

# On the Voltage-Based Control of Robot Manipulators

Mohammad Mehdi Fateh

**Abstract:** This paper presents a novel approach for controlling electrically driven robot manipulators based on voltage control. The voltage-based control is preferred comparing to torque-based control. This approach is robust in the presence of manipulator uncertainties since it is free of the manipulator model. The control law is very simple, fast response, efficient, robust, and can be used for high-speed tracking purposes. The feedback linearization is applied on the electrical equations of the dc motors to cancel the current terms which transfer all manipulator dynamics to the electrical circuit of motor. The control system is simulated for position control of the PUMA 560 robot driven by permanent magnet dc motors.

**Keywords:** High-speed tracking, manipulator, PUMA 560, uncertainty, voltage-based control.

---

## 1. INTRODUCTION

Many advanced control strategies were proposed to control robot manipulators by controlling joint torques. However, the inability of commercial robots to control joint torques is a well-known problem [1-2]. The torque-based control command becomes complex due to the complexity of the dynamic equations of manipulator. For instance, the complicated control laws were proposed in robust control [3-4], adaptive control [5], and intelligent control [6-9]. In practice, implementing the torque control command is a control problem since it cannot be applied directly to the inputs of actuators for driving the manipulator. It is necessary to provide another control law for the actuators so that the proposed torques are provided on manipulator joints [10-11]. The limit values and dynamics of actuators should be regarded to perform a torque-based control strategy. Actually, for a manipulator driven by dc motors, the currents of dc motors are directly controlled to implement the torque control law [12-13]. Consequently, the torque-based control strategies have the drawbacks of involving the complexity of manipulator dynamics, practical problems and excluding the role of actuators. Therefore, in this paper, the voltage-based control is preferred comparing to torque-based control of electrically driven manipulators.

The fact is that the manipulator is driven by actuators. Therefore, in order to control the manipula-

tor, its actuators must be controlled. This view takes the control problem to the field of actuator control. The electrically driven manipulators are really good examples to show the art of control in the field of robot control since the electrical motors have an advantage of high controllability as compared with other actuators. The electrical motors are controlled by voltages applied on the motor inputs. This is why the voltage-based strategy is proposed to control electrically driven manipulators. This view obtains the simplicity, accuracy, speed of calculation and robustness to the manipulator control system. Indeed, the electrical equation of motor is much simpler than the dynamic equation of manipulator.

On the other hand, a control law based on manipulator model is not obtained simply since the dynamic model of manipulator is very complex, large, and involved many calculations and included uncertainties. Moreover, based on the voltage control strategy, the independent joint strategy can be applied perfectly without involving the manipulator dynamic terms. The simplicity of the independent joint strategy is that each joint of manipulator is controlled separately. Also, we can handle the manipulator dynamics by the torque inputs of manipulator through the voltages of motors which regulate the currents of motors. It is stated by the electrical equations of the permanent magnet dc motor in which the motor current is proportional to the motor torque which provides the input torque of manipulator.

So far, most industrial robots are controlled by independent joint control strategy while robots are high nonlinear multi-input/multi-output systems with complex couplings [14]. In industry, based on "teach and play back" technique, the point-to-point motion control is applied on the industrial robots to perform robotic tasks. This approach is operated by the independent joint strategy using simple controllers

---

Manuscript received April 17, 2007; revised December 29, 2007; accepted April 25, 2008. Recommended by Editorial Board member Dong Hwan Kim under the direction of Editor Jae-Bok Song.

Mohammad Mehdi Fateh is with the Department of Electrical and Robotic Engineering, Shahrood University of Technology, Shahrood, Iran (e-mail: mmfateh@shahroodut.ac.ir).

such as PID controller or PD controller with gravity compensation. However, the point-to-point motion control can be used only for regulating purposes. Anyway, the industrial robot is controlled point-to-point in the joint space by independent joint strategy and we can see that a globally asymptotically stable closed loop system can be achieved while the torques remain within specified limits [15]. Of course, the main reason for applying independent strategy successfully is that the industrial robots are highly reliable, precise, and repeatable with suitable resolution.

The performance of the independent control strategy is degraded for tracking purposes at high velocities. This is because of the dynamical terms such as Coriolis and centrifugal and the coupling effects which are related to the velocities of joints. The control system is also dependent on the selected trajectory and the disturbances during the tracking operation. If the dynamical terms are not compensated in the control law, the performance will not be satisfactory in the high speed applications. In addition, the effects of link interactions and frictional forces are considered as disturbances in the dependent joint control strategy. Therefore, the capability of the control law for rejecting disturbances is of great technical point.

For coping with low speed motion of manipulators, motors are equipped by the high reduction gears. The reduction gears reduce the dynamical effects transferred to the motors and as a result, rejecting external disturbances. However, gears involve the compliance, backlash, friction and inaccuracy problems. A PD control law and three model based control laws were implemented on a six-degree-of-freedom PUMA 560 arm and the control algorithms were compared for tracking a circle in the workspace [16]. It was seen that the performance of PD control law can be improved using the higher gains, and performance of the model based control approaches were related to the accuracy of model. It was reported that the specific manipulator dynamic was experimentally determined and the feedforward and computed-torque controllers were used to compensate its dynamics [17]. However, selecting the higher gains produces higher voltage on motors which may cause damages. And, a very accurate model may not be available and if so it may involve high computing and time consuming as it cannot be applied for high speed applications.

Although the independent joint controllers involve link interaction problems, efforts lead to improve them for controlling the high-speed robots [18]. The independent strategy has been considered in the field of intelligent control, as well. There are many developments but we address some of them in the independent joint strategy. A combined computed

torque control and fuzzy control was suggested for flexible-joint manipulators [19]. A model-free control was proposed using integrated PID-type learning and fuzzy control for flexible-joint manipulators [20]. Neural networks can be used to approximate the dynamic equations and estimate the coefficients.

In this paper, the independent strategy is applied in a manner which can solve the coupling dynamic problem to achieve a well tracking. The feedback linearization is then applied in the electrical circuit of the dc motor to cancel the electrical current which includes all dynamical terms of manipulator. This is a novel point of view as we can simply control the manipulator without use of its model. This control approach has the following advantages: first of all, the feedback linearization of system requires only a feedback of motor current to omit the current terms while the control approaches based on manipulator dynamics require more feedbacks. The obtained system after feedback linearization is a linear decoupled system as a first integrator. Secondly, the feedback linearization requires only the electrical coefficients of dc motor. The control law is free of manipulator model, so that the control approach is robust in the presence of manipulator uncertainties. Third, the design is simple and fast response using the independent joint strategy as it is proposed to apply on industrial robots for tracking purpose.

This paper is organized as follows. In Section 2, the control law is formulated based on feedback linearization and stability of control system is analyzed. The proposed control is then compared with the inverse dynamic control law. The control system is presented in Section 3. Rejecting disturbances is then considered in Section 4. After that the simulation results are shown in Section 5 and finally conclusions are given in Section 6.

## 2. FEEDBACK LINEARIZATION

The electrical circuit of the permanent magnet dc motor provides the following equation [14]

$$v = Ri + L \frac{di}{dt} + k_b \frac{d\theta_m}{dt}, \quad (1)$$

where  $R$  is the armature resistance,  $L$  is the armature inductance,  $k_b$  is the back emf constant,  $v(t)$  is the armature voltage,  $i$  is the armature current and  $\theta_m$  is the rotor position, respectively.

In order to cancel the current terms, a control law is proposed as

$$v = Ri + L \frac{di}{dt} + k_b f, \quad (2)$$

where  $f$  is a new control input. Substituting (2) into

(1) results in a linear time invariant system formed as

$$f = \frac{d\theta_m}{dt}. \quad (3)$$

In the system obtained by (3),  $f$  is the input and the rotor position  $\theta_m$  is the output. The system is known as an integrator that represents an uncoupled linear equation. Therefore, for tracking a desired trajectory we may choose a linear control law as

$$f = \dot{\theta}_{md} + k_p(\theta_{md} - \theta_m), \quad (4)$$

where  $\theta_{md}$  is the desired rotor position,  $\dot{\theta}_{md}$  is the desired rotor velocity, and  $k_p$  is the proportional gain. Substituting (4) into (3), yields

$$\dot{\theta}_{md} - \dot{\theta}_m + k_p(\theta_{md} - \theta_m) = 0. \quad (5)$$

We define  $e = \theta_{md} - \theta_m$  as the tracking error to obtain

$$\dot{e} + k_p e = 0. \quad (6)$$

The closed loop control system is a linear system and it is stable only if we choose  $k_p > 0$ . Then  $e(t) \rightarrow 0$  as  $t \rightarrow \infty$ , or the rotor position  $\theta_m$  converges to the desired rotor position  $\theta_{md}$ .

Feedbacks from the motor current and its derivative are required to implement the control law given by (2). In addition, the control law given by (3) requires the feedback of the motor position. The motor current contains all dynamical effects of manipulator transferred to the motor. This fact is concluded from the dynamic equation of a manipulator driven by dc motors [11], formulated as

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \mathbf{k}_m \mathbf{i}, \quad (7)$$

where  $\mathbf{i}$  is the armature current vector,  $\mathbf{q}$  is the joint variable vector,  $\mathbf{M}$  is the completed inertia matrix,  $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}$  is the centrifugal and Coriolis torque vector,  $\mathbf{g}(\mathbf{q})$  is the gravitational torque vector, and  $\mathbf{k}_m$  is the diagonal matrix of motor torque constant. The dissipative torques such as frictional torques and the provided load torque can be added to the left hand of (7) to complete the equation.

The proposed control law given by (2) would be advantageous in comparison with the inverse dynamic control. The inverse dynamic control law is defined as

$$\boldsymbol{\tau} = \mathbf{M}(\mathbf{q})\mathbf{a} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}), \quad (8)$$

$$\boldsymbol{\tau} = \mathbf{k}_m \mathbf{i}, \quad (9)$$

where  $\boldsymbol{\tau}$  is the torque vector inserted to the manipulator joints and  $\mathbf{a}$  is the input vector of the

obtained system after feedback linearization as

$$\mathbf{a} = \ddot{\mathbf{q}}. \quad (10)$$

This control law can transform the nonlinear system given by (8) to a new system shown in (10) which is linear and decoupled. However, there are some problems for implementing the control law presented by (8). This control law is not complete since some terms such as frictional torques have been omitted for simplicity and reducing the computing time, and some terms are not precise. Therefore, applying this control law cannot provide a perfect linear and decoupled system, and due to inaccuracy in model, errors will be produced. Moreover, implementing of the control law requires feedbacks of all joint positions and their derivatives. Also, the control strategy is complex since the system is highly coupled and multi-input/multi-output. The control law is very large, and time consuming that involves a problem of limit sampling rate. The tracking error increases as velocity increases.

Control law given by (2) is preferred as compared with the control law (8). Because, all feedbacks are belonging to the motor as the control strategy is the independent joint strategy. Also, we do not require the manipulator model to form the control law. As a result, the control law is simple, fast, and more accurate in comparison with (8). The control law requires only a feedback of motor current and electrical coefficients of the motor. Moreover, the electrical signals can be measured more convenient and more precise than mechanical signals. This control law can be used for tracking control of a high-speed robot since this approach is free of manipulator model. In fact, the dynamical effects are compensated by currents of motors in high-speed applications.

### 3. THE CONTROL SYSTEM

In the proposed approach, each joint of the manipulator is driven by a permanent magnet dc motor in the control system. The inserted torque on the joint to drive the manipulator is the load torque of motor, which is considered in a dynamic equation formed as

$$T_m = J_m \ddot{\theta}_m + B_m \dot{\theta}_m + rT, \quad (11)$$

where  $T$  is the load torque,  $T_m$  is the motor torque,  $r$  is the gear reduction coefficient,  $J_m$  is the sum of actuator and gear inertia, and  $B_m$  is the damping coefficient. The reduction gear relates the motor position to the joint position as

$$q = r\theta_m, \quad (12)$$

where  $q$  is the joint position which is the joint angle for a revolute joint or the joint distance for a prismatic

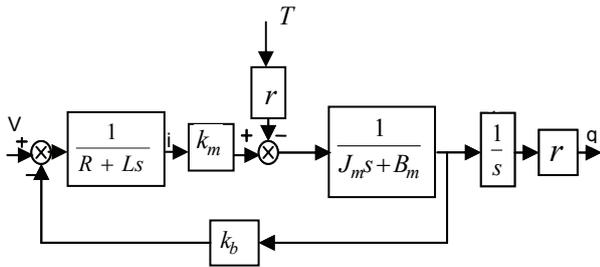


Fig. 1. The permanent magnet geared dc motor.

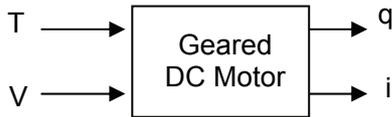


Fig. 2. The motor system.

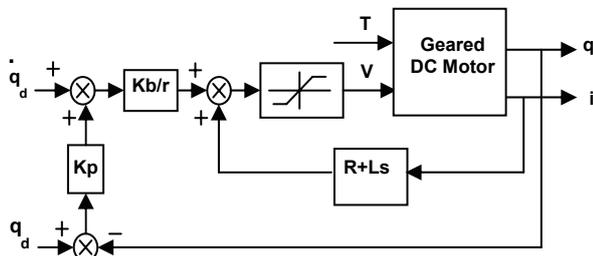


Fig. 3. The motor control system.

joint. The motor torque is proportional to the armature current as

$$T_m = k_m i, \tag{13}$$

where  $k_m$  is the torque coefficient. The torque coefficient is equal to the back emf constant for the permanent magnet dc motor.

$$k_m = k_b. \tag{14}$$

The block diagram of the permanent magnet dc motor is shown in Fig. 1. In the motor system, the motor voltage  $v$  is the control input. The load torque  $T$  is an input, which is not under control. The motor current  $i$  and the joint position  $q$  are given as the motor outputs. The motor system is shown in Fig. 2.

The control system of the permanent magnet dc motor is formed by (1) - (4), as shown in Fig. 3. Only feedbacks of joint position and electrical current of motor are required to form the controller. Moreover, in order to immune motors from over voltages, the motor voltages are limited in the control law. The independent joint control strategy is implemented using a controller to control the motor voltage of each joint as shown in Fig. 4. The desired trajectory of each joint  $q_d$  and its derivative are predetermined for each controller while the joint position  $q$  and the

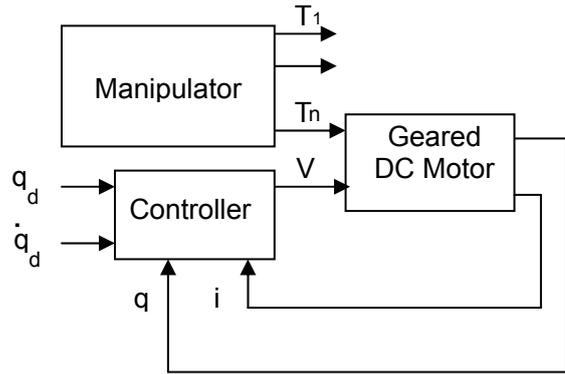


Fig. 4. The manipulator control system.

motor current  $i$  are feedbacks to control in the real time processing.

#### 4. REJECTING DISURBANCES

When an industrial robot transfers objects of different masses, the dynamics of manipulator will be changed. Therefore, the manipulator model involves uncertainties. Moreover, the friction is not included in the inverse dynamic control law (8) since it is not repeatable to have a perfect model of friction. The manipulator may be subject to external disturbances when robot operates in the workspace. We can modify (7) to include friction, external dynamics and other left terms as an global uncertain term denoted by vector  $T_d$ .

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + T_d = k_m i. \tag{15}$$

All dynamical terms are transferred to the electrical currents of motors as stated by (15). We can measure currents of motors precisely by current sensors and then they will be cancelled from the close loop control system by the control law (2). As it is seen from (6), dynamics of error is free of manipulator dynamics so it introduces a robust control system. In this technical point of view, the proposed control law has an advantage to solve such uncertain dynamics which many control approaches are involved. Therefore, the proposed control law is advantageous comparing to the robust control laws which require either the manipulator dynamics or the manipulator parameters. Also, for tracking purpose it is more efficient than the PID control system which it is well known as a simple and useful method to control manipulators. Many industrial robots use a form of so called PID control law [21] as

$$\ddot{q}_d + k_1 e + k_2 \dot{e} + k_3 \int e dt = u, \tag{16}$$

where  $u$  is the control law vector,  $e = q_d - q$  is the

position error vector,  $\mathbf{q}_d$  is the desired joint angle vector,  $\mathbf{q}$  is the joint angle vector,  $\mathbf{k}_1$ ,  $\mathbf{k}_2$  and  $\mathbf{k}_3$  are the diagonal coefficient matrixes. The torque input vector of manipulator is controlled by (15). Thus

$$\begin{aligned} \ddot{\mathbf{q}}_d + \mathbf{k}_1\mathbf{e} + \mathbf{k}_2\dot{\mathbf{e}} + \mathbf{k}_3\int \mathbf{e}dt \\ = \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) + \mathbf{T}_d. \end{aligned} \quad (17)$$

We can arrange (16) in the form of

$$\begin{aligned} \ddot{\mathbf{q}}_d + \mathbf{k}_1\mathbf{e} + \mathbf{k}_2\dot{\mathbf{e}} + \mathbf{k}_3\int \mathbf{e}dt \\ = \ddot{\mathbf{q}} + (\mathbf{M}(\mathbf{q}) - \mathbf{I})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) + \mathbf{T}_d. \end{aligned} \quad (18)$$

Hence

$$\ddot{\mathbf{e}} + \mathbf{k}_1\mathbf{e} + \mathbf{k}_2\dot{\mathbf{e}} + \mathbf{k}_3\int \mathbf{e}dt = \varphi, \quad (19)$$

where  $\varphi$  is

$$\varphi = (\mathbf{M}(\mathbf{q}) - \mathbf{I})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) + \mathbf{T}_d. \quad (20)$$

It can be concluded from (19) that the dynamics of error is subjected to the dynamics of manipulator given by (20). Comparing to the PID control law, the proposed control law is preferred for rejecting disturbances to track a trajectory.

Since the proposed method is a joint control approach it can be applied for a manipulator with a suitable repeatability. Actually, the required data is provided in advance based on ‘‘teach and play back’’ technique and it will be useful if the kinematic parameters to be invariant. Alternatively, a precise kinematic model of manipulator can be used to transfer required data from the task space to the joint space for performing task in the control space.

The proposed control law is stable for  $k_p > 0$  while in the PID control law we cannot make sure about the system stability even in the condition that the gains in (19) are selected for the purpose of locating the poles in the left hand side of  $s$  plane. It is understood from (20) that  $\varphi$  is not an independent input source to the equation (19) and  $\varphi$  is a nonlinear function of system states. Therefore, closed loop control system given by (19) is a nonlinear system and its stability is not proven by this way.

## 5. SIMULATION RESULTS

The shape of trajectory plays a significant role for tracking control of the manipulator. The trajectory is required to be smooth since the first and second time derivatives of the joint positions are appeared in the electrical current as given by (7). If the trajectory is non-smooth, then the system responses may go out of the system specifications. Thus, in this case the

constraints and limits are required to protect the devices. In addition, the derivatives of non-smooth trajectories may cause the infinity problem in simulations.

In order to consider the hard situation, the inductance of armature is included in the motor model. This causes the derivative of motor current as described in (1) and consequently the system involves the third derivative of the joint positions, as well. It is also assumed that the initial tracking error is zero, which is a significant factor to reduce the tracking error. The initial velocity and final velocity are given zero, as well. A formula is proposed to plan a smooth trajectory for tracking purpose which is advantageous for tracking in the joint space. Also, it can be used for the point-to-point motion under the applied limits on the mechanical and electrical system.

$$\begin{aligned} \theta_d(t) = \frac{\theta_f - \theta_0}{2} \left(1 - \cos\left(\frac{\pi}{T_f - T_0}t\right)\right), \\ T_0 \leq t \leq T_f, \end{aligned} \quad (21)$$

where  $\theta_d(t)$  is the desired joint angle,  $T_0$  is the initial time,  $\theta_0$  is the initial joint angle,  $T_f$  is the final time, and  $\theta_f$  is the final joint angle. The desired trajectory begins at  $(T_0, \theta_0)$  and ends at  $(T_f, \theta_f)$ . The duration of operating is  $T_f - T_0$ , and the operating range is  $\theta_f - \theta_0$ . The first and the second time derivatives of the desired position are calculated as

$$\dot{\theta}_d = \frac{\pi}{2} \frac{\theta_f - \theta_0}{T_f - T_0} \sin\left(\frac{\pi}{T_f - T_0}t\right), \quad T_0 \leq t \leq T_f, \quad (22)$$

$$\ddot{\theta}_d = \frac{\pi^2}{2} \frac{\theta_f - \theta_0}{(T_f - T_0)^2} \cos\left(\frac{\pi}{T_f - T_0}t\right), \quad T_0 \leq t \leq T_f. \quad (23)$$

The maximum value of the desired joint velocity,  $\frac{\pi}{2} \frac{\theta_f - \theta_0}{T_f - T_0}$ , and the maximum value of the desired

joint acceleration,  $\frac{\pi^2}{2} \frac{\theta_f - \theta_0}{(T_f - T_0)^2}$ , should be consid-

ered to satisfy by motor specifications.

We simulate the control system shown in Fig. 4 for tracking control of PUMA 560 driven by brushed dc motors with specifications given by [22] as shown in Table 1. Since inductance  $L$  is very small, it was not given in Table 1. However, in the simulations 0.0001H and 0.0002H are given to inductances of the three first motors and three second motors, respectively. The motors which drive joints 1, 2, and 3 are 40V, 160W and the motors of joints 4, 5 and 6 are

40V, 80W [23]. The simulation model of PUMA 560 [24] is used in the control system. The inductances of motors are ignorable but in the simulation the values of 0.0001H are given to the motors 1, 2, 3 and 0.0002H for motors 4, 5, 6. The first three joints are equipped with 24V electromagnetic brakes. The brakes operate when the joints are not driven or when the power is removed.

**Simulation 1:** The desired trajectory used for a normal speed application is drawn in Fig. 5 where the robot moves 2rad for 10sec. The values of  $\theta_f = 2\text{rad}$ ,

$\theta_0 = 0$ ,  $T_0 = 0$ ,  $T_f = 10\text{sec}$  are given to (21) to obtain the desired trajectory as

$$\theta_d(t) = 1 - \cos\left(\frac{\pi}{10}t\right) \quad 0 \leq t \leq 10, \quad (24)$$

To consider the system performance, the desired trajectory is given to every joint of PUMA 560 such that all joints are driven in the same time. The control law (4) is applied with  $k_p = 100$  for all six controllers. Tracking errors of all joints are very small as shown in Fig. 6. The errors are in the range of  $[-2 \ 1.5] \times 10^{-5}\text{rad}$ . The motor voltage and motor current of joint 2 are shown in Figs. 7 and 8, respectively. All of the voltages and currents of motors are located under the valid limited values. Despite the high oscillations in the tracking error, voltage and current of motor 2, the manipulator operates smoothly to follow the desired trajectory. Actually, there are no changes in voltage polarity or current direction, which are good reasons to tolerate the high oscillations. However, the high oscillation may exit the un-modeled dynamics. A low pass filter

Table 1. Parameters of permanent magnet dc motors.

Joint	R	$K_b$	J	B	1/r
1	1.6	0.26	0.0002	0.00148	62.611
2	1.6	0.26	0.0002	0.000817	107.82
3	1.6	0.26	0.0002	0.00138	53.706
4	3.76	0.09	0.000033	0.0000712	76.036
5	3.76	0.09	0.000033	0.0000826	71.923
6	3.76	0.09	0.000033	0.0000367	76.686

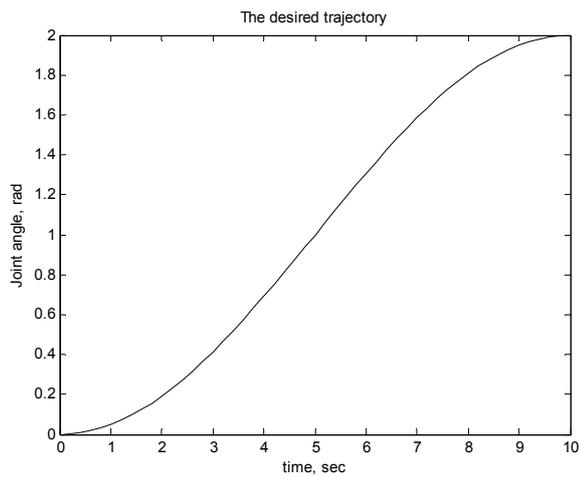


Fig. 5. Desired trajectory.

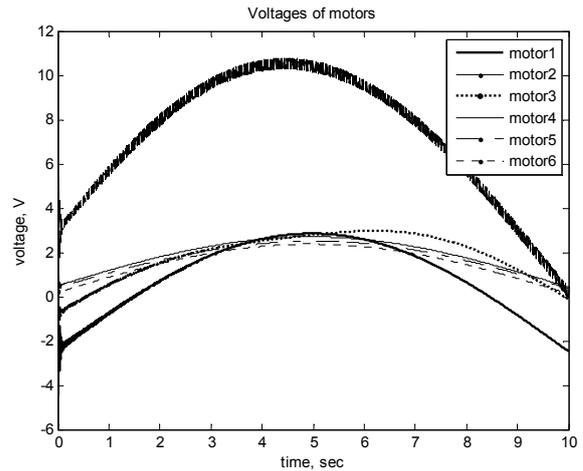


Fig. 7. The voltages of motors.

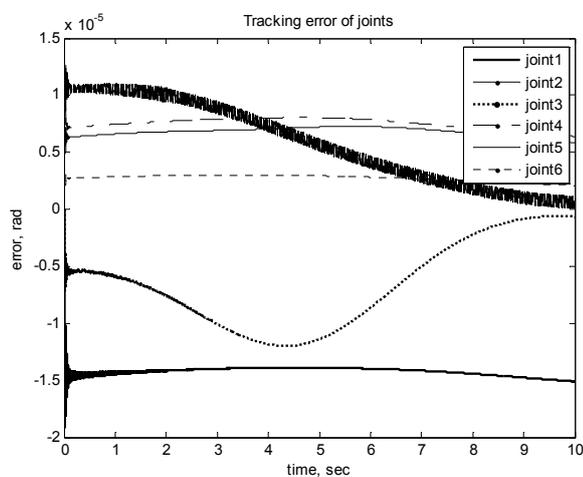


Fig. 6. The tracking error of joints.

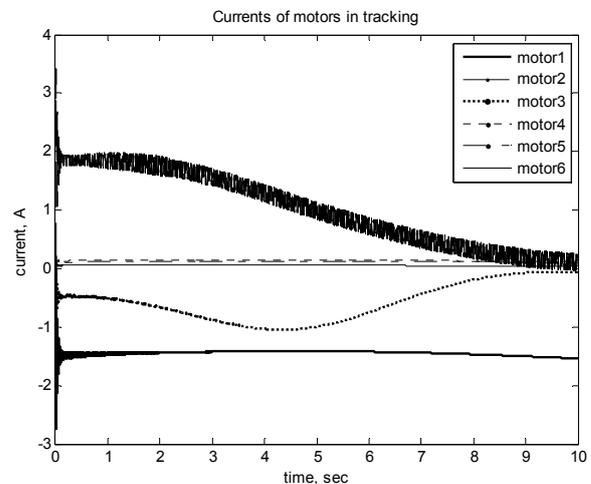


Fig. 8. The currents of motors.

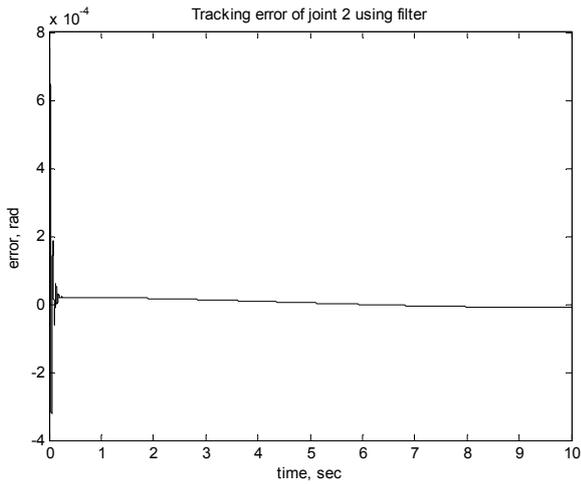


Fig. 9. The tracking error of joint 2 using filter.

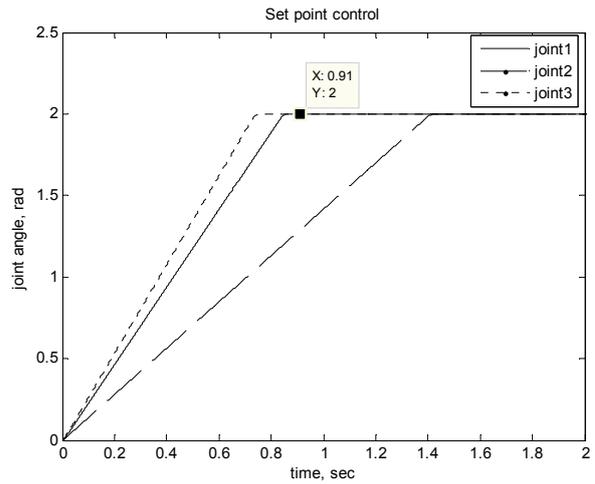


Fig. 11. Joint angles in set point control.

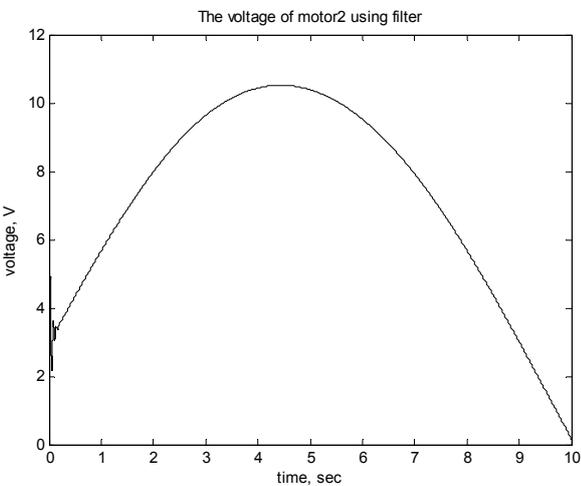


Fig. 10. The voltage of motor 2 using filter.

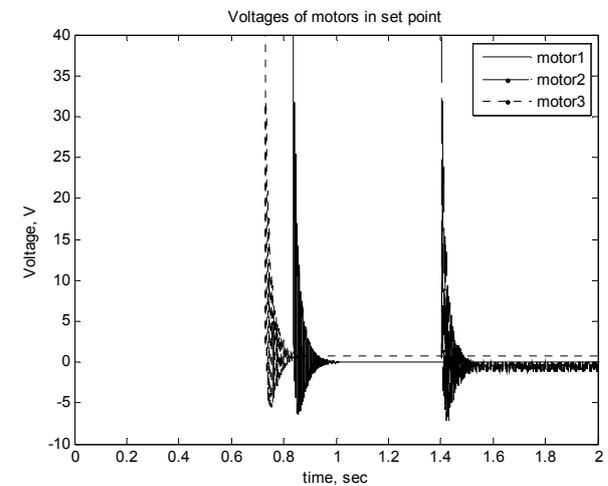


Fig. 12. Voltages of motors in set point control.

is proposed to remove the high oscillation from motor 2. The filter is located after the control law to pass the voltage to the motor.

**Simulation 2:** A filter with transfer function of  $\frac{s}{0.01s + 1}$  is used to the input of the motor 2. The tracking errors were increased to be located in the range of  $[-4 \ 8] \times 10^{-4}$  rad as shown in Fig. 9. And, the high oscillation is removed well from the voltage as shown in Fig. 10.

**Simulation 3:** The industrial robots perform many tasks by point-to-point position control in the task space. The process is performed based on the set point position control. The desired position and orientation of the end effector in the task space is provided by the desired joint angles of the manipulator. In this simulation the desired angles of all joints are given 2rad as a set point to the control law (4) with  $k_p = 100$  for the controllers. The control system responds to 2rad step function while the manipulator

is free of trajectory during operation. The positions of joints approach to the desired value of 2rad without overshoot and steady state error as shown in Fig. 11. The voltages of motors are in the limit value of 40V until the joint angle reach to the set point and then have damped oscillations as shown in Fig. 12. The motors drive manipulator on the maximum allowable voltage which results in undesired effects to stop manipulator. Motor 1, motor 2 and motor 3 went to the set point for 0.85sec, 1.41sec and 0.75sec, respectively. If you like to drive the robot faster, you should use the stronger motors. The motors have different travel times because they have different loads. To adjust the period of operation and moving smoothly, a desired trajectory such as one which was used in Simulation 1 is required. Despite the fact that motors rotate fast, they stop well at the set point. The efforts of control system are seen on the set point by the oscillations of voltages of motors.

**Simulation 4:** the control system is simulated to perform a quick task through the smooth trajectory. A

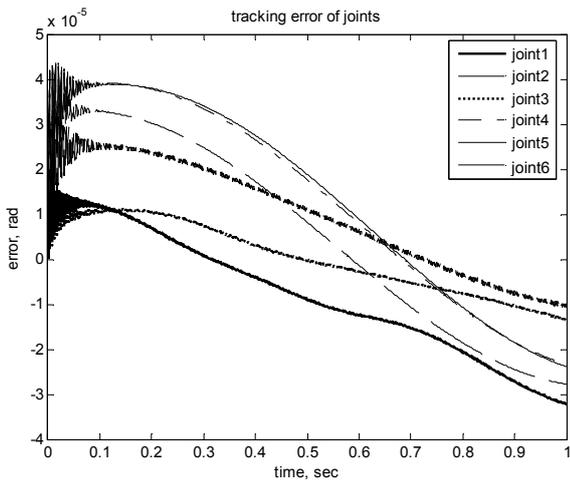


Fig. 13. Tracking errors of joints.

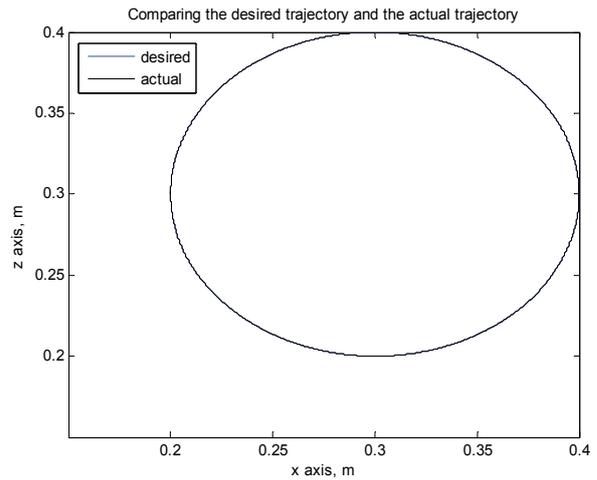


Fig. 15. The tracking of circle.

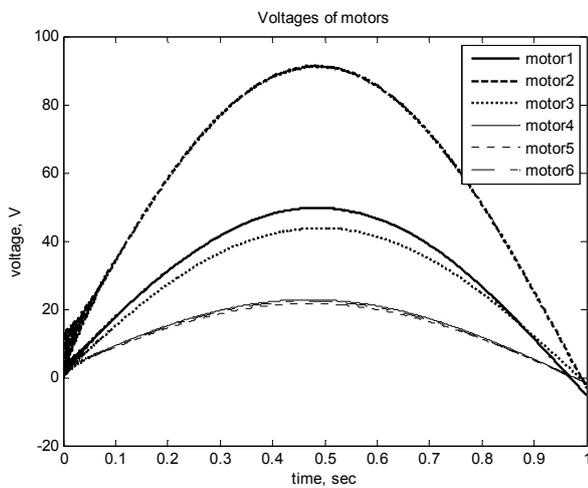


Fig. 14. Voltages of motors.

desired trajectory is obtained by (21) for the values of  $\theta_0 = 0$ ,  $T_0 = 0$ ,  $\theta_f = 2\text{rad}$ ,  $T_f = 1\text{sec}$ .

$$\theta_d(t) = 1 - \cos(\pi t) \quad 0 \leq t \leq 1, \quad (25)$$

The voltage limiters are removed from the control system and the control law (4) with  $k_p = 100$  is performed. The tracking errors of joints show ignorable values in the range of  $[-4 \ 5] \times 10^{-5}$  rad as shown in Fig. 13. The motor 1 and motor 2 are in the condition of over voltages as shown in Fig. 14. Regarding the voltage limit of 40V the motor 1 and motor 2 should be replaced by stronger motors.

**Simulation 5:** In this simulation, the robot is driven to track a circle in the task space. The desired circle is located in the surface  $z = 0$ , with center at (0.3m, 0, 0.3m) and radius of 0.1m. The orientation of the end effector frame is considered to be parallel with the base frame. To have a smooth trajectory along the circle, the central angle of the circle is produced by (21) where the values are  $\theta_0 = 0$ ,  $T_0 = 0$ ,  $\theta_f$

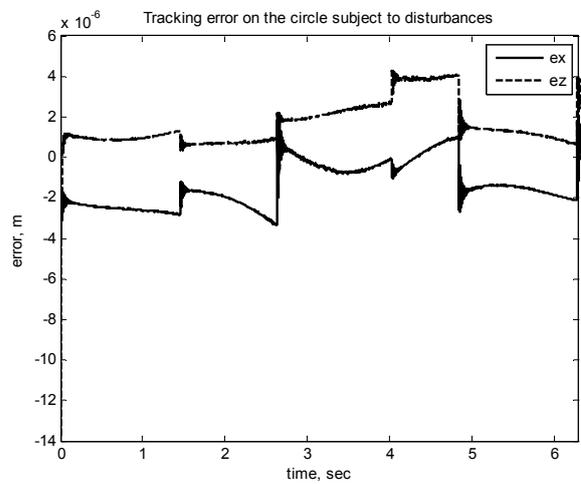


Fig. 16. The tracking error.

$= 6.28\text{rad}$ , and  $T_f = 6.28\text{sec}$ . The trajectory is

$$\begin{aligned} \theta_d(t) &= 3.14(1 - \cos(\frac{\pi}{6.28}t)), & 0 \leq t \leq 6.28, \\ x_d(t) &= 0.3 + 0.1 \cos(\theta_d(t)), \\ z_d(t) &= 0.3 + 0.1 \sin(\theta_d(t)). \end{aligned} \quad (26)$$

We cannot recognize any difference between the desired circle and the output of the system as shown in Fig 15. The control law works well since the tracking errors of the end effector,  $e_x$  and  $e_z$  are in the range of  $[-14 \ 6] \times 10^{-6}$  m as shown in Fig. 16. To perform the task, five joints of manipulator are rotating and only joint 4 is not moving. The voltages and currents of motors are permitted as shown in Figs. 17 and 18.

**Simulation 6:** The ability of control system for rejecting disturbances is simulated. The desired trajectory which was used in Simulation 5 is given to the manipulator and the following disturbance is applied on each joint as external disturbance.

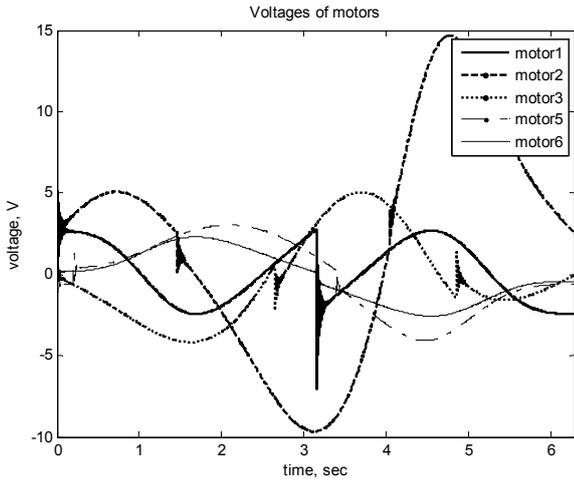


Fig. 17. Voltages of motors.

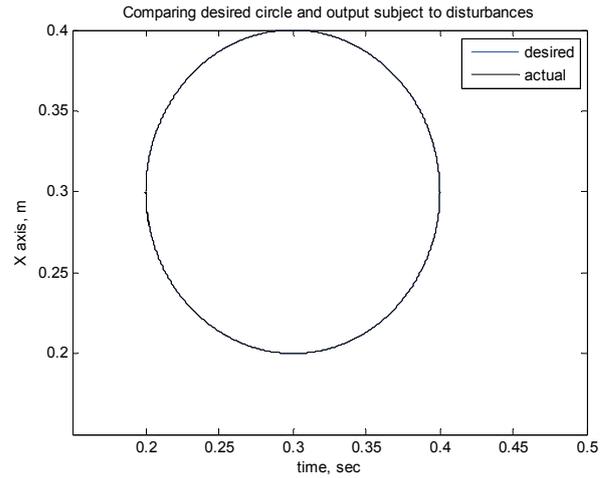


Fig. 19. Tracking subject to disturbances.

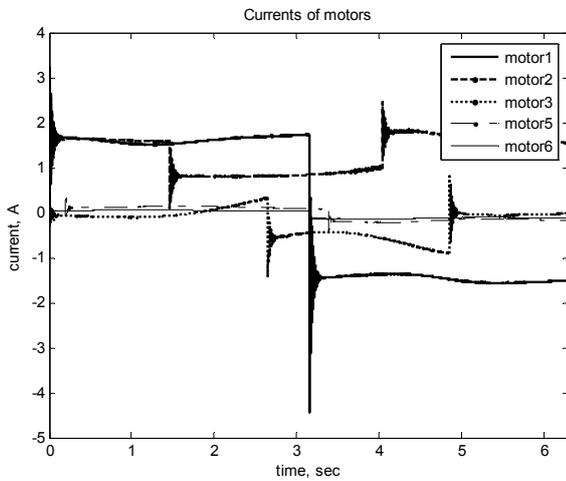


Fig. 18. Currents of motors.

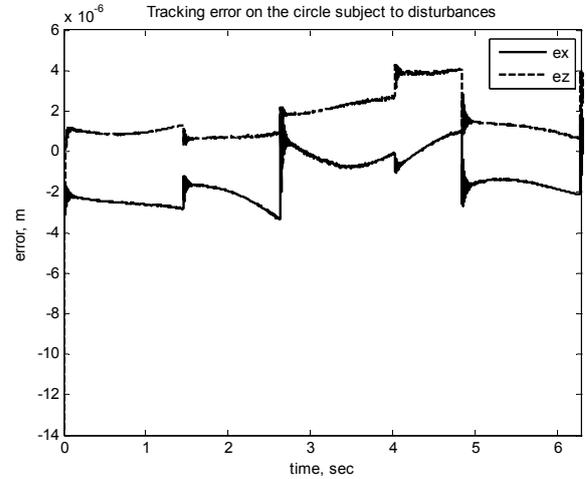


Fig. 20. Tracking error subject to disturbances.

$$T_d = 2 + 2\sin(2t) \text{ Nm} \quad (27)$$

We can see a perfect tracking under disturbances in Fig. 19 where the tracking errors of the end effector,  $e_x$  and  $e_z$  are in the range of  $[-14 \ 6] \times 10^{-6} \text{ m}$ , as shown in Fig. 20. The control system shows a powerful ability of rejecting disturbances if we compare tracking error without disturbances in Fig. 16 and tracking error with disturbances in Fig. 20. It seems that there are no differences.

**Simulation 7:** The PID control law (16) is applied to track the circle under the same condition given in Simulation 5. The coefficients are adjusted by trial and error to present the less error as possible. The PID control law  $V = \ddot{q}_d + k_{1v}\dot{e} + k_{2v}\dot{e} + k_{3v}\int e dt$  with  $k_{1v} = 10000$ ,  $k_{2v} = 200$ ,  $k_{3v} = 10$  is applied on motors except motor 4 which is stop. We can see a good tracking in Fig. 21 where the tracking errors of the end effector,  $e_x$  and  $e_z$  are in the range of  $[-4 \ 4] \times 10^{-4} \text{ m}$  in Fig. 22. However, the PID

control law shows a maximum tracking error of 78 times of the proposed control law.

**Simulation 8:** We compare the PID control law and the proposed control law for tracking the circle in a higher speed so that the period of operation is adjusted 1sec for one turn on circle through the smooth curve.

$$\begin{aligned} \theta_d(t) &= 3.14(1 - \cos(\pi t)) \quad 0 \leq t \leq 1, \\ x_d(t) &= 0.3 + 0.1 \cos(\theta_d(t)), \\ z_d(t) &= 0.3 + 0.1 \sin(\theta_d(t)). \end{aligned} \quad (28)$$

The tracking errors are in the range of  $[-3 \ 3] \times 10^{-3} \text{ m}$  for PID control system while they are in the range of  $[-3 \ 3] \times 10^{-5} \text{ m}$  for the proposed control law. The proposed control law shows much less error than the PID control system by comparing the results.

## 6. CONCLUSIONS

A novel control approach has been proposed based

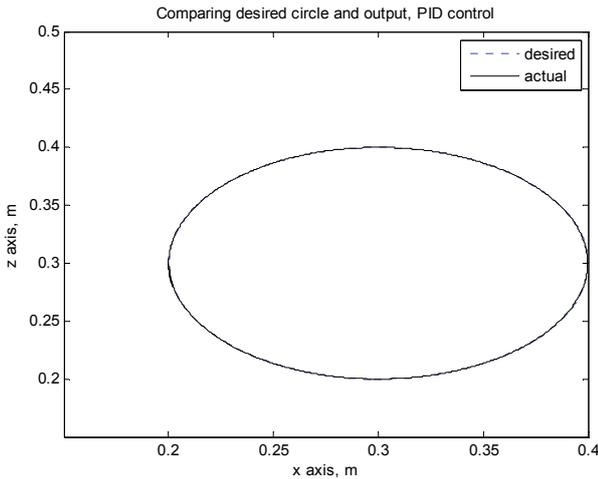


Fig. 21. Tracking the circle by PID control.

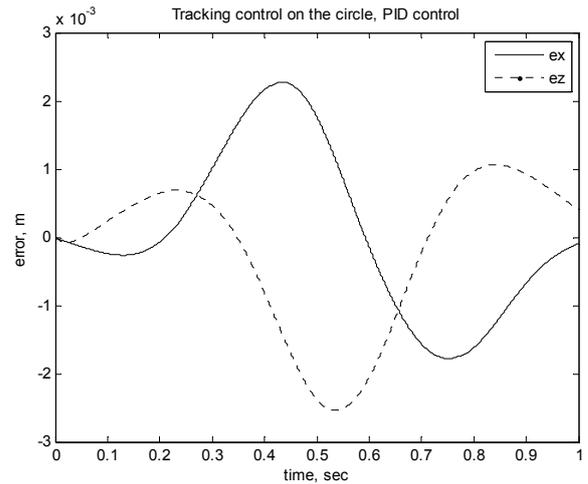


Fig. 23. Tracking on the circle using PID control.

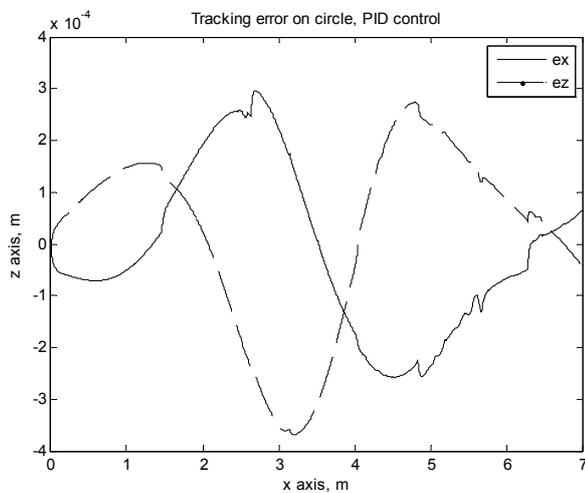


Fig. 22. Tracking error on circle by PID control.

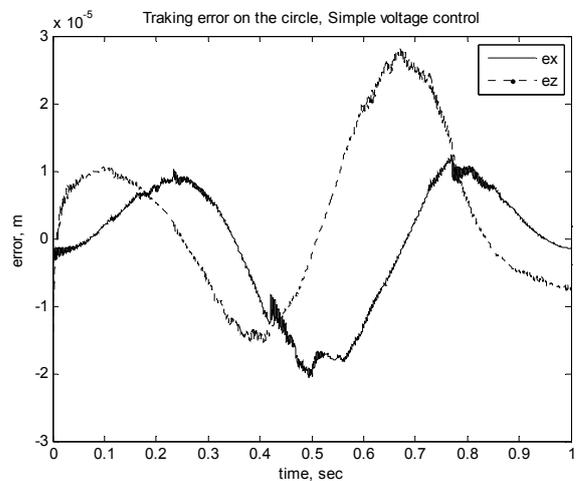


Fig. 24. Tracking on the circle using simple control.

on electrical view on the motor circuit for a manipulator driven by permanent magnet dc motors. The motor current has been considered for compensating all dynamic torques provided by the manipulator. The control approach is based on feedback linearization to omit the current terms and providing a linear decoupled system in the form of first integrator. The control law is free of the manipulator model, so the control approach is robust subject to manipulator uncertainties and external disturbances. And, the design is simple using the independent joint strategy and the control algorithm has a fast response with few calculations. Therefore, the proposed control approach can be used for high-speed operation, as well. A smooth trajectory is proposed to reduce the dynamical reactions of manipulator when starting and stopping under the allowing values of motor inputs. Tracking a specific trajectory provides satisfactory responses in the case of point-to-point control, as well. The simulation results show the satisfactory responses of the control

system in the case of rejecting disturbances, tracking, set point and high-speed applications. The proposed control law shows a very small tracking error in comparing to PID control system. The proposed method is robust, simple, accurate, with less computing as compared with inverse dynamic control system.

**REFERENCES**

[1] C. An, C. Atkeson, and J. Hollerbach, *Model Based Control of a Robot Manipulator*, MIT Press, Cambridge, MA, 1988.  
 [2] Z. Schiller, "Time-energy optimal control of articulated systems with geometric path constraints," *Journal of Dynamic Systems, Measurement and Control, Trans. ASME*, vol. 118, pp. 139-143, 1996.  
 [3] S. Torres, J. A. Mendez, L. Acosta, and V. M. Becerra, "On improving the performance in robust controllers for robot manipulators with parametric disturbances," *Control Engineering*

- Practice*, vol. 15, pp. 557-566, 2007.
- [4] M. W. Spong, "On the robust control of robot manipulators," *IEEE Trans. on Automatic Control*, vol. 37, no. 11, pp. 1782-1786, November 1992.
- [5] C. S. Chiu, K. Y. Lian, and T. C. Wu, "Robust adaptive motion/force tracking control design for uncertain constrained robot manipulators," *Automatica*, vol. 40, pp. 2111-2119, 2004.
- [6] Y. Tang, F. Sun, and Z. Sun, "Neural network control of flexible-link manipulators using sliding mode," *Neurocomputing*, vol. 70, pp. 288-295, 2006.
- [7] L. Tian and C. Collins, "Adaptive neuro-fuzzy control of a flexible manipulator," *Mechatronics*, vol. 15, pp. 1305-1320, 2005.
- [8] Y. Li, "Robust neural networks compensating motion control of reconfigurable manipulator," *Proc. of the First Int. Conf. on Innovative Computing, Information and Control*, vol. 2, pp. 388-391, 2006.
- [9] S. Purwar, I. N. Kar, and A. N. Jha, "Adaptive control of robot manipulators using fuzzy logic systems under actuator constraints," *Fuzzy Sets and Systems*, vol. 152, pp. 651-664, 2005.
- [10] C. Liu, C. C. Cheah, and J. J. E. Slotine, "Adaptive Jacobian tracking control of rigid-link electrically driven robots based on visual task-space information," *Automatica*, vol. 42, pp. 1491-1501, 2006.
- [11] M. W. Spong, *Motion Control of Robot Manipulators*, in *Handbook of Control*, W. Levine, editor, CRC Press, pp. 1339-1350, 1996.
- [12] J. V. Miro and A. S. White, "Modelling an industrial manipulator a case study," *Simulation Practice and Theory*, vol. 9, pp. 293-319, 2002.
- [13] F. Reyes and R. Kelly, "Experimental evaluation of model-based controllers on a direct-drive robot arm," *Mechatronics*, vol. 11, pp. 267-282, 2001.
- [14] M. W. Spong and M. Vidyasagar, *Robot Dynamics and Control*, John Wiley and Sons, Inc, 1989.
- [15] R. Kelly, V. Santibiflez, and H. Berghuis, "Point-to-point robot control under actuator constraints," *Control Engineering Practice*, vol. 5, no. 1, pp. 1555-1562, 1997.
- [16] T. J. Tarn, A. K. Bejczy, G. T. Marth, and A. K. Ramadorai, "Performance comparison of four manipulator servo schemes," *IEEE Control Systems Magazine*, vol. 13, no. 1, pp. 22-29, Feb. 1993.
- [17] M. B. JR Leahy, and G. N. Saridis, "Compensation of industrial manipulator dynamics," *The International Journal of Robotics Research*, vol. 8, no. 4, pp. 73-84, 1989.
- [18] P. Chiacchio, F. Pierrot, L. Sciavicco, and B. Siciliano, "Robust design of independent joint controllers with experimentation on a high-speed parallel robot," *IEEE Trans. on Industrial Electronics*, vol. 40, pp. 393-403, 1993.
- [19] L. L. Chang and C. C. Chuan, "Rigid model-based fuzzy control of flexible-joint manipulators," *Journal of Intelligent and Robotic Systems*, vol. 13, pp. 107-126, 1995.
- [20] L. C. Lin and T. E. Lee, "Integrated PID-type learning and fuzzy control for flexible-joint manipulators," *Journal of Intelligent and Robotic Systems*, vol. 18, no. 1, pp. 47-66, 1997.
- [21] J. J. Craig, *Introduction to Robotics, Mechanics and Control*, Addison-Wesley, 1989.
- [22] P. I. Corke and B. Armstrong-Hlouvry, "A search for consensus among model parameters reported for the PUMA 560 robot," *Proc. IEEE Int. Conf. Robotics and Automation*, vol. 1, pp. 1608-1613, San Diego, 1994.
- [23] G. F. Wyeth, J. Kennedy, and J. Lillywhite, "Distributed digital control of a robot arm," *Proc. of the Australian Conf. on Robotics and Automation*, Melbourne, pp. 217-222, August 30 - September 1, 2000.
- [24] P. I. Corke, "Robotics toolbox for MATLAB," *IEEE Robotics & Automation Magazine*, vol. 3, no. 1, pp. 24-32, 1996.



**Mohammad Mehdi Fateh** received the B.S. and M.Sc. degrees in Electrical Engineering from Isfahan University of Technology and Tarbiat Modares University in Iran, respectively. He received the Ph.D. degree in Robotic Engineering from Southampton University, U.K. in 2001. He is with the Department of

Electrical and Robotic Engineering at Shahrood University of Technology in Iran. His research interests include robot control and modelling.