Aircraft CAS Design with Input Saturation Using Dynamic Model Inversion

Sangsoo Lim and Byoung Soo Kim

Abstract: This paper presents a control augmentation system (CAS) based on the dynamic model inversion (DMI) architecture for a highly maneuverable aircraft. In the application of DMI not treating actuator dynamics, significant instabilities arise due to limitations on the aircraft inputs, such as actuator time delay based on dynamics and actuator displacement limit. Actuator input saturation usually occurs during high angles of attack maneuvering in low dynamic pressure conditions. The pseudo-control hedging (PCH) algorithm is applied to prevent or delay the instability of the CAS due to a slow actuator or occurrence of actuator saturation. The performance of the proposed CAS with PCH architecture is demonstrated through a nonlinear flight simulation.

Keywords: Stability and control, handling qualities, guidance and control.

1. INTRODUCTION

Flight control system design for fighter aircraft continues to be one of the most demanding problems in the world of automatic control. The magnitude of the problem is driven by the nonlinear and uncertain nature of aircraft dynamics. Linear models of these systems are only valid for small regions in trim conditions. The conventional flight control designs for this problem were to perform point designs for a large set of trim conditions and then construct a gain schedule by interpolating gains with respect to flight conditions. This procedure is time consuming and expensive, but is well accepted and has yielded satisfactory results for many aircraft. A more mathematical response to the issue of gain scheduling had appeared to overcome these disadvantages. In reference 1, the systematic approach for automating gain schedule calculations has been described. This approach guarantees both stability and performance. An alternative to gain scheduling is to use control design methods that directly consider the nonlinear nature of the problem. One such alternative method is dynamic model inversion (DMI). DMI has been applied successfully to a number of flight control problems [2-4].

In this paper, a CAS based on DMI for high performance aircrafts is designed to enable the pilot to directly command the angle of attack (or normal acceleration) and roll rate while ensuring the stability of the airframe [5,6]. In the application of DMI, significant problems of instability arise due to limitations on the aircraft inputs, such as actuator time delay based on slow dynamics and input saturation based on actuator displacement limit. These unstable AOA responses of the CAS which remain untreated in the DMI method have been generated and recently developed by pseudo control hedging (PCH) methodology in references [7,8], and is employed to protect the system from unstable response in the presence of slow actuation, actuator saturation and failures.

This paper is organized as follows. Section 2 presents a design approach for α-CAS based on DMI. Section 3 outlines the application of the PCH algorithm to the proposed CAS. The evaluation of the CAS with PCH through the simulation is summarized in Section 4. Conclusions are given in Section 5.

2. DMI BASED α-CAS

Fig. 1 depicts the architecture of a command augmentation system (CAS) design based on DMI for a high performance aircraft. The CAS is also required to stabilize the airframe while following normal acceleration (a_n) command and roll rate (P) command from the pilot. The CAS is responsible for maintaining zero angle of sideslip (or side acceleration) during maneuvers. The CAS consists of two subsystems, the attitude orientation system and command augmentation logic. The former is an inner stabilization loop responsible for stabilizing the airframe while following the commanded body rates. The latter acts as an outer-loop for

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tracking pilot commands. References 5 and 6 detail the design of the command augmentation logic, the command transformation from body rates to Euler angle rates, and the model inversion.

In the CAS of Fig. 1, the angle of attack (α) or pitch rate (Q) command can be used instead of a_n-command according to the region of flight envelope or configuration change.

In this paper, the focus is on a high-α maneuver therefore, the α-CAS following the α-command (rather than the a_n-command) is described. The α command augmentation logic producing pitch rate command, Q_c from a commanded angle of attack, α_c of the pilot is based on the expression of (1).

$$\dot{W} + VP - UQ \;=\; -a_n g_0 + g\cos\phi\cos\theta\,, \qquad (1)$$

where a_n is the acceleration along the normal direction of the reverse Z-axis of the aircraft body axes. By approximating that $\dot{W} \approx U\dot{\alpha}$, $\dot{\alpha}$ can be expressed from (1) as follows:

$$\dot{\alpha} = \frac{1}{U}(UQ - VP - a_n g_0 + g\cos\phi\cos\theta)\,. \qquad (2)$$

Designate the right hand sides of (2) with pseudo-control variable σ , i.e.,

$$\dot{\alpha} = \sigma. \qquad (3)$$

Proportional plus integral control laws of the form will be used to follow the commanded AOA as in the following (4).

$$\sigma = K_{p\alpha}(\alpha_c - \alpha) + K_{i\alpha}\int_0^t(\alpha_c - \alpha)d\tau\,. \qquad (4)$$

The controller gains ($K_{p\alpha}$ and $K_{i\alpha}$) are to be selected on the basis of the speed of AOA response and tracking error. The required pith rate to follow a specified α-command can be derived from (2) using (3) and (4).

$$Q_c = K_{p\alpha}(\alpha_c - \alpha) + K_{i\alpha}\int(\alpha_c - \alpha)d\tau + \frac{1}{U}(VP + a_n g_0 - g\cos\phi\cos\theta). \qquad (5)$$

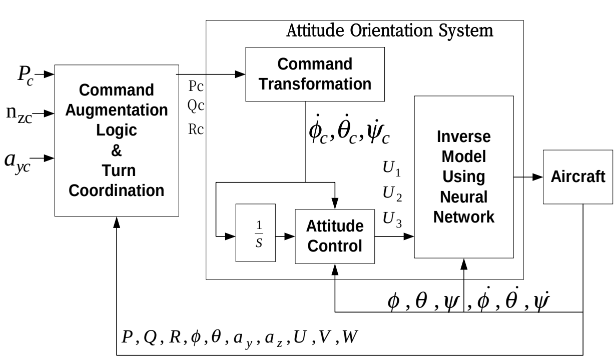

Fig. 1. Command augmentation system.

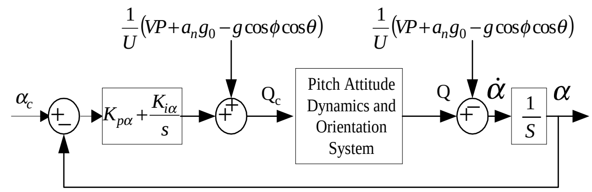

Fig. 2. Outer-loop of α-CAS for stability analysis.

It is usual to design the inner-loop dynamics of the attitude orientation system to be much faster than that of the outer-loop command augmentation tracking loop. Moreover it is assumed that the commanded body angular rates are exactly followed. These assumptions within the bounds of an infinitely fast and stable inner-loop (i.e., a perfect inner-loop is assumed) makes outer-loop analysis possible.

Fig. 2 represents the outer-loop of the α-CAS by assuming that the inner-loop is much faster than the outer-loop and thus Q_c=Q.

From Fig. 2, we can obtain the closed-loop transfer function as

$$\frac{\alpha(s)}{\alpha_c(s)} = \frac{K_{p\alpha}s + K_{i\alpha}}{s^2 + K_{p\alpha}s + K_{i\alpha}}\,. \qquad (6)$$

In this study, a 1% settling time of the inner-loop (t_{Qs}) is designed 3 times faster than that of the outer-loop (t_{α_s}). The controller gains are selected to satisfy the Handling Qualities Specification [9]. That is, t_{α_s} = 2.4 sec and t_{Qs} = 0.8 sec correspond to the short period natural frequency and damping ratio of 2.7 (rad /sec) and 0.707, respectively. As a result, controller gains are chosen as

$$K_{p\alpha} = 3.83\ (\mathrm{sec}^{-1}),\quad K_{i\alpha} = 7.35\ (\mathrm{sec}^{-1}). \qquad (7)$$

The only pitch axis is treated from following [6] because this paper is concerned with AOA command augmentation system and AOA input-output.

The attitude orientation system provides stabilization of the airframe and a tracking function of the attitude rates. In order to enable the use of DMI, the vehicle attitude rate commands in pitch axes must be derived as follows:

$$\dot{\theta}_c = Q_c\cos\phi - R_c\sin\phi\,. \qquad (8)$$

This is the Euler angle rate equation. The Q_c is the derived input of the attitude orientation system in (5) and it is transformed into $\dot{\theta}_c$ using (8) to make the composition of (11) possible (Fig. 4).

The attitude orientation system will be designed to track these commands with as little error as possible. Fig. 1 contains the attitude orientation system (i.e., the inner stabilization loop of the command augmentation system). Note that integrating attitude commands are consistent with the Euler angle rate command.

Next, the Euler pitch angle rate equations are differentiated with respect to time yielding.

$$\ddot{\theta} = (\dot{Q} - R\dot{\phi})\cos\phi - (Q\dot{\phi} + \dot{R})\sin\phi\,. \tag{9}$$

The moment equation [10] can next be substituted in these expressions when the vehicle pitch rate on the right-hand side is eliminated. If this step is carried out, the second-order nonlinear differential equation for the Euler pitch angle can be obtained as

$$\ddot{\theta} = F_2 + G_4\delta_a + G_5\delta_e + G_6\delta_r\,. \tag{10}$$

Development of the attitude orientation system is based on the second-order nonlinear differential equation (10). These equations can be transformed to a linear, time-invariant form by designating the right-hand side of this equation by pseudo-control variable v_2. Proportional plus derivative control laws can next be designed for each of these systems as

$$v_2 = k_{p\theta}(\theta_c - \theta) + k_{d\theta}(\dot{\theta}_c - \dot{\theta}) + \ddot{\theta}_c\,. \tag{11}$$

$\ddot{\theta}_c$ is neglected during modeling because it is a negligible value.

(10) is transformed into (12), a set of three moment coefficient nonlinear equations [6], using the relationship in Fig. 3. These transformations enable us to apply the DMI process. In this paper only pitch axes are described but (12) is used as a set of three equations in order to make inversion process description possible.

$$\begin{aligned} C_L &= F_1 + G_1\delta_{a\_cmd} + G_2\delta_{e\_cmd} + G_3\delta_{r\_cmd}\,, \\ C_M &= F_2 + G_4\delta_{a\_cmd} + G_5\delta_{e\_cmd} + G_6\delta_{r\_cmd}\,, \\ C_N &= F_3 + G_7\delta_{a\_cmd} + G_8\delta_{e\_cmd} + G_9\delta_{r\_cmd}\,. \end{aligned} \tag{12}$$

The variables F_1, F_2, F_3, G_1, G_2,…,G_9 denote nonlinear state dependent functions. Based on the above relationship, the nonlinear DMI becomes the relation of moment coefficients (C_L, C_M, C_N) to control surface deflections ($\delta_{a\_cmd}, \delta_{e\_cmd}, \delta_{r\_cmd}$) as follows:

$$\begin{bmatrix} \boldsymbol{\delta}_{\mathbf{a}\_cmd} \\ \boldsymbol{\delta}_{\mathbf{e}\_cmd} \\ \boldsymbol{\delta}_{\mathbf{r}\_cmd} \end{bmatrix} = \begin{bmatrix} G_1 & G_2 & G_3 \\ G_4 & G_5 & G_6 \\ G_7 & G_8 & G_9 \end{bmatrix}^{-1} \begin{bmatrix} C_L - F_1 \\ C_M - F_2 \\ C_N - F_3 \end{bmatrix}. \tag{13}$$

It is difficult to prove mathematically that the G_{ij} 's are invertible because every aircraft has a different G_{ij} value respectively. However, we can figure out that there is a 1:1 correspondence relationship that exists between control surface ($\delta_e, \delta_a, \delta_r$) and actual C_L, C_M, C_N. The $\delta_e, \delta_a, \delta_r$ which have dominant effects along pitch, roll and yaw axes relating to C_L, C_M, C_N, bring G^{-1} into existence.

The specific process for DMI used in this paper is shown in Fig. 3. The second-order nonlinear equations for the Euler angles are transformed to a linear, time invariant form of the three pseudo-control variables (v_1, v_2, v_3). Then, kinematic relations become moment equations (L, M, N). Next, moment equations are transformed to moment coefficient equations. The last step in this process is the commanded control input derivation process, which is the major newly designed logic in this paper.

A detailed block diagram of the pitch pseudo-control loop that controls stability and a part of the attitude orientation system is given in Fig. 4. This block diagram can be reduced to yield the pitch pseudo-control loop transfer function which is a standard form as

$$\frac{\theta(s)}{\theta_c(s)} = \frac{k_{d\theta}s + k_{p\theta}}{s^2 + k_{d\theta}s + k_{p\theta}}\,. \tag{14}$$

It can track a step and a ramp pitch rate commands with zero steady state error. The feedback gains $k_{p\theta}, k_{d\theta}$ can be related to the natural frequency ω_n and the damping ratio ς as

$$k_{p\theta} = {\omega_n}^2\,,\; k_{d\theta} = 2\varsigma\omega_n\,. \tag{15}$$

3. PSEUDO-CONTROL HEDGING ARCHITECTURE

Input saturation presents a significant problem for control. It also implies controllability and invertibility issues during saturation. This temporary loss of control effectiveness violates the necessary conditions for

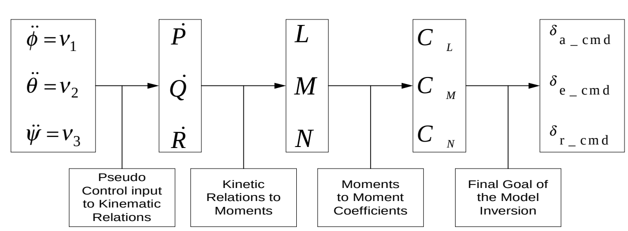

Fig. 3. DMI process.

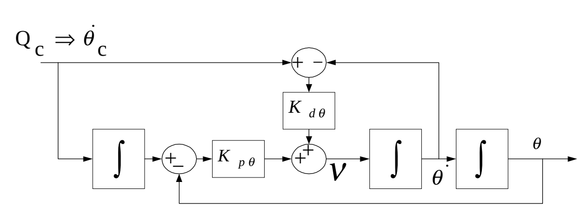

Fig. 4. Pitch orientation pseudo control loop.

DMI. Avoiding saturation is important for systems where the open-loop plants are highly unstable, but otherwise introduces conservatism.

The method described in this section is Pseudo-Control Hedging (PCH) for the CAS based on DMI. The conceptual description of this method is that the PCH signal subtracts the over controlled actuator signal backwards (hedged). This over controlled signal is originated based on input saturation or time-delay in the actuator. The purpose of the PCH is to prevent the aircraft from entering unstable flight conditions while the input is saturated or delayed. Thus, the PCH algorithm is applied only to the attitude orientation system responsible for inner-loop stability of the CAS.

The longitudinal α-CAS with PCH is illustrated in Fig. 5. Where f denotes transformation of commanded pseudo control input (ν) to Proportional plus Derivative control logic.

An approximate dynamic inversion element is developed to determine actuator commands of the form

$$\delta_{cmd} = \hat{f}^{-1}(x, \dot{x}, \nu), \tag{16}$$

where ν is the pseudo-control signal and represents a desired $\ddot{\theta}$ that is expected to be approximately achieved by δ_{cmd}. This DMI block is designed without consideration of the actuator model. This command (δ_{cmd}) will not necessarily equal the actual control (δ) due to the actuator.

The PCH signal (ν_h) is composed of the difference between the commanded pseudo-control input (ν) and the achieved pseudo-control input ($\hat{\nu}$). The commanded pseudo control input (ν) of the output of the PD controller is expressed as follows:

$$\nu = k_{p\theta}(\theta_c - \theta) + k_{d\theta}(\dot{\theta}_c - \dot{\theta}). \tag{17}$$

To obtain the estimated actual pseudo control input ($\hat{\nu}$), reverse calculations from actual elevator deflection angle input (δ) to the pitch angular acceleration ($\ddot{\theta}$) are expressed as follows:

$$\hat{\nu} = \ddot{\theta} = \hat{f}(x, \dot{x}, \delta). \tag{18}$$

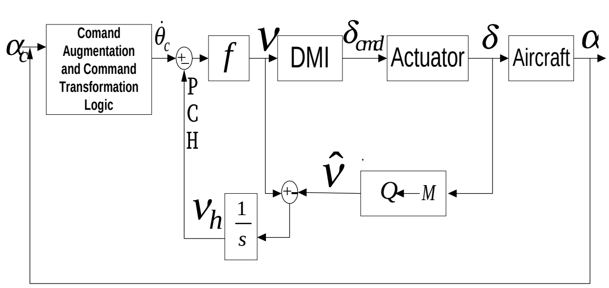

Fig. 5. DMI based α-CAS with PCH compensation.

Therefore, the PCH signal ($\nu_h = \nu - \hat{\nu}$) can be written as follows:

$$\nu_h = \nu - \hat{\nu} = f(x, \dot{x}, \delta_{cmd}) - \hat{f}(x, \dot{x}, \hat{\delta}). \tag{19}$$

This PCH signal should be subtracted from the command signal of $\ddot{\theta}_c$ because the pseudo control signal corresponds to the angular acceleration. However, the attitude orientation system in Fig. 5 uses $\dot{\theta}_c$ as the input instead of $\ddot{\theta}_c$ because this value is negligible. Thus, after integrating this PCH signal (ν_h), it was subtracted (hedged) from $\dot{\theta}_c$.

4. SIMULATION RESULTS AND DISCUSSION

The performance of the α-CAS using the DMI described in previous sections was analyzed through simulation. The F-16 model in [10] was chosen as a mathematical model in the simulation. The simulation focuses only on the longitudinal aircraft motion.

4.1. The influence of slow actuator dynamics

The CAS using DMI is very sensitive to the variance of the actuator dynamics since the actuator dynamics is not considered in the DMI method. To show this, the stabilizer actuator dynamics was modeled as a first order system and two time constants in the actuator model were tested. One (0.02 sec) corresponds to fast actuator dynamics, and the other (0.1 sec) represents slow actuator dynamics.

In this simulation an α-command (α_c = 5 deg) was given to the CAS from the level flight trim condition of Mach 0.55 at an altitude of 10,000 ft. The angle of attack response of the aircraft without the PCH architecture is given in Fig. 6. Fig. 6(a) and 6(b) represent the α-responses with fast and slow actuators, respectively. As shown in Fig. 6, the stability of the CAS using DMI becomes degraded as the actuator dynamics slows down.

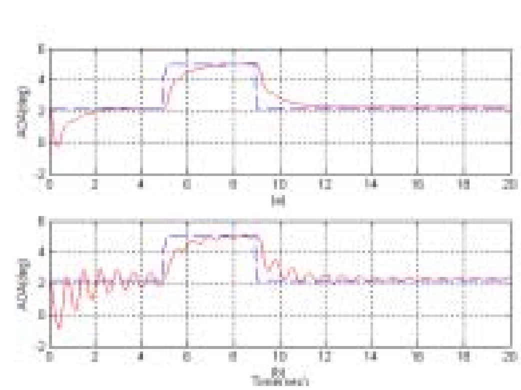

Fig. 6. Effects of slow or fast actuator dynamics.

Fig. 7 depicts the α-response with the compensation of the PCH result in Fig. 6(b). The compensation of the PCH makes the CAS stable despite the slow actuator, as shown in Fig. 7.

4.2. Influence of actuator saturation

Actuator saturation or control surface deflection limit usually occurs in a high α maneuver in a low dynamic pressure region. In the simulation in Fig. 8, a high α-command (α_c = 20 deg) was given to the CAS from the low dynamic pressure trim condition of Mach 0.33 and altitude of 15,000 ft.

As we expected, the stabilizer deflection was saturated with this high α-command input by the pilot.

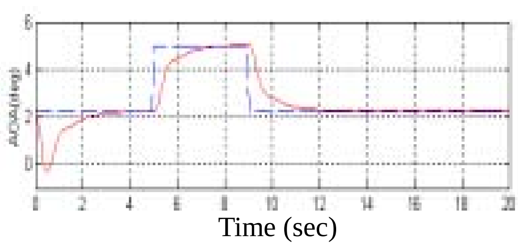

Fig. 7. Compensation of PCH for slow actuator.

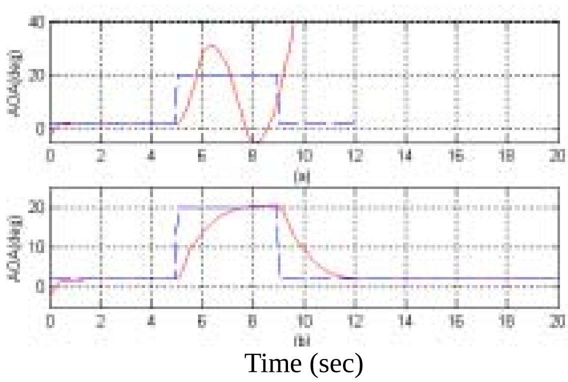

Fig. 8. Effect of PCH in case of actuator saturation.

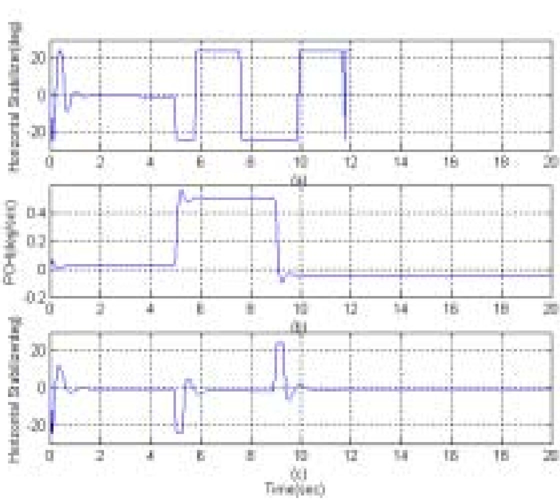

Fig. 9. Stabilizer deflection response with or without PCH.

Fig. 8(a) without PCH shows that the system is extremely unstable due to actuator saturation, while Fig. 8(b) with PCH illustrates that PCH works well even despite actuator saturation.

Fig. 9(a) displays the stabilizer angle corresponding to Fig. 8(a). It is saturated to the displacement limit for a large time period and then demonstrates a bang-bang type response, indicating instability. However, Fig. 9(b) represents the stabilizer angle response with PCH, corresponding to Fig. 8(b). From the PCH signal of Fig. 9(c) and the stabilizer deflections in Fig. 8, it is known that some amount of excessive command input generated in the control loops is subtracted by the PCH signal to prevent or delay the instability of the CAS due to a slow actuator or occurrence of actuator saturation.

6. CONCLUSION

The CAS using DMI has advantages compared to a linear system based CAS, for example, it does not need gain scheduling and is very robust on aircraft considering gravity variation. However, this DMI based CAS makes the closed-loop system very unstable with slow actuator dynamics or in the instance of actuator saturation. In this paper the structure of the α-CAS using DMI is described and the architecture of the PCH is suggested to compensate for the instability tendency related to the actuator. Through the simulation it is demonstrated that the PCH applied to the CAS is very effective to compensate for the slow actuator dynamics or actuator saturation.

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