Morphological Clustering Filter for Wavelet Shrinkage Improvement

Jinsung Oh, Heesoo Hwang, Changhoon Lee, and Younam Kim

Abstract: To classify the significant wavelet coefficients into edge area and noise area, a morphological clustering filter applied to wavelet shrinkage is introduced. New methods for wavelet shrinkage using morphological clustering filter are used in noise removal, and the performance is evaluated under various noise conditions.

Keywords: Morphological filter, wavelet shrinkage, noise reduction.

1. INTRODUCTION

The common method for de-noising in the wavelet domain is to compute the wavelet decomposition of the noisy signal and then to manipulate the wavelet coefficients at each level. The manipulation can be done using hard or soft thresholding methods, called wavelet shrinkage [1-4], in which the significant wavelet coefficients that are supposed to be affected by noise are replaced by zero or another suitable value. Since the wavelet transform allows exceptional localization in both the time and frequency domains, the selective filtering techniques using edge information obtained by the correlation technique [5], the probability density function [6] or zero-crossing [7] can improve the de-noising performance. The de-noising performance depends on finding the location of edges, i.e., separation of significant wavelet coefficients due to edges from wavelet coefficients as result of noise. To classify the significant wavelet coefficients into edge area and noise area, this paper proposes a morphological clustering filter that can be applied to wavelet shrinkage.

2. GEOMETRICAL INTERPRETATION OF WAVELET COEFFICIENTS

Fig. 1 shows the two-dimensional wavelet transform representation of the "House" image using Daubechies wavelet D_3 [8]. As shown in Fig. 1, a coherent edge appearance in the different wavelet domain is observed and the high bands seem to reveal the edge characteristics. As pointed out in [9,10], however, the high bands are mostly speckled or have speckle-like edges with the loss of structured geometry, which means the geometrical filtering techniques on the high bands, for instance, the median filter, are inappropriate since speckled components are removed by the nonlinear filtering technique. Fig. 1 shows the location of significant coefficients with respect to an absolute threshold of 20 in the wavelet domain for the "House" image. As one can see, spatial clustering of significant coefficients in the high bands is observed. Therefore, it is suggested that a morphology-based nonlinear partitioning of the wavelet coefficients into edge area and non-edge area can be applied since the non-clustering area can be easily suppressed by the use of binary mathematical morphology. The first attempt using morphology for the application of image compression has been proposed in [11]. In [11], the regions having significant coefficients (indicating the presence of edges) are obtained by hard thresholding and then dilation operation is applied. Using dilation, the area of the single significant coefficient is enlarged to include neighborhoods, which can alleviate the distortion of edges for reconstruction of the compressed image. However, this simple dilation operation is not useful when the input image is corrupted by noise. In the case of a noisy image, we need to develop a sophisticated morphological operator that can preserve the significant coefficients of the edges with the removal of the significant coefficient due to the noise.

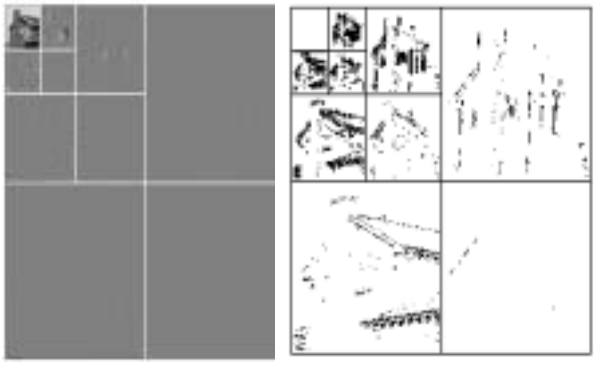

Fig. 1. 2-D wavelet transform of the "House" image and its location of significant coefficients.

Manuscript received May 28, 2002; accepted September 25, 2002.

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3. MORPHOLOGICAL CLUSTERING FILTER

The multiresolution support [3] indicates where the significant wavelet coefficients are and permit the recognition of all information concerning the given images (edge, noise, etc.). The multiresolution support can be easily obtained by thresholding the wavelet coefficients. The multiresolution support at the level l, denoted as S^l, is given by

$$S^l(\underline{n}) = \begin{cases} 1, & |w_l(\underline{n})| > T \\ 0, & |w_l(\underline{n})| \le T \end{cases} \quad (1)$$

where $\underline{n}$ denotes two-dimensional variables, $w_l(\underline{n})$ is the wavelet coefficients at level l, and T is the threshold value. Fig. 2 show a two-dimensional wavelet transform representation of a noisy "House" image corrupted by zero-mean Gaussian noise with a variance of σ_n^2 and its multiresolution support where T is $2\sigma_n$. As can be seen, the location of significant coefficients by the edges remains with the spatial clustering. However, the location of the significant coefficients due to the noise is also detected but randomly distributed. Clearly, the multiresolution support representation has the geometrical information of significant coefficients by the edges, which indicates that geometrical filtering techniques using binary mathematical morphology can be applied to the multiresolution support.

The definition of morphological clustering filter (MCF) applied on the multiresolution support S^l is as follows:

$$\left\{ \max_{i\in(1,\cdots,M)} \left[\left(S^l \Theta B_i \right)(\underline{n}) \right] \right\} \oplus B_F(\underline{n}), \quad (2)$$

where B_i is a flat structuring element in one of four possible directions and B_F is the union of the B_i s (see Fig. 3). Specifically, the morphological clustering filter has a maximum of directional erosions (Θ) followed by dilation ($\oplus$) with the flat structuring element. The directional erosion can remove isolated noises, but preserve directional patterns. And the dilation operation can widen directional area output filtered by erosion. It is shown that these directional morphological filtering techniques [12] remove the noise effectively while preserving detail of the edges.

Using 4 directional structuring elements each with a size of 1x3, the morphological clustering filtered output applied on $S^{LH2}(\underline{n})$ is shown in Fig. 4. As can be seen in Fig. 4, the morphological clustering filter removes the location of the significant coefficients

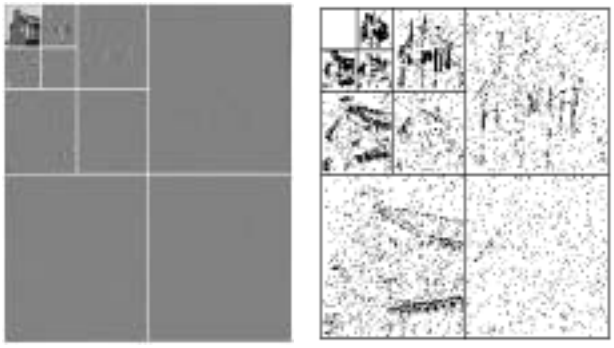
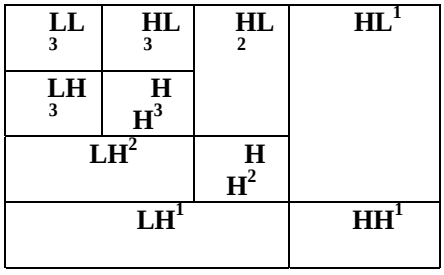


Fig. 2. 2-D wavelet transform of noisy "House" image and its multiresolution support ($T=2\sigma_n$).

where l =



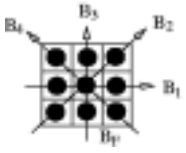


Fig. 3. 4 directional structuring elements each with a size of 1x3.

falsely detected by the noise with the preservation of the spatial clustering of significant coefficients by the edges. Based on this observation, new methods for wavelet shrinkage combined with morphological clustering filter are proposed as follows:

Method 1

(1) Compute the wavelet decomposition of the noisy image. (2) By hard thresholding with threshold T_1, $S^l(\underline{n})$ is obtained. (3) Apply the MCF on $S^l(\underline{n})$:

$$\tilde{S}^l(\underline{n}) = \left\{ \max_{i\in(1,\cdots,M)} \left[\left(S^l \Theta B_i \right)(\underline{n}) \right] \right\} \oplus B_F(\underline{n}).$$

(4) Mani-pulate the wavelet coefficient by $w_l^{new}(\underline{n}) = \tilde{S}^l(\underline{n}) w_l(\underline{n})$. (5) Reconstruct the image by inverse wavelet transform.

When step 3 is omitted, method 1 becomes the hard thresholding technique used in [3, 4].

Method 2

(1) Compute the wavelet decomposition of the noisy

image. (2) By hard thresholding with threshold T_{21}, $S^l(\underline{n})$ is obtained. (3) Apply the MCF on $S^l(\underline{n})$:

$$\tilde{S}^l(\underline{n}) = \left\{ \max_{i\in(1,\cdots,M)} \left[\left(S^l \Theta B_i \right)(\underline{n}) \right] \right\} \oplus B_F(\underline{n}).$$

(4) Mani-pulate the wavelet coefficient by $w_l^{new}(\underline{n}) = \tilde{S}^l(\underline{n}) w_l(\underline{n})$. (5) Re-apply soft thresholding with threshold T_{22}(<T_{21}) on $w_l^{new}(\underline{n})$. (6) Reconstruct the image by inverse wavelet transform.

Note that the soft thresholding used in wavelet shrinkage [2] is given by

$$N_s(w,T) = \mathrm{sgn}(n)\left(|w| - T\right)_+,$$

where $(x)_+$ takes the value x for positive x and zero otherwise.

When step 5 is omitted, method 2 is equivalent to method 1. As can be seen, these new methods can be easily implemented by inserting morphological clustering filtering into the wavelet shrinkage algorithm.

4. SIMULATION RESULTS

The performance of new methods is evaluated by signal-to-noise ratio (SNR) gain. The SNR gain is given by

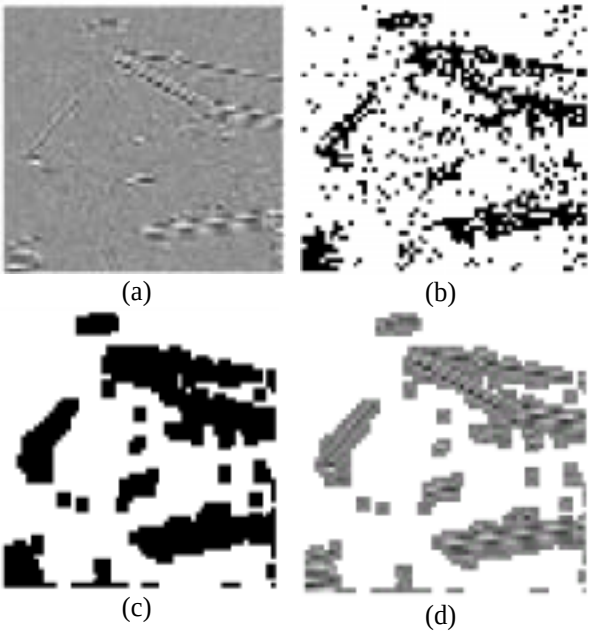


Fig. 4. MCF applied on S^{LH2}: (a) LH2, (b) S^{LH2}, (c) MCF output, (d) edge area.

$$\begin{aligned} SNR_{gain} &= SNR_{output} - SNR_{input} \\ &= 10 \cdot \log_{10} \frac{\sigma_s^2}{\sigma_r^2} - 10 \cdot \log_{10} \frac{\sigma_s^2}{\sigma_n^2} \\ &= 10 \cdot \log_{10} \frac{\sigma_n^2}{\sigma_r^2}, \end{aligned} \qquad (3)$$

where σ_n^2 is the variance of the additive white Gaussian noise, σ_s^2 is the variance of the original signal and σ_r^2 is the variance of the remaining noise, i.e., the variance of error between the original and the reconstructed signal. Since the threshold value is known to be related to noise variance, in this simulation, the thresholds are set to $T=T_{12}=T_{21}=2\sigma_n$ and $T_{22}=\sigma_n/2$ for performance comparison. The value σ_n is directly obtained from input noise. And Daubechies wavelet D_3 [8] is used.

Table 1 shows the gain in SNR obtained with denoising methods for varying input SNR of given noisy "House" and "Pepper" images. Clearly, new methods combined with the morphological clustering filter outperform all other methods (hard and soft thresholding). It is concluded that the noise suppression performance of thresholding-based de-noising methods in the wavelet domain can be increased using the morphological clustering filter. The restored images by new and other methods are shown in Figs. 5 and 6. The images restored by new methods clearly are less noisy than those obtained by hard and soft thresholding methods. It is observed that the restored images (see Fig. 5 (e)) contain spurious cross-like features especially around the edges. These features are due to unsuppressed noisy wavelet coefficients in the high level [6]. Since the morphological clustering filter with the same size of structuring element is applied on all multiresolution supports S^l and a single coefficient in level l, corresponds to 4 coefficients in level l+1, the location of a significant coefficient in the higher level is greatly expanded compared to the lower level. Thus, there is a possibility that the significant coefficients detected in the higher level contain more noise components. Therefore, to reduce spurious cross-like features, the morphological clustering filter having a small sized structuring element in the higher level might be needed.

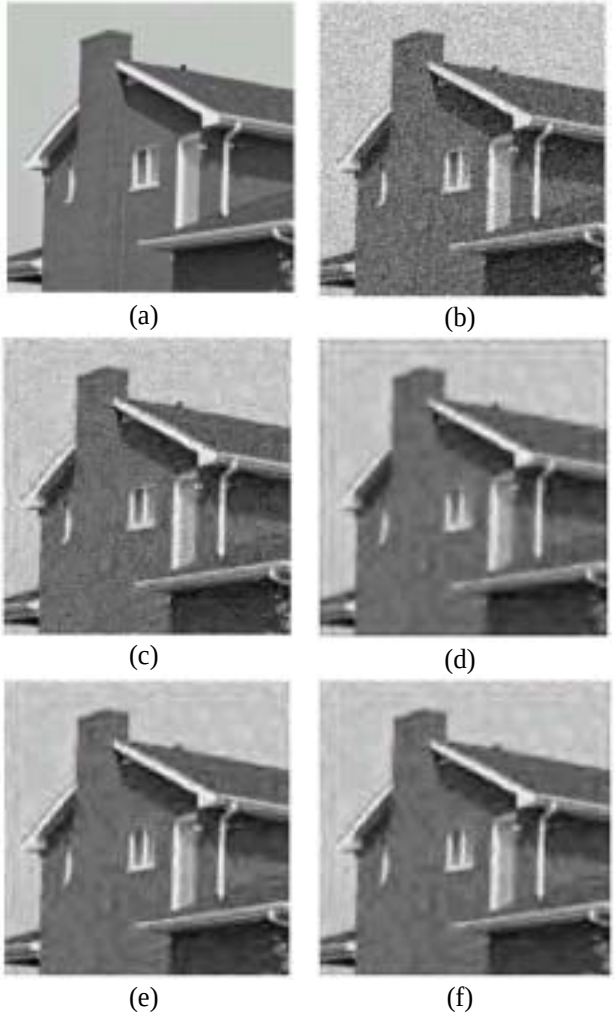
5. CONCLUSIONS

In this paper, the morphological clustering filtering techniques applied to wavelet shrinkage are presented. It is shown that a simple morphological clustering filter enables us to separate the significant wavelet coefficients corresponding to the image component from those corresponding to the noise. The simulation results indicate that noise suppression performance of wavelet shrinkage can be increased using the proposed morphological filter.

Table 1. Comparative de-noising results.

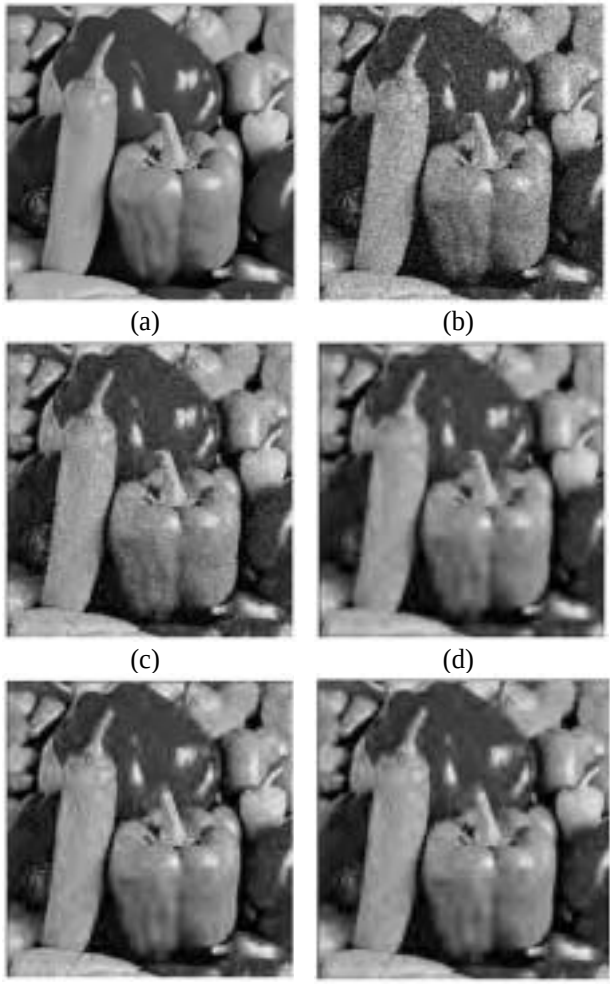
"House" image	Hard Threshold	Soft Threshold	Proposed Method1	Proposed Method2
SNRinput (dB)	SNR gain (dB)			
0	4.77	9.71	10.34	10.81
5	4.11	7.21	8.17	8.50
10	3.28	4.87	6.00	6.37
15	2.43	2.62	3.96	4.25
20	1.20	0.47	2.02	2.28

"Pepper" image	Hard Threshold	Soft Threshold	Proposed Method1	Proposed Method2
SNRinput (dB)	SNR gain (dB)			
0	4.47	9.04	9.08	9.46
5	3.81	6.36	6.77	7.08
10	2.86	3.86	4.59	4.93
15	1.80	1.58	2.60	2.86
20	0.52	-0.61	0.78	1.04



(a) (b) (c) (d) (e) (f)

Fig. 5. Restored "House" images: (a) Original image, (b) Noisy image (SNR input=5 dB), (c) Hard threshold, (d) Soft threshold, (e) Method 1, (f) Method 2.



(a) (b) (c) (d) (e) (f)

Fig. 6. Restored "Pepper" images: (a) Original image, (b) Noisy image (SNR input=5 dB), (c) Hard threshold, (d) Soft threshold, (e) Method 1, (f) Method 2.

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