

# Pre-processing Faded Measurements for Bearing-and-Frequency Target Motion Analysis

Man Hyung Lee, Jeong-Hyun Moon, In-Soo Kim, Chang Sup Kim, and Jae Weon Choi

**Abstract:** An ownship with towed array sonar (TAS) has limited maneuvers due to its dynamic feature, bearing and frequency measurements of a target which are not detected continuously but are often lost in ocean environment. We propose a pre-processing algorithm for the faded bearing and frequency measurements to solve the BFTMA problem of TAS under limited detection conditions. The proposed pre-processing algorithm to restore the faded bearing and frequency measurements is implemented to perform a BFTMA filter even if the measurements of a target are not continuously detected. The Modified Gain Extended Kalman Filter (MGEKF) method based on the Interacting Multiple Model (IMM) structure is applied for a BFTMA filter algorithm to estimate the target. Simulations for the various conditions were carried out to verify the applicability of the proposed algorithms, and confirmed superior estimation performance compared with the existing Bearings-Only TMA (BOTMA).

**Keywords:** Bearing-and-frequency target motion analysis (BFTMA), bearings-only TMA (BOTMA), modified gain extended Kalman filter (MGEKF), towed array sonar (TAS).

## 1. INTRODUCTION

Towed array sonar (TAS) is a passive sonar system that is loaded behind an ownship and detects a target from its radiated noise. TAS has a long-range detection performance and can detect submarines traveling in both deep and shallow waters since it can freely adjust its operating depth, therefore, it is very useful in detecting submarines maneuvering in a shadow zone and is an essential sonar system for submarines. TAS depends on only the radiated noise from a target and can not directly detect the motion information (range, speed, course) of a target, therefore, target motion analysis (TMA), which is able to estimate and analyze the target motion, is essentially required.

Due to the important role of passive sonar systems, research and development for sonar systems have been actively carried out. The linear Kalman filter

theory was applied to Bearings-Only TMA (BOTMA) by using the Taylor series expansion for the nonlinear measurement equation of bearing [1]. In [2], the extended Kalman filter (EKF) was used in BOTMA with the nonlinear equation of bearing measurements. The BOTMA problem of modified polar coordinates was replaced with Cartesian coordinates to improve stability and convergence of EKF in [3]. To linearize the nonlinear measurement equations, the pseudo-measurement filter (PMF) structure that uses the characteristics of trigonometric function was suggested for BOTMA [4,5]. Since the filter gain and the measurement residues of the PMF are a function of past and present bearings with measurement noises, the PMF has biased estimate results. So the modified gain EKF (MGEKF) theory, whose gain is a function of only past bearings, was suggested to reduce interrelation between measurements and its residues and to solve the problem of the biased estimates caused by measurement noises [6]. An ownship with a TAS system should maneuver to increase the observability of BOTMA. However, the ownship has many restrictions in maneuvering and low bearing rate due to the TAS system, it causes poor performance of BOTMA which depends only on bearing measurements.

There have been many researches on the bearing-and-frequency TMA (BFTMA) that estimates target motion by bearing and frequency measurements [7-9]. Especially, frequency measurements are very important data for convergence and stability of a BFTMA filter, which include Doppler shift that occurs

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from the relative maneuver between an ownship and a target. We must consider the maneuver restrictions of an ownship due to the long array sensors, the detecting and fading conditions of bearing and frequency measurements that are affected by the ocean environment, low bearing rate caused by long range detection of a target with low speed, and an uncertain maneuver of a target during target motion analysis.

Our aim is to establish a practical pre-processing algorithm for bearing and frequency measurements to improve convergence and stability of a BFTMA filter. Bearing and frequency measurements are not continuously detected by a TAS system and frequently fade depending on the underwater environment. A BFTMA filter can not perform continuously with the faded measurements and it is the primal poor performance of BFTMA. We suggest a pre-processing algorithm for BFTMA where the faded measurements are restored by a data interpolation method which uses the prior and posterior of the detected measurements. It especially considers mutual dependent features between the detected and faded measurements.

A multiple model structure based Interacting Multiple Model (IMM) algorithm was applied to a BFTMA filter [10-13]. For the filtering method, the MGEKF algorithm was used for the nonlinear measurement equations.

This paper is organized as follows. In Section 2, we analyze the mathematical modeling and the observability of BFTMA. In Section 3, we deal with the theoretical aspect of the MGEKF, the structure of the IMM, and the construction of a BFTMA filter. We describe the pre-processing algorithm for measurements in Section 4. Finally, Section 5 deals with the performance of the pre-processing algorithms through simulations.

## 2. BFTMA SYSTEM MODELING

Two assumptions are necessary for BFTMA to

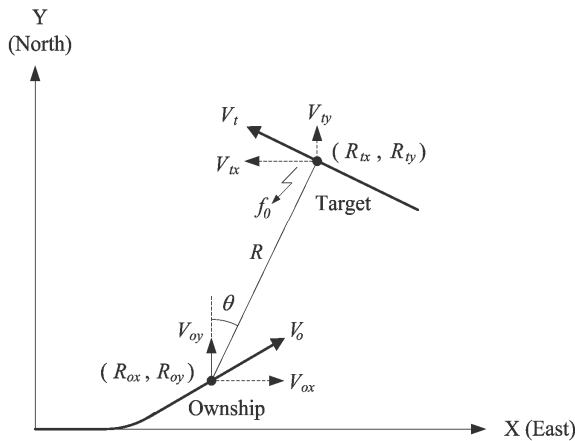


Fig. 1. Geometry of ownship and target for BFTMA.

estimate the relative range between an ownship and a target, the target speed and course using bearing and frequency measurements. As shown in Fig. 1, an ownship and a target are assumed to move in the same two-dimensional plane. The target is assumed to maintain a constant speed and a constant course while carrying out BFTMA. Under these assumptions, the state vector of the target for the estimation problem is defined as a target frequency, the relative ranges and velocities between ownship and target.

$$\underline{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{bmatrix} \triangleq \begin{bmatrix} R_x(t) \\ R_y(t) \\ V_x(t) \\ V_y(t) \\ f_0 \end{bmatrix} = \begin{bmatrix} R_{tx}(t) - R_{ox}(t) \\ R_{ty}(t) - R_{oy}(t) \\ V_{tx}(t) - V_{ox}(t) \\ V_{ty}(t) - V_{oy}(t) \\ f_0 \end{bmatrix}, \quad (1)$$

where the ownship and the target are assumed to travel on the same plane,  $R_x(t)$ ,  $R_y(t)$  and  $V_x(t)$ ,  $V_y(t)$  are given as the relative ranges and velocities along the  $X$ ,  $Y$  axes, respectively.  $f_0$  is the frequency constantly radiated from the target.  $R_{tx}(t)$ ,  $R_{ty}(t)$  and  $V_{tx}(t)$ ,  $V_{ty}(t)$  are the target positions and velocities along the  $X$ ,  $Y$  axes, respectively.  $R_{ox}(t)$ ,  $R_{oy}(t)$  and  $V_{ox}(t)$ ,  $V_{oy}(t)$  are the ownship's positions and velocities along the  $X$ ,  $Y$  axes, respectively.

To derive a model that estimates the relative range of a target from an ownship, target speed and course, the time derivatives, the state variables of (1) are obtained by

$$\underline{\dot{x}}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \\ \dot{x}_5(t) \end{bmatrix} = \begin{bmatrix} V_x(t) \\ V_y(t) \\ A_x(t) \\ A_y(t) \\ 0 \end{bmatrix} = \begin{bmatrix} x_3(t) \\ x_4(t) \\ A_{tx}(t) - A_{ox}(t) \\ A_{ty}(t) - A_{oy}(t) \\ 0 \end{bmatrix}, \quad (2)$$

where  $A_x(t)$ ,  $A_y(t)$  are the relative accelerations,  $A_{tx}(t)$ ,  $A_{ty}(t)$  and  $A_{ox}(t)$ ,  $A_{oy}(t)$  are the target and the ownship's accelerations along the  $X$ ,  $Y$  axes, respectively.

If  $\underline{w}(t)$  is assumed to be a white Gaussian noise with zero-mean and variance of  $\sigma_w^2(t)$ , the state equation of BFTMA is derived as

$$\dot{\underline{x}}(t) = A\underline{x}(t) + B\underline{a}(t) + B\underline{w}(t), \quad (3)$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -1 & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix}, \underline{a} = \begin{bmatrix} A_{ox}(t) \\ A_{oy}(t) \end{bmatrix}.$$

$$(4)$$

The measurement equation of BFTMA given by the target bearing  $\theta(t)$  and the target frequency  $f(t)$  including Doppler shift which is expressed as the nonlinear equation of the relative position and velocity is obtained by

$$\begin{aligned} \underline{z}(t) &= \begin{bmatrix} \theta(t) \\ f(t) \end{bmatrix} + \underline{v}(t) \\ &= \begin{bmatrix} \tan^{-1} \frac{R_x(t)}{R_y(t)} \\ f_0 \left( 1 - \frac{V_x(t) \sin \theta(t) + V_y(t) \cos \theta(t)}{c} \right) \end{bmatrix} \\ &\quad + \begin{bmatrix} v_\theta(t) \\ v_f(t) \end{bmatrix}, \end{aligned} \quad (5)$$

where  $c$  is the sound speed,  $v_\theta(t)$  and  $v_f(t)$  are the bearing and frequency measurements noise, respectively.  $v_\theta(t)$  and  $v_f(t)$  are assumed to be white Gaussian noises with zero-mean and variances of  $\sigma_\theta^2(t)$  and  $\sigma_f^2(t)$ , respectively.

### 3. BEARING-AND-FREQUENCY TMA FILTER DESIGN

#### 3.1. IMM algorithm

A BFTMA filter is designed under the assumption that a target keeps constant speed and course. Since TAS is a long range detection system for detecting a target with low speed, the BFTMA filter must be designed with the consideration of an uncertain target's maneuver throughout BFTMA. When a filtering technique with a single model is applied to a filter structure of BFTMA in TAS, the performance of the filter, such as convergence, estimation error, is affected by the dynamic characteristics of TAS and the uncertain maneuvers of a target. Therefore, a filter algorithm-based multiple model structure is required for BFTMA in TAS to minimize the estimation error of the filter due to the uncertain target's maneuver.

To enhance the performance of BFTMA under such uncertain target maneuvering, the IMM algorithm [10,12] which sets up multiple filter models is applied to correspond to the target's different maneuvers, associates the probabilities of the filter models, and finally estimates the target's motion. The outline of the IMM algorithm using  $r$  number of models is shown in Fig. 2.

The IMM algorithm is based on the interaction stage between the state estimate and the error covariance estimate of each filter model and the mode probability update stage where the probability of a filter model, which is the probability that each filter

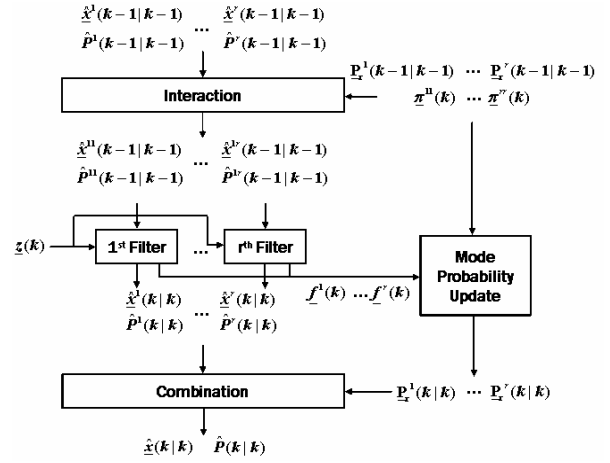


Fig. 2. The IMM structure.

model is correct after updating the state estimate and error covariance estimate, is calculated. The  $j$ -th filter is assumed to have the probability that the  $j$ -th hypothesis (model) is correct during the interaction process. The estimate results of each filter are given by interacting the filters with the updated state estimate  $\hat{\underline{x}}_{k-1}^j$ , the error covariance estimate  $\hat{P}_{k-1}^j$ , and the mode transition probability  $\underline{\pi}$  at  $t_{k-1}$ .

If the state and the error covariance estimates of each filter are represented as  $\tilde{\underline{x}}_{k-1}^j$  and  $\tilde{P}_{k-1}^j$ , then we have

$$\tilde{\underline{x}}_{k-1}^i = \frac{\sum_{j=1}^r \hat{\underline{x}}_{k-1}^j \underline{\pi}_{ij} \underline{P}_r(M_{k-1}^j | Z_{k-1})}{\sum_{j=1}^r \underline{\pi}_{ij} \underline{P}_r(M_{k-1}^j | Z_{k-1})}, \quad (6)$$

$$\tilde{P}_{k-1}^i = \frac{\sum_{j=1}^r (\hat{P}_{k-1}^j + \hat{\underline{x}}_{k-1}^j \hat{\underline{x}}_{k-1}^{jT}) \underline{\pi}_{ij} \underline{P}_r(M_{k-1}^j | Z_{k-1})}{\sum_{j=1}^r \underline{\pi}_{ij} \underline{P}_r(M_{k-1}^j | Z_{k-1})} \quad (7)$$

$$- \tilde{\underline{x}}_{k-1}^i \tilde{\underline{x}}_{k-1}^{iT}, \quad \underline{\pi}_{ij} = \underline{P}_r(M_k^j | M_{k-1}^i), \quad (8)$$

where  $M_k^j$  is the  $j$ -th filter model,  $\underline{P}_r(M_{k-1}^j | Z_{k-1})$  is the mode probability, which is the probability that the  $j$ -th filter model is correct for the detected measurements on  $[t_0 t_{k-1}]$ .  $\underline{\pi}_{ij}$  is the mode transition probability, which is the transition probability from the  $j$ -th filter model at  $t_{k-1}$  to the  $i$ -th filter model at  $t_k$ .

$\underline{P}_r(M_{k-1}^j | Z_{k-1})$  is updated with the measurement  $\underline{z}_k$  on  $[t_0 t_k]$  as follows:

$$\begin{aligned} & P_r(M_k^i | Z_k) \\ &= \frac{f(\underline{z}_k | M_k^i, Z_{k-1}) \sum_{j=1}^r \pi_{ij} P_r(M_{k-1}^j | Z_{k-1})}{\sum_{j=1}^r f(\underline{z}_k | M_k^j, Z_{k-1}) \sum_{j=1}^r \pi_{ij} P_r(M_{k-1}^j | Z_{k-1})}, \end{aligned} \quad (9)$$

where  $f(\underline{z}_k | M_k^i, Z_{k-1})$  is the conditional probability density function of the estimates for the state variables at  $t_k$  given by  $\bar{x}_k^i$  and the propagation equation of the  $i$ -th filter.  $S_k^i$  is the covariance of the innovation process of the  $i$ -th filter as shown for measurement equation  $\underline{h}(\underline{x}_k)$  at  $t_k$ .

$$\begin{aligned} & f(\underline{z}_k | M_k^i, Z_{k-1}) \\ &= \frac{1}{2\pi\sqrt{S_k^i}} \exp\left(-\frac{1}{2}(\underline{z}_k - \underline{h}(\bar{x}_k^i))^T S_k^{i-1} (\underline{z}_k - \underline{h}(\bar{x}_k^i))\right), \end{aligned} \quad (10)$$

$$S_k^i = \left( \frac{\partial \underline{h}(\bar{x}_k^i)}{\partial \bar{x}_k^i} \right) \bar{P}_k^i \left( \frac{\partial \underline{h}(\bar{x}_k^i)}{\partial \bar{x}_k^i} \right)^T + R_k. \quad (11)$$

The combination process that computes the estimates of the state variables and the error covariance from the product of the estimates for the state variables of  $r$  filters by the mode probability is described as follows with the output  $\hat{x}_k^j$  and  $\hat{P}_k^j$  of each filter.

$$\underline{x}_k^j = \sum_{i=1}^r \bar{x}_k^i P_r(M_k^i | Z_k), \quad (12)$$

$$\hat{P}_k^j = \sum_{i=1}^r (\hat{P}_k^i + \hat{x}_k^i \hat{x}_k^{iT}) P_r(M_k^i | Z_k) - \hat{x}_k^j \hat{x}_k^{jT}. \quad (13)$$

### 3.2. Design of BFTMA Filter

In this section, the BFTMA filter is designed with the IMM structure which is based on a multiple filter model, and employed the MGEKF algorithm which is suitable for nonlinear systems such as BFTMA. The state equation and the measurement equations are represented as (3) and (5) are as follows:

$$\dot{\underline{x}}(t) = A\underline{x}(t) + B\underline{a}(t) + B\underline{w}(t), \quad (14)$$

$$\begin{aligned} \underline{z}(t) &= \begin{bmatrix} \theta(t) \\ f(t) \end{bmatrix} + \underline{v}(t) \\ &= \begin{bmatrix} \tan^{-1} \frac{R_x(t)}{R_y(t)} \\ f_0 \left( 1 - \frac{V_x(t) \sin \theta(t) + V_y(t) \cos \theta(t)}{c} \right) \end{bmatrix} + \underline{v}(t). \end{aligned} \quad (15)$$

The state vector  $\underline{x}(t)$  is given by (16) and the measurement equations of (15) are rewritten by (17).

$$\begin{aligned} \underline{x}(t) &= \begin{bmatrix} \frac{f_0}{c} R_x(t) & \frac{f_0}{c} R_y(t) & \frac{f_0}{c} V_x(t) & \frac{f_0}{c} V_y(t) & \frac{f_0}{c} \end{bmatrix}^T \\ &= [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5], \end{aligned} \quad (16)$$

$$\begin{aligned} \underline{z}_k &= \begin{bmatrix} \tan^{-1} \frac{x_1}{x_2} \\ cx_5 - \frac{x_1 x_3}{\sqrt{x_1^2 + x_2^2}} - \frac{x_2 x_4}{\sqrt{x_1^2 + x_2^2}} \end{bmatrix} + \underline{v}_k \\ &= \begin{bmatrix} h_1(x_k) \\ h_2(x_k) \end{bmatrix} + \underline{v}_k. \end{aligned} \quad (17)$$

The MGEKF propagation equations of the state estimate and the error covariance that are applied in BFTMA are obtained by

$$\bar{x}_k = \Phi(t_k, t_{k-1}) \hat{x}_{k-1} + \int_{t_{k-1}}^{t_k} \Phi(t_k, \tau) B \underline{a}(\tau) d\tau, \quad (18)$$

$$\bar{P}_k = \Phi(t_k, t_{k-1}) \hat{P}_{k-1} \Phi^T(t_k, t_{k-1}) + Q(t_k), \quad (19)$$

$$\text{where } \Phi(t, t_0) = \begin{bmatrix} 1 & 0 & t-t_0 & 0 & 0 \\ 0 & 1 & 0 & t-t_0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

The updated equations of the state estimate  $\underline{x}_k$  and the error covariance  $P_k$ , the measurements  $\underline{z}_k$  and the filter gain  $K_k$  are obtained by

$$\hat{x}_k = \bar{x}_k + K_k (\underline{z}_k - \underline{h}(\bar{x}_k)), \quad (20)$$

$$\begin{aligned} \hat{P}_k &= (I - K_k M(\underline{z}_k, \bar{x}_k)) \bar{P}_k (I - K_k M(\underline{z}_k, \bar{x}_k))^T \\ &\quad + K_k P_k K_k^T, \end{aligned} \quad (21)$$

$$K_k = \bar{P}_k C^T(\bar{x}_k) (C(\bar{x}_k) \bar{P}_k C^T(\bar{x}_k) + R_k)^{-1}, \quad (22)$$

$$C(\bar{x}_k) = \left. \frac{\partial \underline{h}(\underline{x}_k)}{\partial \underline{x}_k} \right|_{\underline{x}_k = \bar{x}_k}. \quad (23)$$

To design the BFTMA filter with the IMM structure, we established the covariance of process noise according to the two models of a target maneuver - the covariance  $Q^1$  for an uncertain target's maneuver as the upper limit and the covariance  $Q^2$  for a constant speed and course of a target as the lower limit. Using the IMM structure that has the MGEKF models with  $Q^1$  and  $Q^2$ , we suggest a BFTMA filtering technique that considers convergence and stability of the BFTMA filter.

The IMM structure for BFTMA is shown in Fig. 3.

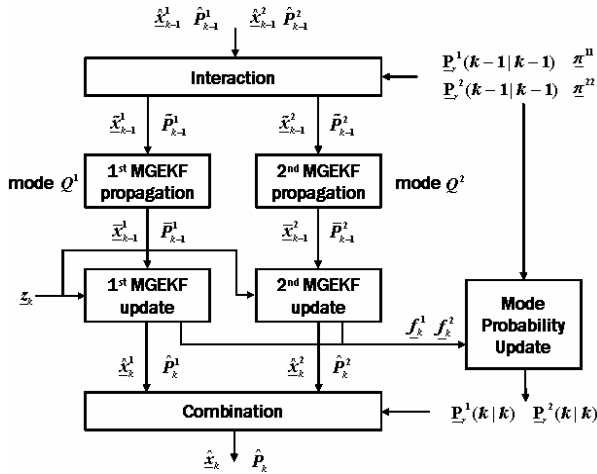


Fig. 3. The IMM structure with two MGEKFs for BFTMA.

The updated state variables and error covariance of two MEGKFs (modes) are mixed through an interaction process and then estimated from the propagation process of two MGEKFs with  $Q^1$  and  $Q^2$ . The estimates of the two MGEKFs are updated with the detected measurements and the state variables and error covariance of BFTMA are finally estimated by combining the two MGEKFs estimates.

#### 4. PRE-PROCESSING FOR BFTMA

For BFTMA in the ocean environment, bearing and frequency measurements of a target are not constantly detected, and frequently it is lost. When the measurements are lost, a BFTMA filter can keep only the propagation process and the state estimates can only be updated after detecting the new measurements, and thus the performance of the BFTMA filter could be degraded and the operation of the BFTMA system could be restricted. To resolve this BFTMA problem, we suggest a pre-processing algorithm for the measurements of a target. The basic concept of the pre-processing algorithm is that we first check whether the measurements are lost or not and then restore the lost measurements with a curve fitting technique that is a data interpolation method to renew the lost measurements with prior detected measurements.

The measurements of a target detected by a passive sonar system are the bearings, which are the directions of the target, and the Doppler shifted which are the noises radiated from the target. In cases where some frequencies are lost but other frequencies are regularly detected, the BFTMA filter can keep tracking the target continuously with the detected measurements of the frequencies. The bearings can not be detected when all the frequencies are lost, in this case the

BFTMA filter only keeps the propagation process.

We suggest a pre-processing algorithm for the bearing and frequency measurements of a target. Particularly, we consider the relation where all the frequencies can be detected or lost independently but the bearing and frequency are detected dependently of each other.

Measurements of a target for BFTMA have the following features:

- As a well known TMA problem of a passive sonar system, measurements for BFTMA are the bearing and the frequencies of a target detected by TAS.
- When the measurements of all the frequencies of a target are not detected by TAS, the bearing of the target is also lost. That is, loss of all the frequencies leads BFTMA to rest.
- When the several parts of the frequency measurements for BFCMA are lost, the detected measurements of other frequencies can be used for BFTMA.

The basic concepts of the pre-processing method for the measurements are as follows:

- When the measurements of the bearing and all the frequencies are not detected, BFTMA is temporarily paused until the detection of target measurements resume to normal.
- When the measurements are newly detected, the information about the loss of the measurements is analyzed and the lost measurements are restored by a curve fitting method with prior detected measurements.
- The number of measurements that are used to restore in the curve fitting method is variable and dependent on the detection condition.
- When the frequency measurements for BFTMA are not detected, the pre-processing algorithm is carried out to restore the lost frequency or change the other detected frequency according to the bearing measurements which are detected to be normal or not.

Particularly, if the frequency measurements for BFTMA are not detected before the initial states of a target are estimated, we must estimate the initial states of the target from only the bearing measurements. This causes unreliable estimation results of the initial states of the target. To resolve this estimation problem, we analyze the features of the normally detected frequency measurements for BFTMA, restore the lost frequency measurements by the pre-processing algorithm, and finally estimate the initial states of the target with the bearing measurements and the restored frequency measurements.

The flowcharts of the suggested pre-processing algorithm for the bearing and frequency measurements are shown in Figs. 4 and 5.

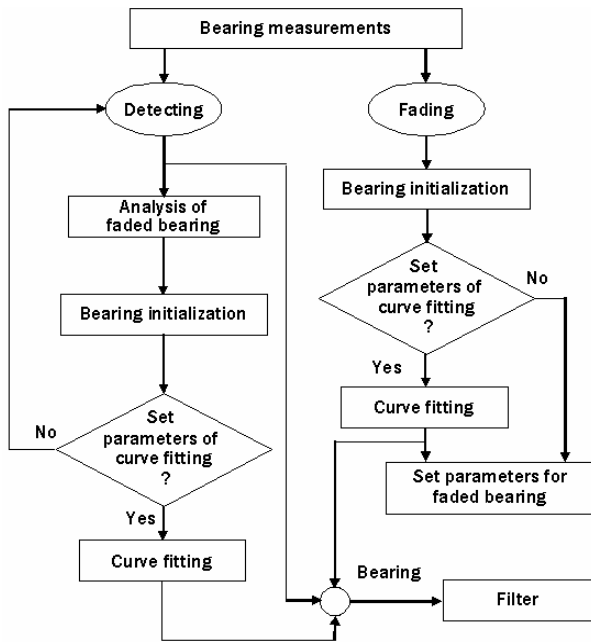


Fig. 4. The flowchart of pre-processing for bearing measurements.

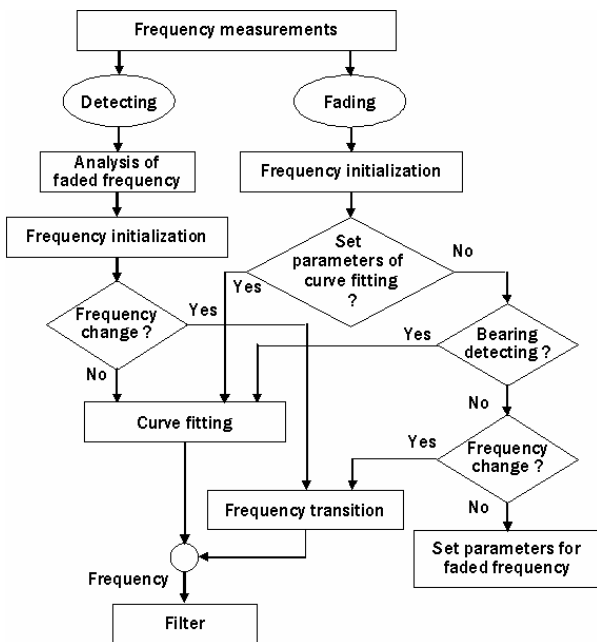


Fig. 5. The flowchart of pre-processing for frequency measurements.

### 5. SIMULATION RESULTS

For the verification of the pre-processing algorithm for the measurements of a target, we present the simulation results for the cases where the bearing and frequency measurements fade independently, and where the frequency measurements used in BFTMA are lost and are not detected any more so only the measurements of the bearing are detected during the estimation of the initial states of a target.

The covariance matrices of the process noise  $Q^1$  and  $Q^2$  which are used in the two filters of IMM are shown as (24) and (25), respectively. The covariance matrices of the bearing and frequency measurements noise  $R_\theta$  and  $R_f$  are assumed to be constant such that  $R_\theta = 5I_2$  and  $R_f = 0.5I_2$ , respectively.

$$Q^1 = \begin{bmatrix} 0.01 & 0 & 0 & 0 & 0 \\ 0 & 0.01 & 0 & 0 & 0 \\ 0 & 0 & 0.001 & 0 & 0 \\ 0 & 0 & 0 & 0.001 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (24)$$

$$Q^2 = \begin{bmatrix} 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0.0001 & 0 & 0 \\ 0 & 0 & 0 & 0.0001 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (25)$$

The filters are initialized with the error covariance  $P_0$ , the mode probability  $Pr_0$ , and the mode transition probability  $\pi$  where

$$P_0 = \begin{bmatrix} 100000 & 0 & 0 & 0 & 0 \\ 0 & 100000 & 0 & 0 & 0 \\ 0 & 0 & 1000 & 0 & 0 \\ 0 & 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 0 & 10 \end{bmatrix}, \quad (26)$$

$$\pi = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}, \quad (27)$$

$$Pr_0 = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}. \quad (28)$$

First, the simulation for verifying the pre-processing algorithm under the condition of the measurements given by Table 2 was carried out. The scenario of the ownship and the target's motions is the same as that in Table 1. It was assumed that the source frequencies of the target are 500Hz and 600Hz including Doppler shift were detected by TAS and the frequency 500Hz was for BFTMA. The results of the pre-processing for the lost measurements are shown in Table 3. The faded measurements of the bearing and frequency prior to estimating the initial states of the target were restored by using the detected measure-

Table 1. The scenario of target's motions.

	Speed[Kts]	Coure[deg]
Target	4	280
Initial range[m]	20000	
Initial bearing[deg]	70	

Table 2. Measurements of the bearing and frequencies algorithms.

Frames [No]	Target measurements			Faded measurements	Estimation conditions
	Bearing [deg]	Frequencies[Hz]			
		1	2		
50	65.1	500.601	601.051		Before estimating the initial states of a target
51	-	-	-	Bearing/ Fre-quencies	
52	-	-	-	Bearing/ Fre-quencies	
53	-	-	-	Bearing/ Fre-quencies	
54	-	-	-	Bearing/ Fre-quencies	
55	-	-	-	Bearing/ Fre-quencies	
56	66.3	500.775	601.942		
57	66.4	500.795	601.934		
58	-	-	-	Bearing/ Fre-quencies	
59	-	-	-	Bearing/ Fre-quencies	
60	62.7	-	600.925	Frequency 1	
61	65.1	-	601.100	Frequency 1	
62	64.1	500.650	601.085		
300	18.6	500.215	600.335		
301	18.2	-	600.360	Frequency 1	
302	18.6	-	600.412	Frequency 1	
303	18.1	-	600.445	Frequency 1	
304	18.5	500.316	600.505		
305	17.3	500.285	-	Frequency 2	
306	17.3	500.240	-	Frequency 2	
307	15.0	500.197	600.102		

ments of the bearing and frequency. The measurements of the frequency 500Hz for BFTMA are lost from 301 to 303 frames but replaced by the detected frequency, 600Hz. The estimates of the initial states of the target by the proposed pre-processing algorithm are shown in Table 4 and the estimates of the range, speed, course of the target are presented in Figs. 6, 7 and 8.

Second, the simulation where the measurements of the frequency 500Hz for BFTMA were lost and are not detected any more prior to estimating the initial

Table 3. Measurements of the bearing and frequencies.

Frames [No]	Target measurements		Restored measurements	Estimation conditions
	Bearing [deg]	Frequency[Hz]		
50	65.1	500.601		Before Estimating the initial states of a target
51	65.4	500.675	Bearing/ Frequency	
52	65.4	500.713	Bearing/ Frequency	
53	65.5	500.735	Bearing/ Frequency	
54	65.6	500.748	Bearing/ Frequency	
55	65.7	500.767	Bearing/ Frequency	
56	66.3	500.775		
57	66.4	500.795		
58	64.3	500.693	Bearing/ Frequency	
59	64.0	500.680	Bearing/ Frequency	
60	62.7	500.672	Frequency	
61	65.1	500.665	Frequency	
62	64.1	500.650		
300	18.6	500.215		
301	18.2	600.360	Replaced by frequency 2	
302	18.6	600.412	Replaced by frequency 2	
303	18.1	600.445	Replaced by frequency 2	
304	18.5	500.316		
305	17.3	500.285		
306	17.3	500.240		
307	15.0	500.197		

Table 4. The estimation results of the initial states of the target.

Target	True values	Estimates	Errors
Initial range[m]	20000	19609.96	390.04
Speed[Kts]	4	3.62	0.38
Course[deg]	280	283.38	-3.38

states of the target was carried out. When the measurements of the frequency for BFTMA are not detected any more prior to estimating the initial states of the target, they must be estimated by only the bearing measurements. However, it affects the estimation performance of the initial states of the target, therefore, it is necessary to restore the lost measurements of the frequency. The two results for the estimation of the initial states of the target, one is the estimation of the initial states of the target with only the bearing measurements and the other is the estimation with the bearing measurements and the restored measurements of the frequency are compared in Table 5. The BFTMA with the measurements of the bearing and the frequency 500Hz and the BOTMA

Table 5. The estimation results of the initial states of the target (BFTMA and BOTMA).

Target	True values	BFTMA		BOTMA	
		Estimates	Errors	Estimates	Errors
Initial range[m]	20000	20896.53	-896.53	32503.38	-12503.38
Speed [Kts]	4	4.04	-0.04	9.64	-5.64
Course [deg]	280	284.79	-4.79	278.57	1.43

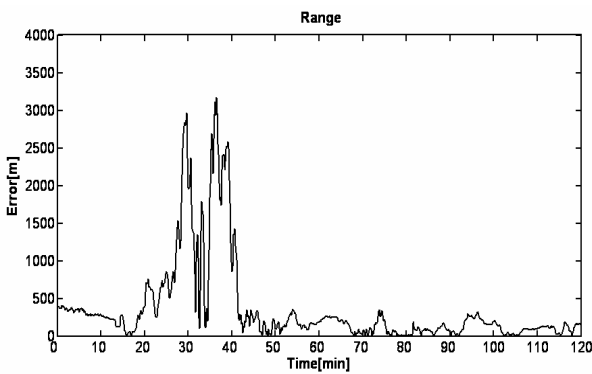


Fig. 6. BFTMA results - range estimate error.

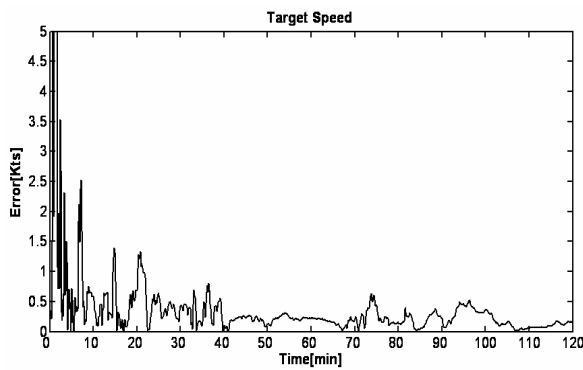


Fig. 7. BFTMA results - speed estimate error.

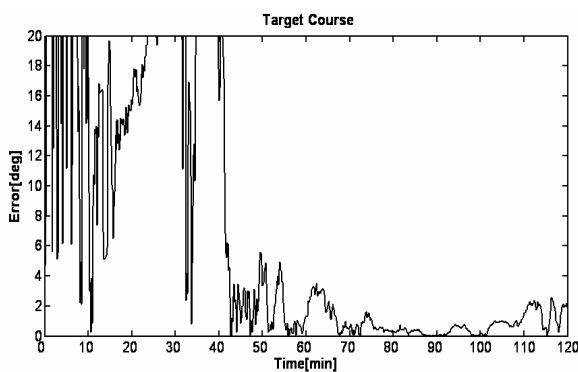


Fig. 8. BFTMA results - course estimate error.

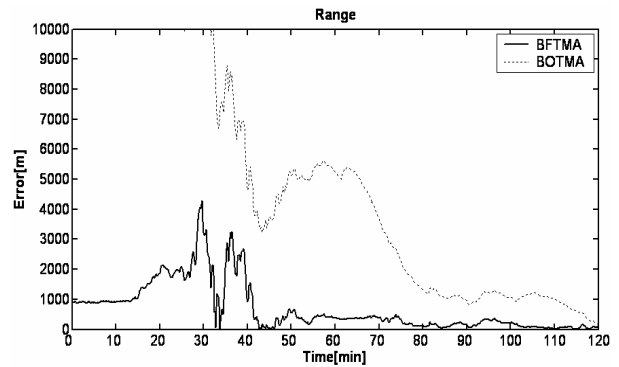


Fig. 9. Range estimates errors (BFTMA and BOTMA).

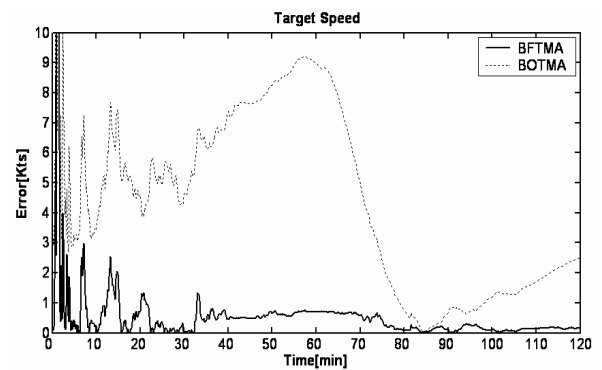


Fig. 10. Speed estimates errors (BFTMA and BOTMA).

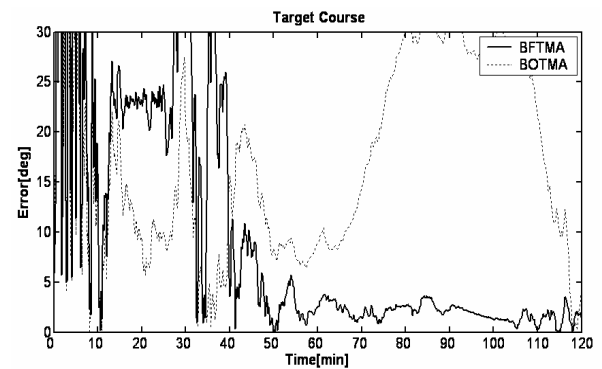


Fig. 11. Course estimates errors (BFTMA and BOTMA).

with the measurements of only the bearing are shown in Figs. 9, 10 and 11. While the BFTMA shows good tracking performance, the BOTMA remains unstable.

## 6. CONCLUSION

In this paper, we focused on the technical research for the performance of BFTMA in towed array sonar. The BFTMA filters were designed by using the MGEKF based on the conventional IMM structure. We suggested algorithms for the better performance of BFTMA, the pre-processing for the measurements of a target to restore the faded measurements and guarantee continuous convergence and stability of BFTMA. Simulations were done under various conditions to verify the validity of the suggested algorithms.

## REFERENCES

- [1] R. C. Kolb and F. H. Hollister, "Bearing-only target estimation," *Proc. of the First Asilomar Conference on Circuits and Systems*, pp. 935-946, 1967.
- [2] V. J. Aidala, "Kalman filter behavior in bearings-only tracking applications," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 15, no. 2, pp. 29-39, January 1979.
- [3] V. J. Aidala and S. E. Hammel, "Utilization of modified polar coordinates for bearings-only tracking," *IEEE Trans. on Automatic Control*, vol. 28, no. 3, pp. 283-294, March 1983.
- [4] A. G. Lindgren and K. F. Gong, *Position and Velocity Estimation via Bearing Observations*, NUSC Technical Report, June 1977.
- [5] A. G. Lindgren and K. F. Gong, "Position and velocity estimation via bearing observations," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 14, no. 4, pp. 564-577, July 1978.
- [6] T. L. Song and J. L. Speyer, "A Stochastic analysis of a modified gain extended Kalman filter with applications to estimation with bearings only measurements," *IEEE Trans. on Automatic Control*, vol. 30, no. 10, pp. 940-949, October 1985.
- [7] R. R. Tenney, R. S. Hebbert, and N. R. Sandell, "A tracking filter for maneuvering sources," *IEEE Trans. on Automatic Control*, vol. 22, no. 2, pp. 246-251, April 1977.
- [8] M. W. Ockeloen and G. A. Willemsen, "Target motion analysis with an extended Kalman-filter using bearing and frequency measurements," *LEOK TR 1982-02*, 1982.
- [9] C. Kent, "Algorithms for Tracking a Sound Source Using Frequency and Bearings," *Report DREV 4199*, 1981.
- [10] H. A. P. Blom and Y. Bar-Shalom, "The interacting multiple model algorithm for systems with Markovian switching coefficients," *IEEE Trans. on Automatic Control*, vol. 33, no. 8, pp. 780-783, August 1988.
- [11] H. A. P. Blom, "Tracking a maneuvering target using input estimation versus the interacting multiple model algorithm," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 25, no. 2, pp. 296-300, March 1989.
- [12] Y. Bar-Shalom and X. R. Li, *Estimation and Tracking Principles, Techniques, and Software*, Artech House, 1993.
- [13] Y. Bar-Shalom and X. R. Li, *Multitarget-Multisensor Tracking: Principles and Techniques*, YBS Publishing, 1995.



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