The RTD Measurement on a Submerged Bio-Reactor using a Radioisotope Tracer and the RTD Analysis

Seungkwon Shin, Jongbum Kim, Sunghee Jung, and Joonha Jin

Abstract : This paper presents a residence time distribution (RTD) measurement method using a radioisotope tracer and the estimation method of RTD model parameters to analyze a submerged bio-reactor. The mathematical RTD models have been investigated to represent the flow behavior and the existence of stagnant regions in the reactor. Knowing the parameters of the RTD model is important for understanding the mixing characteristics of a reactor. The radioisotope tracer experiment was carried out by injecting a radioisotope tracer as a pulse into the inlet of the reactor and recording the change of its concentration at the outlet of the reactor to obtain the experimental RTD response. The parameter estimation was performed by the Levenberg-Marquardt optimization algorithm. The proposed scheme allowed the parameter estimation of RTD model suggested by Adler-Hovorka with very low deviations. The estimation procedure is shown to lead to accurate estimation of the RTD parameters and to a good agreement between experimental and simulated response.

Keywords : Residence time distribution, radioisotope tracer, Adler-Hovorka model, Levenberg-Marquardt's optimization.

1. INTRODUCTION

The residence time distribution (RTD) function was defined by Danckwerts using the application of the population balance principle [1]. The knowledge of the RTD function is very important because it provides information about the fraction of the fluid that spends a certain time in a reactor. A considerable bypassing flow is an indication of poor design, and residual stagnant regions may serve to cut down the effective or useful volume of the reactor. Therefore, a lot of RTD models have been investigated for the last decade as a useful means of understanding mixing characteristics, such as a bypassing flow or the amount of stagnant regions in a reactor [1-4]. The radioisotope tracer is one of the efficient methods for RTD function determination especially in waste water treatment facilities where the waste water is opacified and its color is very dark.

This paper describes a method of obtaining the parameters of RTD model that represents the flow behavior and the mixing characteristics of a reactor. The numerical approach allows the implementation of time domain based parameter estimation for the evaluation of RTD model parameters. Newton's method [5], one of the least squares estimation methods, has several drawbacks that can cause numerical difficulties.

This paper suggests that drawbacks of Newton's method have been improved by introducing Marquardt's constant [6]. In addition, by fitting a RTD model response to the RTD response obtained from the radioisotope tracer experiment, the submerged bio-reactor is analysed and diagnosed.

2. RTD MODEL

A lot of mathematical RTD models have been developed to represent RTD characteristics of a reactor. The RTD model with a stagnant region will be used in this paper to know the stagnant region that may exist the submerged bio-reactor.

2.1. Model with a stagnant region

This model was suggested by Adler and Hovorka [1] and is sketched in Fig. 1.

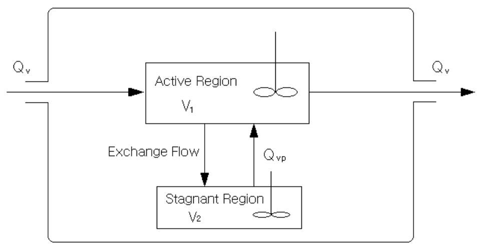

Fig. 1. Model with a stagnant region.

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This model has a slow exchange or cross flow between the fluid in the stagnant region and the active fluid passing through the reactor. The stagnant region is considered as a perfect mixer and slowly interchanges fluid with the active flow region. This model is expressed as the following differential equation form [4].

$$V_1 \frac{dc_1(t)}{dt} = Q_v \delta(t) + c_2(t) Q_{vp} - c_{1(t)} \left[Q_v + Q_{vp} \right], \quad (1)$$

$$V_2 \frac{dc_2(t)}{dt} = Q_{vp} \left[c_1(t) - c_2(t) \right] \quad (2)$$

where V_1 is the volume of the active region (m^3), Q_v is the flow rate of the main flow (m^3/s), V_2 is the volume of the stagnant region (m^3), and Q_{vp} is the flow rate of the exchange flow (m^3/s).

The above equations can be rewritten by dividing (1) by Q_v and (2) by Q_{vp}.

$$t_1 \frac{dc_1(t)}{dt} = \delta(t) + f c_2(t) - (1+f) c_1(t), \quad (3)$$

$$t_2 \frac{dc_2(t)}{dt} = c_1(t) - c_2(t). \quad (4)$$

The t_1 in (3) represents the mean residence time of the active region.

$$t_1 = \frac{V_1}{Q_v} = \frac{\bar{t}}{(1+\alpha)}. \quad (5)$$

The $\bar{t}$ in (5) is the mean residence time of this model and the α represents the volume ratio between the active region and the stagnant region : ($\alpha = \frac{V_2}{V_1}$).

The t_2 in (4) represents the mean residence time of the stagnant region.

$$t_2 = \frac{V_2}{Q_{vp}} = \frac{\alpha}{f} t_1. \quad (6)$$

The f in (6) represents the flow rate ratio between the main flow and the exchange flow: ($f = \frac{Q_{vp}}{Q_v}$).

2.2. Optimization algorithm

The Levenberg-Marquardt Algorithm (LMA) is applied to determine the optimal parameter set of the RTD model with a stagnant region. The error vector $\boldsymbol{F}(k)$ is defined as

$$\boldsymbol{F}_{(k)} = (\boldsymbol{Y}^s - \boldsymbol{Y}^c(k)) \quad (7)$$

where $Y^c(k)$ is the RTD model response obtained from the k-th parameter vector $\boldsymbol{p}(k)$ and $\boldsymbol{Y}^s$ is the output response obtained from an experiment. $\boldsymbol{F}(k)$ can be linearly approximated by Taylor's expansion to compute the ($k+1$)-th parameter vector $\boldsymbol{p}(k+1)$ from the k-th parameter vector $\boldsymbol{p}(k)$.

$$\boldsymbol{F}(k+1) = \boldsymbol{F}(k) + \boldsymbol{J}(k)\left[\boldsymbol{p}(k+1) - \boldsymbol{p}(k)\right] \quad (8)$$

where $\boldsymbol{J}(k)$ is a gradient vector of $\boldsymbol{F}(k)$.

The cost function $f(k+1)$ is defined by using linear approximation of $\boldsymbol{F}(k+1)$

$$\boldsymbol{f}(k+1) = \frac{1}{2} \boldsymbol{F}(k+1)^T \boldsymbol{F}(k+1). \quad (9)$$

$F(k+1)$ in (9) is substituted for (8) and then the quadratic form of (9) becomes

$$f(k+1) = \frac{1}{2}\left[F(k) + J(k)\Delta p(k)\right]^T \left[F(k) + J(k)\Delta p(k)\right] \quad (10)$$

where $\Delta p(k) = p(k+1) - p(k)$.

To calculate the gradient vector of $f(k+1)$, it is differentiated with respect to $\Delta p(k)$

$$\nabla f(k+1) = J(k)^T F(k) + J(k)^T J(k) \Delta p(k). \quad (11)$$

We set $\nabla f(k+1) = 0$ in (11) to determine $p(k+1)$.

$$\Delta p(k) = -\left[J(k)^T J(k)\right]^{-1} J(k)^T F(k). \quad (12)$$

Using the relation of

$$\Delta p(k) = p(k+1) - p(k),$$

$p(k+1)$ becomes

$$p(k+1) = p(k) - \left[J(k)^T J(k)\right]^{-1} \left[J(k)^T F(k)\right]. \quad (13)$$

The Levenberg-Marquardt Algorithm (LMA) adds λ to (13) to improve stability and convergence.

$$p(k+1) = p(k) - \left[J(k)^T J(k) + \lambda(k) I\right]^{-1} \left[J(k)^T F(k)\right] \quad (14)$$

where $\lambda(k)$ is Marquardt's constant and I is an identity matrix. This LMA is executed by the following steps.

Step 1: Set the iteration index as $k = 0$ and select the parameter vector $p(0)$ with an engineering point view. Also, select a tolerance ε as the stoping criterion and $\lambda(0)$ as a constant.

Step 2: Calculate the RTD model response $Y^c(k)$ using the k-th parameter vector $p(k)$, and then calculate the error function $F(k)$ and the cost function $f(k)$.

Step 3: Calculate the gradient vector $J(k)$ of the error function and calculate the ($k+1$)-th parameter vector by (14).

Step 4: If $\|\Delta p(k)\| < \varepsilon$, stop the iterative process. Otherwise, continue.

Step 5: If $f(k+1) < f(k)$, go to Step 6. Otherwise, let $\lambda(k) = 10\lambda(k)$ and go to Step 3.

Step 6: Set $\lambda(k+1) = 0.1\lambda(k)$, let the iteration index be $k = k+1$, and go to Step 2.

3. RADIOISOTOPE TRACER EXPERIMENT

The waste water treatment facility consists of six compartments as shown in Fig. 3. A radioisotope tracer experiment was carried out in the submerged bio-reactor processing a dye-waste water. This paper investigated the first of six compartments. The volume of the submerged bio-reactor is 273.3 (m^3) and the dye-waste water is flowing into the reactor.

3.1. Tracer experiment

Approximately 20 (mCi) of ^{131}I as a tracer was instantaneously injected at the inlet of the submerged bio-reactor. To measure the change of the radioisotope tracer concentration by detecting gamma radiation emitted from the tracer, NaI (Tl) scintillation detectors (SPA-3, Eberline) were installed at the outlets of each compartment.

3.2. Experimental equipments

The NaI(Tl) scintillation detector (SPA-3, Eberline) was used to measure gamma radiation. The gamma radiation is converted into visible light by the scintillation, and the light is converted into electric

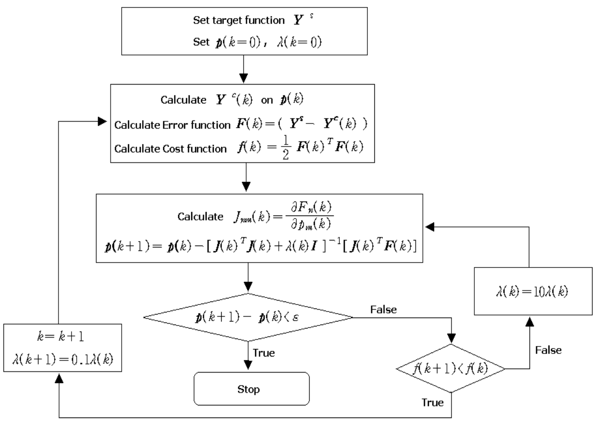

Fig. 2. represents the flow chart of the optimization algorithm using the LMA.

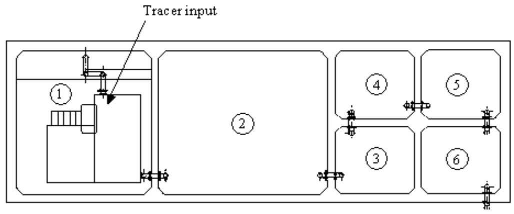

Fig. 3. The waste water treatment facility.

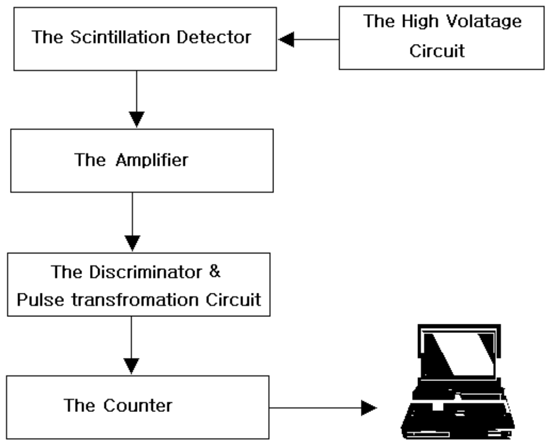

Fig. 4. represents for the measurement device for γ-radiation.

signals by the photo-multiplier. The signals from the photo-multiplier are amplified and transformed into TTL-level pulses by the discriminator and pulse transformation circuit. Then these pulses are counted by the rate meter for a preset counting time and transferred into a notebook computer via serial communication (RS-232C).

4. EXPPERIMENT RESULTT AND RTD SIMULATION

The experimental data was recorded with a sampling time of 60 [s] and the number of recorded data was 3184. To use the experimental data for the RTD analysis of the bio-reactor, the background radiation and the spontaneous decay should be corrected.

4.1. Data correction

The radioisotope tracer concentration obtained from the experiment was corrected to decrease the error arising from the background radiation and the spontaneous decay of the radioisotope. Let $c_m(t)$ be the radioisotope tracer concentration obtained from the experiment and $c_{bc}(t)$ be the radioisotope tracer concentration after subtracting the influence of the background radiation from $c_m(t)$. Then the concentration $c(t)$, which is corrected the effect of spontaneous decay, is given by

$$c(t) = c_{bc}(t) e^{\frac{\ln 2}{T_{half}} t} \tag{15}$$

where $c_{bc}(t) = c_m(t) - c_{\,background}$ and T_{half} is the half-life of the radioisotope.

4.2. Normalization

To compare the RTD response obtained from the experiment with the RTD model response, the radioisotope tracer concentration is subject to normalization.

$$c_0 = \int_0^{\infty} c(t)dt \tag{16}$$

where c_0 is the total area under the concentration- time curve. For idealized instantaneous pulse input, the RTD function or E -curve is then defined as

$$E(t) = \frac{c(t)}{c_0} \tag{17}$$

where the area under the E-curve is equal to one. The mean residence time (MRT) is determined as the first moment [2] and is equivalent to

$$\overline{t} = \frac{\int_0^{\infty} tc(t)dt}{c_0} = \frac{V}{Q}. \tag{18}$$

4.3. Simulation

The radioisotope tracer concentration obtained from the experiment was corrected by (15). Fig. 5 represents the experimental data and the corrected data. In addition, the corrected data was normalized by (17) to compare it with the RTD model response. The RTD model suggested by Adler-Hovorka was used and the differential equation was solved by the four-stage Runge-Kutta method. The parameters of the model is determined by fitting the model response to the response obtained from the experiment. The LMA method was applied to determine optimal parameters.

The initial values were $t_1 = 10000(s), = 10000(s)$, $t_2 = 23000$ (s), $f = 0.4$, and the specified tolerance $\varepsilon = 0.001$ for the stop criterion of RTD simulation. The optimal parameter values are calculated by adjusting the initial parameter values in an iterative procedure. The process continues until the RTD model response matches the output response obtained from the experiment within a specified tolerance. The simulation was iterated forty times to reach convergence. Fig. 6 represents the change in parameters according to the iteration number during the RTD simulation using the LMA. The simulation result is summarized in Table 1.

The RTD simulation result is shown in Fig. 7. The root mean square error between the response of the

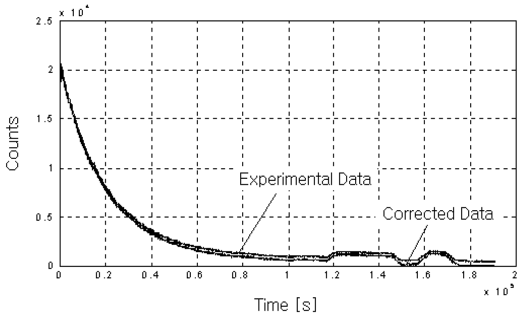

Fig. 5. Comparison between expreimental data and corrected data.

Table 1. The parameters of the RTD model.

MRT t_1	MRT t_2	$f = \frac{Q_{vp}}{Q_v}$
25165 [s]	66347 [s]	0.2943

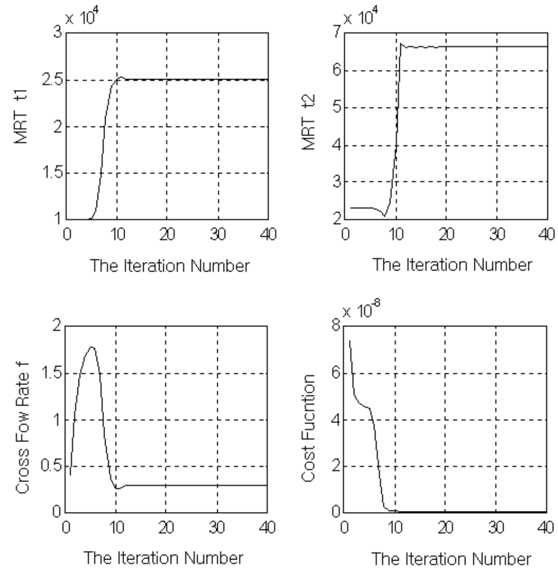

Fig. 6. The parameters change according to the iteration number.

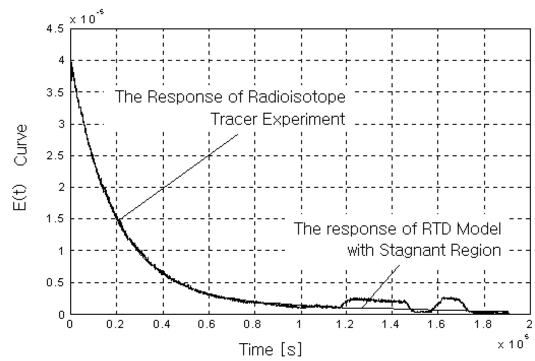

Fig. 7. Comparison between the response of the rtd model and the response of the tracer experiment.

radioisotope tracer experiment and the response of the RTD model is $6.6318*10^{-7}$.
The experimental MRT obtained from the RTD simulation in the submerged bio-reactor is 12.41 (h) from (5). The simulation result shows that the volume ratio between the active region and the stagnant region is 77.59 (%) from (6). This exchange flow is much slower than the main flow, and the flow rate ratio between the main flow and the exchange flow is 0.2943. The bypassing flow was not observed.

5. CONCLUSION

The radioisotope tracer experiment was carried out in the submerged bio-reactor of a wastewater treatment facility to investigate the existence of a stagnant region and the feasibility of bypassing flow. The LMA method was applied to determine the optimal parameters of the RTD model, and was shown to the simulated RTD response fit well in the experimental RTD response with the very low deviation. Also, the submerged bio-reactor was analyzed with the parameters of the RTD model with a stagnant region. This radioisotope tracer method will be effectively utilized in a variety of radioisotope studies.

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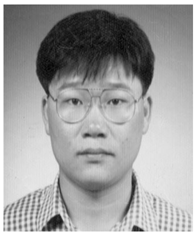

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