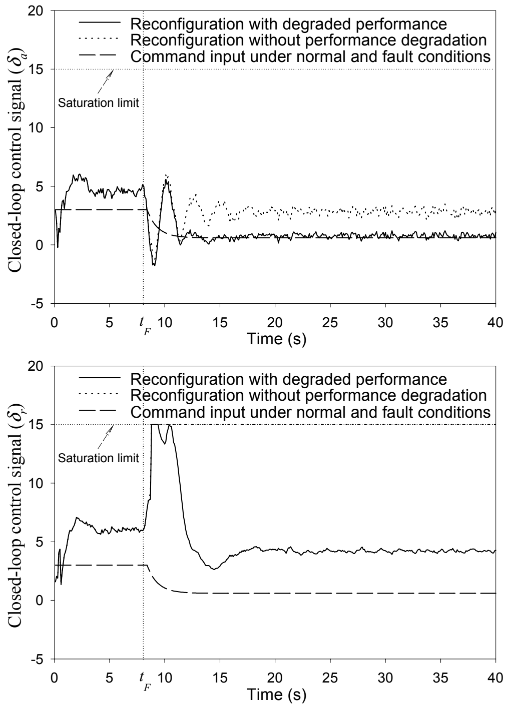

Fig. 7. Control signals.

tion, the reconfigured sideslip response can not track either the fault-free system output response or the expected reference trajectory with the degraded performance. The required control signal in rudder channel exceeds the actuator saturation limit immediately after the fault occurrence.

6.3.3 Further analyses

To demonstrate the role of degraded reference model, Figs. 8 and 9 show the output responses and corresponding control signals using either degraded or desired reference model for control reconfiguration in the event of the 75% aileron actuator fault case. It can be seen that if the same reference model used for normal condition had been used in the case of the fault, faster output responses had been obtained. However, such a performance requires also fast control signals

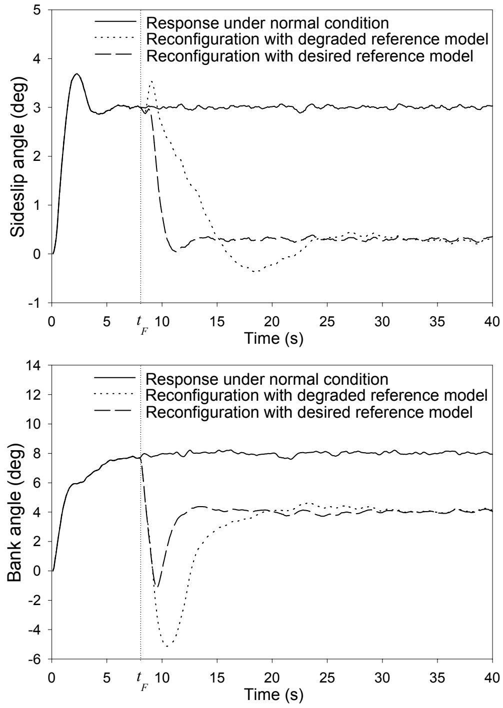

Fig. 8. System outputs using desired and degraded models.

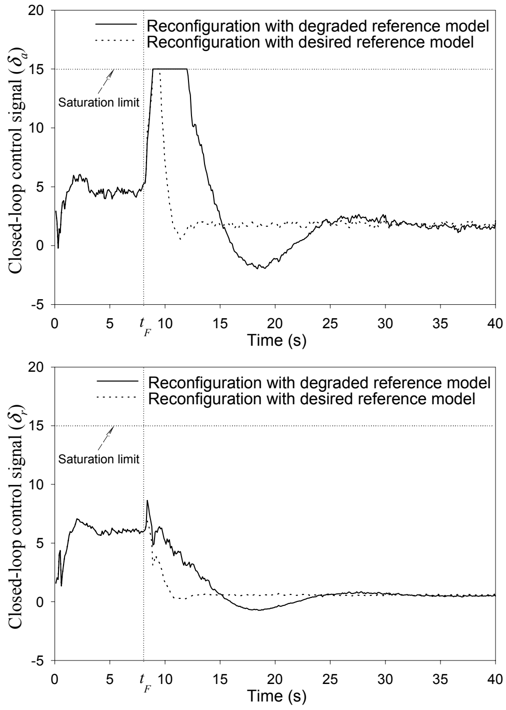

Fig. 9. Control signals.

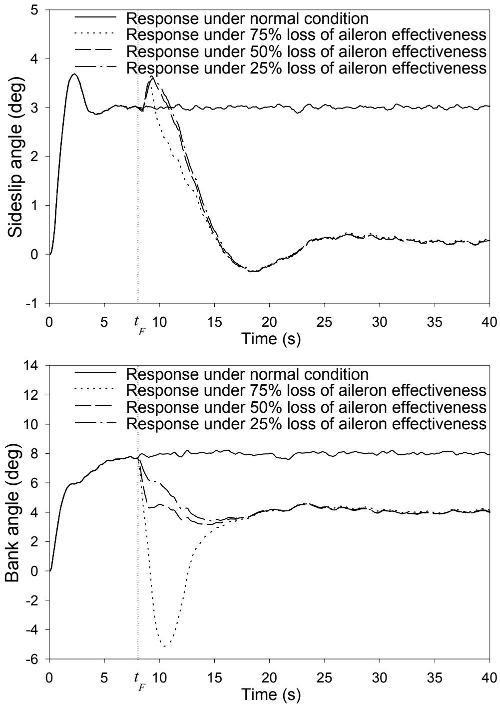

Fig. 10. System outputs under different levels of fault.

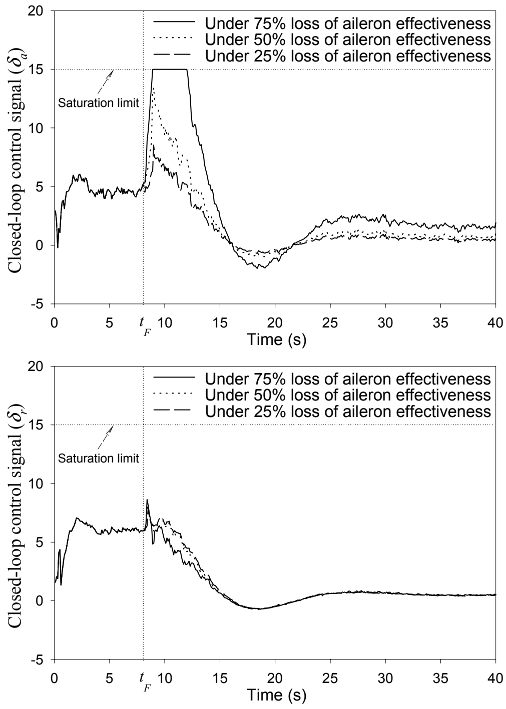

Fig. 11. Control signals under different levels of fault.

as shown in Fig. 9, which may induce actuator slew-rate saturation. It can been observed from Figs. 8 and 9, with only difference by using either desired or degraded reference model, difference in the system responses for the two cases lies in the transient part after the fault occurrence. The steady-state performance is identical. This fact demonstrated clearly the role of the degraded reference model in governing dynamic performance of the post-fault system.

To test the performance of the developed FTCS for handling different levels of fault, different levels, ranging from 0% to 100% control effectiveness loss, of faults for single as well as simultaneous actuator faults have been simulated. For demonstration, results for the following three fault levels, 75%, 50% and 25% loss of the aileron control effectiveness, are shown in Figs. 10 and 11. As can be seen, as the severity of the fault increases, the performance of the reconfigured system decreases. Because of significant change of the system dynamics induced by the 75% aileron fault, the significantly large control signal is needed at the initial period of the system reconfiguration. In fact, the required control signal in the aileron channel has exceeded the saturation limit. Once less severe fault is introduced, the control signal are well within the limits. For different levels of fault, there are also some differences in the steady-state.

As shown in Fig. 11, it should be pointed out that due to the severity of the actuator faults (75% loss of aileron/rudder control effectiveness in these cases) and the requirement of smooth command input switching, a short period of actuator amplitude saturation in the aileron/rudder control channel is observed. To demonstrate the effects and the limitations induced by actuator saturation, system responses and associated control signals without saturation constraint are plotted further in Figs. 12 and 13. It is interested to see that without consideration of performance degradation and actuator saturation constraint, ideally, one is able to recover the original

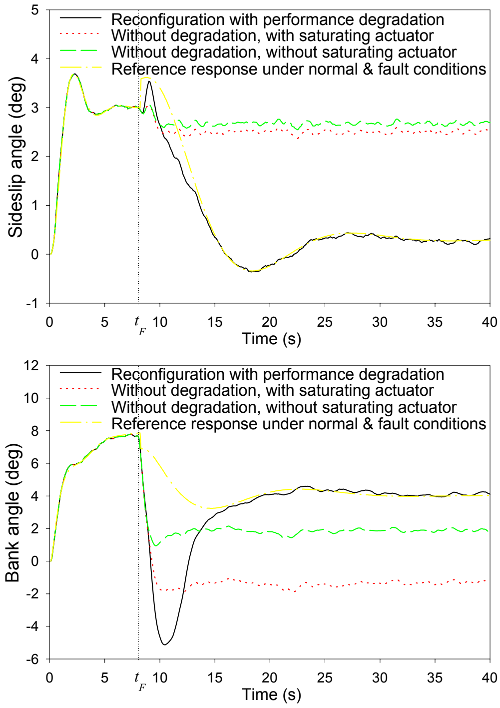

Fig. 12. Effect of actuator saturation on system outputs.

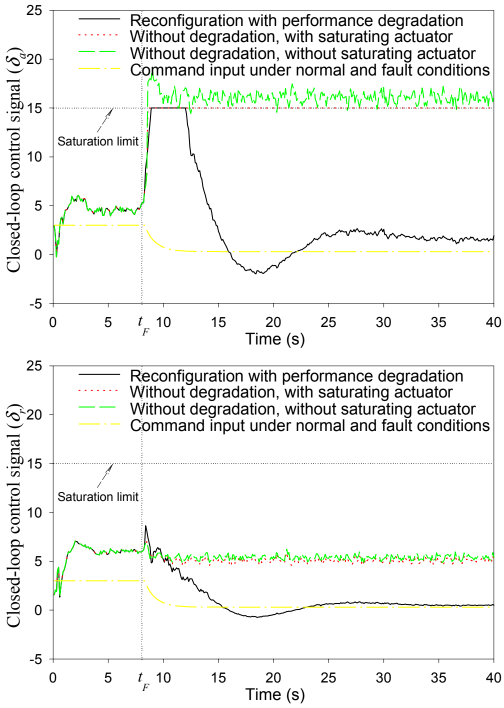

Fig. 13. Effect of actuator saturation on control signals.

performance. However, the price to pay for the demanded performance is that significantly large control signals need to be generated, as can be seen in Fig. 13 for the case of aileron fault condition. Because of such physical demand for a large control signal to recover performance, violent changes in actuator characteristics induced by the fault, smoothen switching in command input in consideration of actuator slew-rate constraint, and the unavailability of an accurate post-fault model for feedback control signal synthesis during initial period of control reconfiguration, such a temporary actuator saturation should be acceptable in the case of severe fault conditions.

7. CONCLUSIONS

To achieve gracefully degraded performance for different actuator faults, a new fault-tolerant control system design method has been developed which can deal with different actuator faults through different degraded reference models. Fault-tolerant control is implemented through a model-following control structure by using these degraded reference models. Furthermore, the control system command inputs are also adjusted accordingly to avoid potential saturation. Simulation results have demonstrated the effectiveness of the proposed scheme using an aircraft example.

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