Robust H_∞ Control Method for Bilinear Systems

Beom-Soo Kim and Myo-Taeg Lim

Abstract: In this paper, we investigate a robust H_∞ state feedback control technique for continuous time bilinear systems with an additive disturbance input. The nonlinear robust H_∞ control for bilinear systems requires a solution to the state dependent algebraic Riccati equation (SDARE). We present a new robust H_∞ control technique based on the successive approximation method for solving the SDARE by converting bilinear systems into time-varying linear systems. The proposed control method guarantees robust stability for closed loop bilinear systems. The proposed algorithm is verified by numerical examples.

Keywords: Robust control, H_∞ control, bilinear systems, optimal control, successive approximation.

1. INTRODUCTION

Bilinear systems represent many real physical processes, thus it is important to understand their real properties, guarantee their global stability, improve their performance by applying various control techniques to bilinear systems themselves rather than their linearized systems since the linearization of bilinear systems loses its natural properties. Indeed, some linearization of a nonlinear system around an equilibrium point yields a bilinear system. Mohler [11] presents detailed reviews of bilinear systems and their control design methods.

During the past twenty years, robust H_∞ optimization became one of the most interesting and challenging areas of optimal control and filtering theories and their application. The main advantage of H_∞ optimization comes from the fact that such obtained controllers and filters are robust to internal and external disturbances. The difficulty associated with the robust H_∞ control for nonlinear or bilinear systems, however, is in solving the state dependent H_∞ algebraic Riccati equation (SDARE).

In optimal regulation problems of bilinear systems, only simple cases can obtain optimal control in the explicit feedback form [2]. In finite-time optimal control problem of continuous time bilinear systems, Hofer et al. [7] and Aganovic et al. [1] propose some numerical procedures. Hofer's main results state that finite-time optimal control of bilinear systems is derived by a sequence of differential Riccati equations [7]. Robust stabilization problems for bilinear systems have been widely studied by many researchers [4-6]. Teolis et al. [13] report robust H_∞ output feedback control for bilinear systems using the information state.

In this paper, we present a robust H_∞ state feedback control technique using van der Schaft's [14] results and Kim and Lim's [10] successive approximation method for solving the SDARE. This control technique has the following steps: (i) obtain the stabilizing robust control for the linear system constructed by ignoring the multiplicative terms of the bilinear systems; (ii) convert the bilinear systems into the time-varying form by using the result of the previous step and solve the algebraic H_∞ Riccati equation derived by updating the performance index containing the L_2-gain concept and the associated Hamilton-Jacobi-Isaacs (HJI) equation; (iii) iterate step (ii) until the convergence of state is satisfied.

2. PRELIMINARIES

We first introduce some necessary notations and quote some relevant results. Let $R^{n\times 1}$ denote the usual n dimensional vector space and let the norm of a vector $x=[x_1 \cdots x_n]^T$ be denoted by

$$\|x\| = \left(\sum_{i=1}^{n} x_i^2\right)^{1/2}. \qquad (1)$$

The norm of $A \in R^{n\times m}$ is defined by

$$\|A\| = \left(\sum_{i=1}^{n}\sum_{j=1}^{m} a_{ij}^2\right)^{1/2}, \qquad (2)$$

Manuscruipt received ; July 15, 2002 accepted October 25, 2002.

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where a_{ij} is an (i, j) -th element of A and the weighted 2-norm $\|x\|_Q^2$ is defined by $x^T Qx$. The following lemma and corollary are used to prove the convergence of the proposed algorithm (Their proofs can be found in [8]).

Lemma 1 [8]**:** Considering the vector $x \in R^{n\times 1}$ and matrix $M_i \in R^{n\times m}$ with $i = 1, 2, \cdots, n$, then the following relation holds:

$$\sum_{i=1}^{n} x_i M_i = \sum_{i=1}^{n} X_i N_i \tag{3}$$

with

$$N_i = \begin{bmatrix} n_{i1}^1 & n_{i2}^1 & \cdots & n_{im}^1 \\ n_{i1}^2 & n_{i2}^2 & \cdots & n_{im}^2 \\ \vdots & & \ddots & \\ n_{i1}^n & n_{i2}^n & \cdots & n_{im}^n \end{bmatrix} \tag{4}$$

where n_{ij}^k is an (i, j) -th element of the M_k matrix. An i -th row of $X_i \in R^{n\times n}$, $i = 1, 2 \cdots n$ is x^T and the other rows of X_i are zero vectors.

Corollary 1[8]**:** The norm for equation (3) has the relation

$$\left\| \sum_{i=1}^{n} x_i M_i \right\| < \|x\| \sum_{i=1}^{n} \|N_i\| \tag{5}$$

where N_i is defined in equation (4).

We consider the robust H_∞ control problem of continuous time bilinear systems with an additive disturbance given by

$$\begin{aligned} \dot{x} &= Ax + \left(B + \sum_{i=1}^{n} x_i M_i \right) u + Ew, \quad x(t_0) = x_0, \\ z &= Cx + Du, \end{aligned} \tag{6}$$

where $x \in R^{n\times 1}$ represents a state vector, $u \in R^{m\times 1}$ is a control vector, $w \in R^{l\times 1}$ denotes the disturbance input with $w \in L_2\left[t_0, t_f\right]$, $z \in R^{q\times 1}$ is the penalty function to be used in the cost function, and $A, B, C, D, E,$ and M_i are constant matrices of appropriate dimensions.

van der Schaft's nonlinear robust H_∞ control problem [14] designs a controller that robustly stabilizes the closed loop in the sense of the L_2 -gain concept since the H_∞ norm on the transfer function matrix of linear systems equals the L_2 -induced system norm on its impulse response matrix [12]. Thus, the nonlinear robust H_∞ control guarantees that the performance index shown in equation (7) below remains within an upper bound for a given positive number .

$$\begin{aligned} J(x(t_o), u, w, t_0, t_f) &= \frac{1}{2} \left\| x(t_f) \right\|_s^2 \\ &+ \frac{1}{2} \int_{t_o}^{t_f} \left\{ \|z\|^2 - \gamma^2 \|w\|_W^2 \right\} dt, \end{aligned} \tag{7}$$

with $t_0 \le t_f \le \infty$ and $S, W > 0$. The Hamiltonian H corresponding to the system in equation (6) and performance index in equation (7) is

$$\begin{aligned} H &= \frac{1}{2} \left\{ \|z\|^2 - \gamma^2 \|w\|_W^2 \right\} \\ &+ \lambda^T \left[Ax + Ew + \left(B + \{xM\} \right) u \right], \end{aligned} \tag{8}$$

where $\{xM\} = \sum_{i=1}^{n} x_i M_i$ and $\lambda \in R^{n\times 1}$ is the Lagrangian multiplier. Then the optimal control is

$$u = -R^{-1} \tilde{B}^T \frac{\partial J(x)}{\partial x}, \tag{9}$$

and the worst disturbance input is

$$w = \frac{1}{\gamma^2} W^{-1} E^T \frac{\partial J(x)}{\partial x}, \tag{10}$$

where $\tilde{B} = B + \sum_{i=1}^{n} x_i M_i$, and $J(x)$ satisfies the following HJI equation:

$$\begin{aligned} 0 &= \frac{1}{2} x^T C^T Cx - \frac{1}{2} \left(\frac{\partial J}{\partial x} \right)^T \tilde{B} R^{-1} \tilde{B}^T \frac{\partial J}{\partial x} \\ &+ \frac{1}{2\gamma^2} \left\| W^{-1} E^T \frac{\partial J}{\partial x} \right\|_W^2 + \left(\frac{\partial J}{\partial x} \right)^T Ax, \end{aligned} \tag{11}$$

under the assumption of $D^T C = 0$ and $R = D^T D > 0$ [12].

3. MAIN RESULTS

The HJI equation shown in equation (11) has the solution $J = \frac{1}{2} x^T Px$ in the sense of the Lyapunov function and the L_2 -gain stability [14]. By substituting $J = \frac{1}{2} x^T Px$ into the HJI equation (11), we get the following SDARE:

$$0 = C^T C + PA + A^T P - P\left(\tilde{B}R^{-1}\tilde{B}^T - \frac{1}{\gamma^2} EW^{-1}E^T \right)P, \quad (12)$$

under the assumption that W is a unitary matrix. Using the solution to the SDARE and $J = \frac{1}{2}x^T Px$, we can obtain the optimal control as

$$u = -R^{-1}\tilde{B}^T Px, \quad (13)$$

which is necessary to implement the robust H_∞ state feedback controller for the bilinear systems. Since the SDARE includes the unknown state variable x, we adopt the successive approximation method for solving the functional equation of dynamic programming [1,9]. The proposed algorithm is composed of the following steps under the assumption that

$$\left| w(t) \right| \le W_m, \quad (14)$$

for the known bound $0 < W_m < \infty$.

Step 1: Find $P^{(0)}$, which is the solution of the Riccati equation with $J^{(0)} = \frac{1}{2}x^{(0)^T} P^{(0)} x^{(0)}$,

$$0 = C^T C + P^{(0)}A + A^T P^{(0)} - P^{(0)}(BR^{-1}B^T - \frac{1}{\gamma^2}EW^{-1}E^T)P^{(0)}, \quad (15)$$

corresponding to the linear constraint with an additive disturbance,

$$\dot{x}^{(0)} = Ax^{(0)} + Bu^{(0)} + EW_m,\ x^{(0)}(t_0) = x_0, \quad z^{(0)} = Cx^{(0)}Du^{(0)}. \quad (16)$$

Then we can obtain the stabilizing linear robust H_∞ control

$$u^{(0)}\left(x(t)\right) = -R^{-1}B^T P^{(0)} x^{(0)}\left(t\right). \quad (17)$$

Step 2: Convert the bilinear system shown in equation (6) into the following linear time varying form by using the result of Step 1.

$$\dot{x}^{(k)} = Ax^{(k)} + \tilde{B}^{(k-1)}u^{(k)} + EW_m, \quad z^{(k)} = Cx^{(k)} + Du^{(k)}, \quad \tilde{B}^{(k-1)} = B + \sum_{i=1}^{n} x_i^{(k-1)} M_i, \quad k = 1, 2, \cdots. \quad (18)$$

The initial value and the disturbance input are frozen in this iteration step. Then we can find the expression for $\left(\frac{\partial J}{\partial x}\right)^{(k)}$ from the following performance index and the partially frozen Hamiltonian.

$$J^{(k)}\left(x^{(k)}, u^{(k)}, w, t_0, t_f\right) = \frac{1}{2}\left\| x\left(t_f\right)\right\|_s^2 + \frac{1}{2}\int_{t_0}^{t_f} \left\{ \left\| z^{(k)} \right\|^2 - \gamma^2 \left\| w \right\|^2 w \right\} dt, \quad (19)$$

$$H = \frac{1}{2}\left\{ \left\| z^{(k)} \right\|^2 - \gamma^2 \left\| w \right\|_W^2 \right\} + \left(\frac{\partial J}{\partial x}\right)^{(k)^T} \left[Ax^{(k)} + Ew + \tilde{B}^{(k-1)}u^{(k)} \right]. \quad (20)$$

From equation (20) and a solution, $J^{(k)} = \frac{1}{2}x^{(k)^T} P^{(k)} x^{(k)}$, of the corresponding HJI equation, the algebraic H_∞ Riccati equation becomes

$$0 = -P^{(k)}(\tilde{B}^{(k-1)}R^{-1}\tilde{B}^{(k-1)^T} - \frac{1}{\gamma^2}EW^{-1}E^T)P^{(k)} + P^{(k)}A + A^T P^{(k)} + C^T C. \quad (21)$$

By solving equation (21), one can obtain the optimal control as

$$u^{(k)}\left(x(t)\right) = -R^{-1}\tilde{B}^{(k-1)^T} P^{(k)} x^{(k)}(t). \quad (22)$$

Step 3: To obtain the stabilizing robust state feedback control subjected to an L_2-gain for closed loop bilinear systems, iterate Step 2 by increasing $k = k + 1$ until the inequality

$$\left\| x^{(k)}\left(t\right) - x^{(k-1)}\left(t\right) \right\| \le \beta \quad (23)$$

is satisfied for a given small positive parameter β.

4. CONVERGENCE PROOF

The convergence proof for the proposed algorithm consists of two parts: (i) find the differences of $x^{(k+1)}(t) - x^{(k)}\left(t\right)$; (ii) develop the inequality that relates to this iterative relation by using norms.

By substituting equation (22) into equation (18), one can write the following identity, which relates the error of two consecutive iterations:

$$\frac{d}{dt}(x^{(k+1)}(t) - x^{(k)}(t)) = \overline{A}^{(k)}\left(x^{(k+1)}\left(t\right) - x^{(k)}\left(t\right)\right) + \left(\overline{A}^{(k)} - \overline{A}^{(k-1)}\right)x^{(k)}(t), \quad (24)$$

where $\overline{A}^{(k)} = A - \tilde{B}^{(k-1)} R^{-1} \tilde{B}^{(k-1)^T} P^{(k)}$. Then the solution of equation (24) is

$$x^{(k+1)}(t) - x^{(k)}(t) = \Psi(t)\int_{t_0}^{t} \Psi^{-1}(s)(\overline{A}^{(k+1)} - \overline{A}^{(k)}) \times\left(\varphi(s)\int_{t_0}^{s}\varphi^{-1}(\tau)Ew(\tau)d\tau\right)ds + \Psi(t)\int_{t_0}^{t}\Psi^{-1}(s)\left(\overline{A}^{(k+1)} - \overline{A}^{(k)}\right)x^{(k)}(s)ds, \quad (25)$$

where the initial condition for $x^{(k+1)}(t_0) - x^{(k)}(t_0) = 0$ and $\Psi(t)$ is the fundamental matrix of system $\Psi(t) = \overline{A}^{(k+1)}\Psi(t)$, satisfying $\Psi(t_0) = I$ of the dynamic equation shown in equation (24). Taking the norm of both sides of equation (25) and using the norm properties, we get

$$\left\|x^{(k+1)} - x^{(k)}\right\| \le \int_{t_0}^{t} \alpha_1 \left\|\overline{A}^{(k+1)} - \overline{A}^{(k)}\right\| ds, \quad (26)$$

where

$$\alpha_1 = \left\|\Psi(t)\Psi^{-1}(s)\right\|(\left\|\varphi(s)x_0\right\| + \left\|\varphi(s)\int_{t_0}^{s}\varphi^{-1}(\tau)Ew(\tau)d\tau\right\|), \quad (27)$$

and

$$\overline{A}^{(k+1)} - \overline{A}^{(k)} = (S^{(k)} - S^{(k-1)})P^{(k+1)} + S^{(k-1)}\left(P^{(k+1)} - P^{(k)}\right), \quad (28)$$

with $S^{(k)} = -\tilde{B}^{(k)} R^{-1} \tilde{B}^{(k)^T}$. $\varphi(t)$ is the fundamental matrix for $\dot{\varphi}(t) = \overline{A}^{(k)}\varphi(t)$ of the dynamic system $\dot{x}^{(k)}(t) = \overline{A}^{(k)}x^{(k)}(t) + Ew(t)$. Using Lemma 1 and Corollary 1, the norm of $S^{(k+1)} - S^{(k)}$ and $S^{(k-1)}$ have the following relations:

$$\left\|S^{(k+1)} - S^{(k)}\right\| < \left\|BR^{-1}\right\|\left\|\left\{\left(x^{(k)} - x^{(k-1)}\right)M\right\}^T\right\| + \left\|\left\{\left(x^{(k)} - x^{(k-1)}\right)M\right\}\right\|\left\|R^{-1}B^T\right\| + \left\|\left\{\left(x^{(k)} - x^{(k-1)}\right)M\right\}\right\|\left\|R^{-1}\right\|\left\|\left\{x^{(k)}M\right\}^T\right\| + \left\|\left\{x^{(k-1)}M\right\}\right\|\left\|R^{-1}\right\|\left\|\left\{\left(x^{(k)} - x^{(k-1)}\right)M\right\}^T\right\|^T \le s_1\left\|R^{-1}\right\|\left\|x^{(k)} - x^{(k-1)}\right\|,$$

$$\left\|S^{(k-1)}\right\| \le \left\|R^{-1}\right\| s_2,$$

where

$$\begin{aligned} s_1 &= \|B\| M^\Sigma + M^\Sigma \left\|B^T\right\| + M^\Sigma \left\|x^{(k)}\right\| M^\Sigma + \left\|x^{(k-1)}\right\| M^\Sigma M^\Sigma, \\ s_2 &= \|B\|\left\|B^T\right\| + \|B\|\left\|x^{(k-1)}\right\| M^\Sigma + \left\|x^{(k-1)}\right\| M^\Sigma \left\|B^T\right\| + \left\|x^{(k-1)}\right\| M^\Sigma \left\|x^{(k-1)}\right\| M^\Sigma, \\ M^\Sigma &= \sum_{i=1}^{n}\|M_i\|. \end{aligned} \quad (29)$$

Hence,

$$\begin{aligned} \left\|x^{(k+1)} - x^{(k)}\right\| &\le \int_{t_0}^{t}(m_1\left\|x^{(k)} - x^{(k-1)}\right\| + m_2\left\|P^{(k+1)} - P^{(k)}\right\|)ds, \\ m_1 &= \alpha_1\left\|R^{-1}\right\|\left\|P^{(k+1)}\right\| s_1, \\ m_2 &= \alpha_1\left\|R^{-1}\right\| s_2. \end{aligned} \quad (30)$$

Using the results of Bruni et al. [3], we get

$$\lim_{k\to\infty}\left\|x^{(k+1)} - x^{(k)}\right\| = 0, \quad (31)$$

which also implies that

$$\lim_{k\to\infty}\left\|P^{(k+1)} - P^{(k)}\right\| = 0. \quad (32)$$

5. NUMERICAL EXAMPLES

The proposed control scheme is illustrated by the following numerical examples.

Example 1: The bilinear model of a chemical reactor [1] is given by

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = A\begin{bmatrix}x_1\\ x_2\end{bmatrix} + (B + x_1M_1 + x_2M_2)u + Ew,$$

$$z = Cx + Du,$$

where

$$A = \begin{bmatrix}3/16 & 5/12\\ -50/3 & -8/3\end{bmatrix},\ B = \begin{bmatrix}-1/8\\ 0\end{bmatrix},\ E = \begin{bmatrix}0\\ 1\end{bmatrix},$$

$$M_1 = \begin{bmatrix}-1\\ 0\end{bmatrix},\quad M_2 = \begin{bmatrix}0\\ 0\end{bmatrix},\quad W = [1],$$

$$D = \begin{bmatrix}1\\ 0\end{bmatrix},\quad C = \begin{bmatrix}0 & 0\\ 1 & 2\end{bmatrix},\ F = \begin{bmatrix}10 & 0\\ 0 & 10\end{bmatrix},$$

$$Q = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\quad x_0 = \begin{bmatrix}1\\ 0\end{bmatrix},$$

and x_1 and x_2 represent the temperature and concentration of a chemical reaction while u represents the coolant flow rate around the reactor. We choose $\gamma = 0.5$ and the disturbance input as $w = 3$ for $t = 0.5, 2.5, 4.5...$ and $w = 0.8 \times \sin(5\pi t)$ otherwise.

In this example, the matrix A has stable eigenvalues $-1.2396 + i2.2154$ and $-1.2396 - i2.2154$. The optimal state and control trajectories are shown in Figs. 1, 2, and 3. In the figures, the dashed lines represent the initial trajectories, the dotted lines the first iteration trajectories, the dashed-and-dotted lines the second iteration, and the solid lines the third iteration.

In the case of stable bilinear systems with additive disturbance, the equilibrium state approaches 0.

Example 2: We consider robust H_∞ control for unstable bilinear systems with additive disturbance input.

$$A = \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 9 \\ 1 \end{bmatrix}, M_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, M_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, x_0 = \begin{bmatrix} 4 \\ 0 \end{bmatrix}.$$

The matrix A has a stable eigenvalue at -3 and an unstable eigenvalue at 1. The other parameters are the same as those of Example 1.

The simulation results are shown in Figs. 4, 5, and 6. Observe that the states and input of the unstable bilinear systems with additive disturbance oscillate around the equilibrium point. Figs. 1-6 show that the proposed algorithm has good convergence for both stable and unstable bilinear systems with additive disturbance.

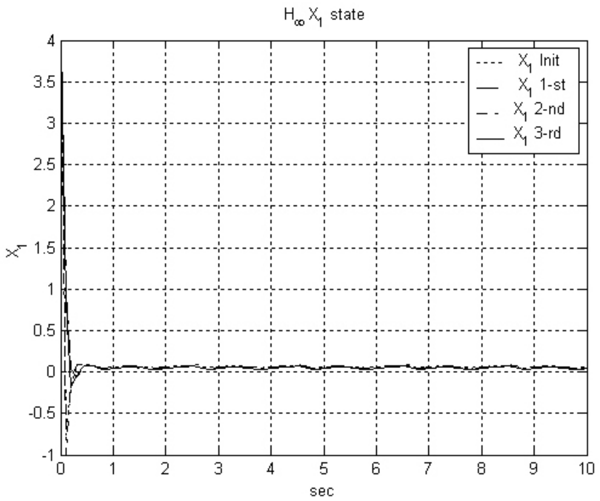

Fig. 1. Trajectories of temperature (Example 1).

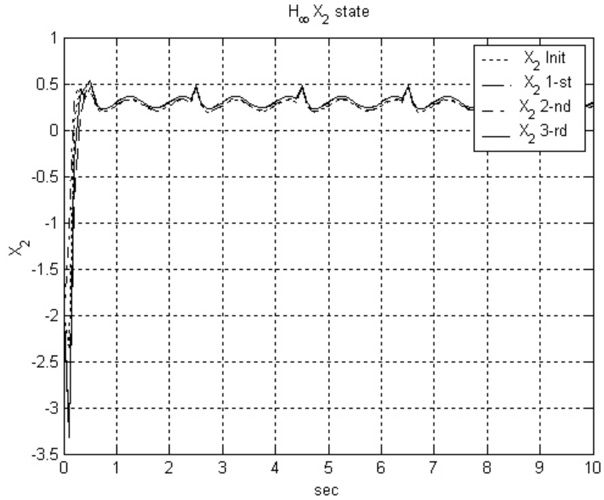

Fig. 2. Trajectories of concentration. (Example 1).

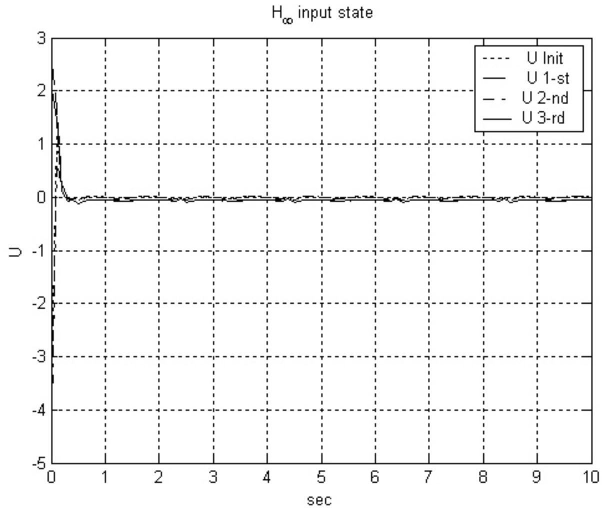

Fig. 3. Trajectories of input. (Example 1).

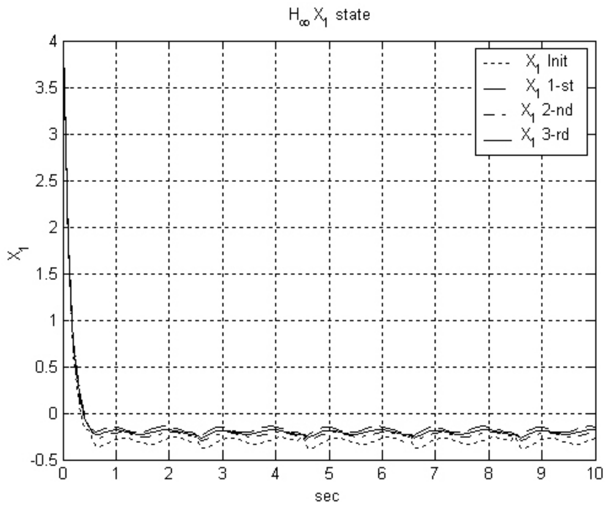

Fig. 4. Trajectories of temperature (Example 2).

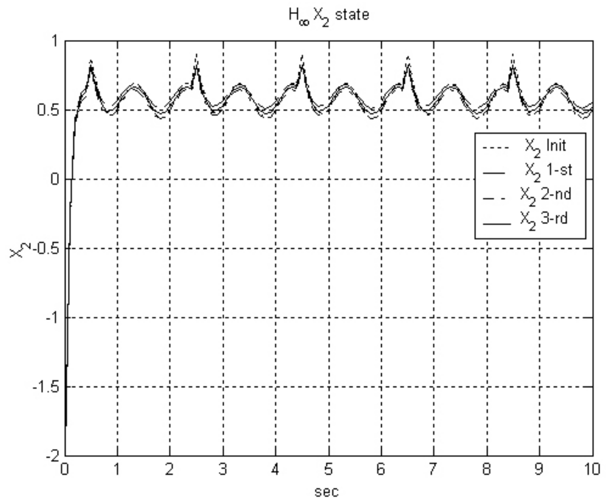

Fig. 5. Trajectories of concentration (Example 2).

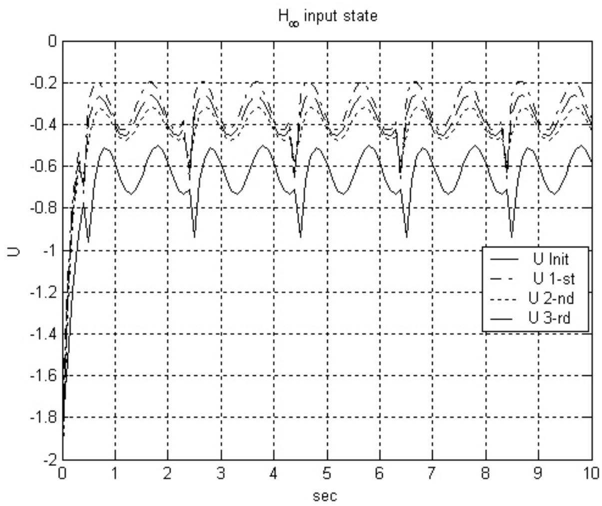

Fig. 6. Trajectories of input (Example 2).

6. CONCLUSIONS

In this paper, we propose a new robust H_∞ control technique for bilinear systems with additive disturbance. The proposed algorithm, based on the successive approximation method, offers a simple method for solving the SDARE. We also provide the proof of convergence. The effectiveness and convergence of the proposed algorithm is demonstrated in the numerical examples and indicates that this robust control scheme is applicable to bilinear systems.

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